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58.1 Introduction

Transportation plays an enormous role in our everyday lives. Each of us travels somewhere almost every day, whether it be to get to work or school, to go shopping, or for entertainment purposes. In addition, almost everything we consume or use has been transported at some point.

For a variety of reasons that are beyond the scope of the *Handbook,* many of the transportation services that affect our lives are provided by the public sector (rather than the private sector) and, hence, come under the aegis of civil engineering. This portion of *The Civil Engineering Handbook* deals with the role that *transplantation planners* play in the provision of those services.

What Is Transportation Planning?

It is somewhat difficult to define **transportation planning** since the people who call themselves transportation planners are often involved in very different activities. For the purposes of the *Handbook* the easiest way to define transportation planning is by comparing it to other public sector activities related to the provision of transportation services. In general, these activities can be characterized as follows:

Management/Administration: Activities related to the transportation organization itself.
Operations/Control: Activities related to the provision of transportation services when the system is in a stable (or relatively stable) state.
Planning/Design: Activities related to changing the way transportation services are provided (i.e., state transitions).

Transportation planning activities are often characterized as being either strategic (i.e., with a fairly long time horizon) or tactical (i.e., with a fairly short time horizon).

Unfortunately, these definitions, in and of themselves, are not really enough to characterize transportation planning activities. To do so requires some concepts from systems theory.

A **system,** as defined by Hall and Fagen [1956], is a set of objects (the parameters of the system), their attributes, and the relationships between them. Any system can be described at varying levels of **resolution.**

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The resolution level of the system is, loosely speaking, defined by its elements and its environment. The environment is the set of all other systems, and the elements are treated as "black boxes" (i.e., the details of the elements are ignored; they are described in terms of their inputs and their outputs).

Thus, it is possible to talk about a variety of different transportation systems including (in increasing order of complexity):

1. Car
2. Driver + Car
3. Road + Driver + Car
4. Activities generating flows + Road + Driver + Car
5. Surveillance and control devices + Activities generating flows + Road + Driver + Car

Transportation planning is concerned with the fourth system listed above, treating the Road + Driver + Car subsystem as a black box. For example, transportation planners are interested in how activities generating flows interact with this black box to create congestion, and how congestion influences these activities. In contrast, automotive engineering is concerned with the vehicle as a system, human factors engineering is concerned with the Driver + Vehicle system, geometric design and infrastructure management are concerned with the Road or the Road + Car systems, and highway traffic operations and Intelligent Vehicle Highway Systems are concerned with the surveillance and control systems and how they interact with the Activities + Road + Driver + Car subsystem.

The Transportation Planning Process

The transportation planning process almost always involves the following six steps (in some form or another):

1. Identification of goals/objectives (anticipatory planning) or problems (reactive planning)
2. Generation of alternative methods of accomplishing these objectives or solving these problems
3. Determination of the impacts of the different alternatives
4. Evaluation of different alternatives
5. Selection of one alternative
6. Implementation

Some people have argued that this process is/should be completely "rational" or "scientific" and hence that the above steps are/should be performed in order (perhaps with a loop between evaluation of alternatives and generation of alternatives).[1] However, many others argue that the transportation planning process is not nearly this scientific. For example, Grigsby and Bernstein [1993] argue that there are a variety of factors that shape the transportation planning process:

Societal Setting: The laws, regulations, customs, and practices that distribute decision-making powers and that set limits on the process and on the range of alternatives.

Organizational Setting: The orbit and administrative rules and practices that distribute decision-making powers and that set limits on the process and on the range of alternatives.

Planning Situation: The number of decision makers, the congruity and clarity of values, attitudes and preferences, the degree of trust among decision makers, the ability to forecast, time and other resources available, quality of communications, size and distribution of rewards, and the permanency of relationships.

For these and other reasons, a variety of other "less-than-rational" descriptions of the planning processes have been presented. For example, Lindblom [1959] described what he called the "science of muddling through," in which planners build out from the current situation by small degrees rather than

[1]This is sometimes called the *3C process:* continuing, comprehensive, and coordinated.

starting from the fundamentals each time. Etzioni [1967] described a mixed scanning approach which combines a detailed examination of some aspects of the "problem" with a truncated examination of others.

Fortunately, the exact process used has little impact on the day-to-day tasks that transportation planners are involved in. Transportation planners typically evaluate alternative proposals and sometimes generate alternative proposals. Hence, the transportation planner's job is primarily to determine the demand for the proposed alternatives (i.e., how the proposed alternatives affect the activities which generate flows).

Given that transportation planners are principally concerned with determining the demand changes that result from proposed projects, it would be natural to assume that they use the tools of the micro-economist (i.e., models of consumer and producer behavior). While this is true in some sense, the generic demand models used in microeconomics are usually not powerful enough to support the transportation planner. That is, for most applications it is not possible to reliably estimate the demand for a project/facility as a function of the attributes of that project/facility. This is because of the complex interactions that exist between different people and different facilities. Instead, transportation planners use a variety of different models depending on the specific decision they are trying to predict.

58.2 Transportation Planning Models

In order to determine the demand for a transportation project/facility the transportation planner must answer the following questions:

- Who travels?
- Why do they travel?
- Where do they travel?
- When do they travel?
- How do they travel?

The who and why questions are actually fairly easy to answer. In general, transportation planners need to distinguish between commuters, shoppers, holiday travelers, and business travelers. To answer the where, when, and how (and the aggregate question "how much") transportation planners develop theories and models of the decision-making processes that different travelers go through.

To do so, the transportation planner considers the following:

- The decision to travel
- The choice of a destination (and/or an origin)
- The choice of a mode
- The choice of a path (or route)
- The choice of a departure time

Models of the first four of these decisions are traditionally referred to as **trip generation**, **trip distribution**, **modal split**, and **traffic assignment** models. These types of models have been widely studied and applied. Departure-time choice has, for the most part, been ignored or handled in an ad hoc fashion.

It is important to observe that not all of these models need to be applied in all situations. In practice, the models used should depend on the time frame of the forecast being generated. For example, in the very short run, people are not likely to change where they live or where they work, but they may change their mode and/or path. Hence, when trying to predict the short-run reactions of commuters to a project, it does not make sense to run a trip generation or trip distribution model. However, it is important to run both the modal split and the traffic assignment models.

It is also important to note that it is often necessary to combine different models, and this can be done in one of two ways. Continuing the example above, if the choice of mode and path are tightly intertwined, then it may make sense to solve/run the two models simultaneously. If, on the other hand, people first choose a mode (based on some estimate of the costs on the two modes) and then choose the path on that mode, then it may make sense to solve/run the models sequentially. This will be discussed more fully below. For the time being, the models will be presented as if they are used sequentially.

The subsections that follow contain some of the more common models of each type. It is important to recognize at the outset that some of these models are very **disaggregate** while others are quite **aggregate** in nature. Disaggregate models consider the behavior of individuals (or sometimes households). They essentially consider the **choices** that individuals make among different **alternatives** in a given situation. Aggregate models, on the other hand, consider the decisions of a group in total. The groups themselves can be based either on geography (resulting in zonal models) of socioeconomic characteristics.[2]

Though each of the decisions that travelers make are modeled differently in the subsections that follow, it is important to realize that many of the techniques described in one subsection may be appropriate in others. In general, they are all models of how people make choices. Hence, they are applicable in a wide variety of different contexts (both inside and outside of transportation planning).

The Decision to Travel

In general, trip generation models relate the number of trips being taken to the characteristics of a "group" of travelers. The models themselves are usually statistical in nature. Zone-based models use aggregate data while household-based models use disaggregate data. These models typically fall into two groups: linear regression models and category analysis models. The output of a trip generation model is either **trip productions** (the number of trips originating from each location), **trip attractions** (the number of trips destined for each location), or both.

These models have, in general, received very little attention in recent years. That is, the techniques have not changed much in the past twenty years; only new parameters have been estimated. This is, in large part, because transportation planners have traditionally been concerned with congestion during the peak period, and it is relatively easy to model the decision to travel for work trips (i.e., everyone with a job takes a trip). However, this is beginning to change for several reasons:

- Congestion is increasingly occurring outside of the traditional morning and evening peaks. Hence, more attention needs to be given to nonwork trips.
- New technologies are changing the way in which people consider the decision to travel. The advent of **telecommuting** means that people may not commute to work every day. Similarly, **teleshopping** and **teleconferencing** can dramatically change the way people decide to take trips.

These trends have created a great deal of renewed interest in trip generation models.

Linear Regression Models

In a liner regression model a statistical relationship is estimated between the number of trips and some characteristics of the zone or household. Typically, these models take the form

$$Y = \beta_0 + \beta_1 X_1 + \cdots + \beta_n X_n + \epsilon \tag{58.1}$$

where Y (called the dependent variable) is the number of trips, $X_1, X_2, \ldots, X_n$ (called the independent variables) are the n factors that are believed to affect the number of trips that are made, $\beta_1, \beta_2, \ldots, \beta_n$ are the coefficients to be estimated, and ϵ is an error term. Such models are often written in vector notation as $Y = \beta_0 + \beta X' + \epsilon$ where $\beta = (\beta_1, \ldots, \beta_n)$ and $X = (X_1, \ldots, X_n)$. Clearly, since $\beta_i = \partial Y / \partial X_i$, the coefficients represent the contribution of the independent variables to the magnitude of the dependent variable.

In a disaggregate model, Y is normally measured in trips (of different trips) per household, whereas in an aggregate model it is measured in trips per zone. In general, the independent (or explanatory)

[2]In some cases, disaggregate models are statistically estimated using aggregate data and knowledge of the distributional of the groups.

variables should not be (linearly) related to each other, but should be highly correlated with the dependent variable. The selection of which dependent variables to include is part of the "art" of developing such models.

Such models are traditionally estimated using a technique known as *least squares estimation.* This technique determines the parameter estimates that minimize the sum of the squared differences between the observed and the expected values of the observations. It is described in almost every book on econometrics (see, for example, Theil [1971]).

In general, it is important to realize that linear regression models are much more versatile than one might immediately expect. In particular, observe that both the independent variables and the dependent variable can be transformed in nonlinear ways. For example, the model

$$\log(Y)j = \beta_0 + \beta_1 \log(X_1) + \cdots + \beta_n \log(X_n) + \epsilon \tag{58.2}$$

can be estimated using ordinary least squares. In this case, the values of the coefficients can be interpreted as elasticities since

$$\frac{\partial \log(Y)}{\partial \log(X_i)} = \beta_i \Rightarrow \beta_i = \frac{\partial Y/Y}{\partial X_i/X_i} \tag{58.3}$$

Category Analysis Models

In category analysis, a mean trip rate is determined for different types (i.e., categories) of people and trips. The categories are typically based on social, economic, and demographic characteristics. The resulting models are nonparametric and have the following form (see, for example, Doubleday, [1977]):

$$\Omega_{zc}^{p} = \frac{\Sigma_{r \in z} O_{rc}^{p}}{n_z} \tag{58.4}$$

where Ω_{zc}^{p} denotes the trip rate for people in category z for purpose p during time period c, O_{rc}^{p} denotes the number of trips by person r for purpose p during time period c, and n_z denotes the number of people in category z.

Models of this type are generally presented in tabular form as follows:

	Trip Type 1	Trip Type 2	…	Trip Type k
Category 1				
Category 2				
M				
Category m				

where the entries in the table would be the trip rates. These trip rates can then be used to predict future trip attractions and productions simply by predicting the number of people in each category and multiplying.

Origin and Destination Choice

Of course, each trip that a person takes must have an origin and a destination. For commuters, this origin/destination choice process is fairly long-term in nature. For morning trips to work, the origin is usually the person's place of residence and the destination is usually the place of work, and for evening trips from work it is exactly the opposite. Hence, for commuting trips the origin and destination choice processes are tantamount to the residential location and job choice processes. For shopping trips, the origin choice process is long-term in nature (i.e., the choice of a residence), but the destination choice process is very short-term in nature (i.e., where to go shopping for this particular trip). For holiday travel things are somewhat more confusing. However, in many cases we can treat holiday travel as if it involves short-term origin and short-term destination choices. For example, consider the holiday travel that occurs on Thanksgiving. You know that your family is going to get together, but where? Hence, the origin/destination choice process corresponds to determining where you will meet and, hence, who will be traveling from where and to where.

There are two widely used types of trip distribution models: gravity models and Fratar models. Gravity models are typically used to calculate a trip table from scratch, whereas Fratar models are used to adjust an existing trip table. Both types of models are aggregate in nature and use trip production and/or trip attractions to determine specific trip pairings (often called a **trip table**).

Gravity Models

The most popular models of origin/destination choice are collectively called *gravity models* (see, for example, Hua and Porell [1979]), Erlander and Stewart [1989], and Sen and Smith [1994]). These models get their name because of their similarity to the Newtonian model of gravity. At the most basic level, these models assume that the movements of people tend to vary directly with the size of the attraction and inversely with the distance between the points of travel. So, for example, one could have a gravity model of the following kind:

$$T_{ij} = \alpha \frac{M_i M_j}{d_{ij}^2} \tag{58.5}$$

where T_{ij} denotes the number of trips between origin zone i and destination zone j, M_i denotes the population of zone i, M_j denotes the population of zone j, d_{ij} denotes the distance between i and j, and α is the so-called demographic gravitational constant.

Many models of this kind have been estimated and used over the years. However, they have also received a great deal of criticism. First, there is no particular reason to use d_{ij}^2 in the denominator; this seems to be carrying the Newtonian analogy farther than is justified. Second, there is no reason to use $M_i M_j$ in the numerator; it makes just as much sense to weight each of these terms (e.g., to use $w_i M_i^\beta u_j M_j^\lambda$). Finally, these models suffers from a small distance problem: as the distance between the origin and destination decreases, the number of trips increases without bound (i.e., as $d_{ij} \to 0$, $T_{ij} \to \infty$).

These criticisms led researchers to try many other forms of the gravity model. One of the more general specifications was given by Hua and Porell [1979]:

$$T_{ij} = A(i)B(j)F(d_{ij}) \tag{58.6}$$

where $A(i)$ and $B(j)$ are weighting functions and $F(d_{ij})$ is a distance deterrence function. Most of the variants of this model have differed in the form of the deterrence functions used. For example, the classical doubly constrained gravity models is given by

$$T_{ij} = A_i B_j O_i D_j f(c_{ij}) \tag{58.7}$$

where O_i is the number of trips originating at , D_j is the number of trips destined for j, and A_i and B_j are defined as follows:

$$A_i = \left[\sum_j B_j D_j f(c_{ij}) \right]^{-1} \tag{58.8}$$

$$B_i = \left[\sum_i A_i O_i f(c_{ij}) \right]^{-1} \tag{58.9}$$

Though these variants have been motivated in a number of different ways, some formal and others more ad hoc (see, for example, Stouffer [1940], Niedercorn and Bechdolt [1969], and T. E. Smith [1975, 1976a, 1976b, 1988]), perhaps the most appealing to date are those based on the most probable state approach (see, for example, Wilson [1970] and Fisk [1985]).

In this approach, each individual is assumed to choose an origin and/or destination (the set of such choices are referred to as the **microstates** of the system). Any particular microstate will have associated with it a **macrostate,** which is simply the number of trips to and/or from each zone. A macrostate is feasible if it reproduces known properties referred to as **system states** (e.g., total cost of travel, total number of travelers). Letting $\mathcal{F}$ denote the set of feasible macrostates and $W(n)$ the number of microstates that are consistent with macrostate n, then the total number of possible microstates is given by

$$\Omega = \sum_{n \in \mathcal{F}} W(n) \tag{58.10}$$

Finally, if each microstate is equally likely to occur then the probability of a particular (feasible) macrostate is

$$P(n) = \frac{W(n)}{\Omega} \tag{58.11}$$

To develop specific gravity models using the most probable state approach one need simply derive an expression for $W(n)$ and then find the macrostate which maximizes (58.11). Fisk [1985] discusses several such models.

For shopping trips (from given origins), the following gravity model can be derived:

$$T_j = N \frac{D_j \exp(-\beta c_j)}{\sum_k D_k \exp(-\beta c_k)} \tag{58.12}$$

where T_j denotes the number of trips to destination j, N is the total number of travelers, D_j is the number of possible stores at destination j, and β is a parameter of the model. In general, β is expected to be negative.

For commuting trips, the following gravity model can be derived (assuming that the number of jobs is shown and that one trip end is permitted per job):

$$T_j = \frac{D_j}{z^{-1} \exp(\beta c_j) + 1} \tag{58.13}$$

where D_j is the number of jobs at location j, and z^{-1} is found by substituting this expression into the equations defining the system states. For example, if the total number of travelers, N, and the total travel cost, C, are both known, z^{-1} would be obtained using

$$\sum_i n_i = N \tag{58.14}$$

$$\sum_i n_i c_i = C \tag{58.15}$$

These models are typically estimated using maximum likelihood techniques. These techniques attempt to find the value of the parameters that make the observed sample most likely. That is, a likelihood function is formed which represents the probability of the sample conditioned on the parameter estimates, and this likelihood function is then maximized using techniques from mathematical programming.

Fratar Models

A popular alternative to gravity models are Fratar models. While not as theoretically appealing, the Fratar model is sometimes used to adjust existing trip tables. The "symmetric" Fratar model, which is the only one presented here, requires that the number of trips from i to j equals the number of trips from j to i (i.e., $T_{ij} = T_{ji}$).

Letting T^0 denote the original trip table and O denote the future trip-end totals, this approach can be summarized as follows:

Step 0: Set the iteration counter to zero (i.e., $k = 0$).
Step 1: Calculate trip production totals. That is, set $P_i^k = \sum_j T_{ij}^k$.
Step 2. Set $k = k + 1$ and calculate the adjustment factors $f_i^k = O_i/P_i^{k-1}$ for all i. If $f_i^k \approx 1$ for all i then STOP.
Step 3. Set $N_{ij}^k = (T_{ik}^{k-1} f_j^k / \sum_n T_{in}^{k-1} f_n^k)O_i$.
Step 4. Set $T_{ij}^k = (T_{ij}^k + T_{ji}^k)/2$ and GOTO step 1.

Note that this algorithm does not always converge and that it cannot be used at all when the number of zones changes.

Mode Choice

Mode choice models are typically motivated in a disaggregate fashion. That is, the concern is with the choice process of individual travelers. As might be expected, there are many theories of individual choice that can be applied in this context.

One of the most successful theories of individual choice is the classical microeconomic theory of the consumer. This theory postulates that an individual chooses the consumption bundle that maximizes his or her utility given a particular budget. It assumes that the alternatives (i.e., the components of the consumption bundle) are continuously divisible. For example, it assumes that individuals can consume 0.317 units of good x, 5.961 units of good y, and 1.484 units of good z. As a result, it is not possible to directly apply this theory to the typical mode choice process in which travelers make discrete choices (e.g., whether to drive, take the bus, or walk).

Of course, one could modify the traditional theory of the consumer to incorporate discrete choices. In fact, such models have received a great deal of attention. The goal of these models is to impute the weights that an individual gives to different attributes of the alternatives based on the choices that are observed (again assuming that the individual chooses the alternative with the highest utility).

Unfortunately, however, these models do not always work well in practice. There are at least two reasons for this. First, individuals often select different alternatives when faced with (seemingly) identical choice situations. Second, individuals sometimes (seem to) make choices (or express preferences) that violate the **transitivity of preferences**. That is, they choose A over B, choose B over C, but choose C over A.

Two explanations have been given for these seeming inconsistencies. Some people, so-called random utility theorists, have argued that we (as observers) are unable to fully understand and measure all of the relevant factors that define the choice situation. Others, so-called constant utility theorists, have argued that decision makers actually behave based on choice probabilities. Both theories result in probabilistic models of choice rather than the deterministic models discussed thus far.

In the discussion that follows, a probabilistic model of choice will be motivated using random utility theory. However, it could just as easily have been motivated using constant utility theory. For the purposes of this *Handbook,* the end result would have been the same.

A General Probabilistic Model of Choice

Following the precepts of random utility theory, assumes that individual n selects the mode with the highest utility but that utilities cannot be observed with certainty. Then, from the analyst's perspective, the probability that individual n chooses mode i given choice set C_n is given by

$$P\left(i \middle| C_n\right) = \text{Prob}\left[U_{in} \geq U_{jn}, \forall j \in C_n\right] \tag{58.16}$$

where U_{in} is the utility of mode i for individual n. In other words, the probability that n chooses mode i is simply the probability that i has the highest utility.

Now, since the analyst cannot observe the utilities with certainty they should be treated as random variables. In particular, assume that

$$U_{in} = V_{in} + \epsilon_{in} \tag{58.17}$$

where V_{in} is the systematic component of the utility and ϵ_{in} is the random component (i.e., the disturbance term). Combining (58.16) and (58.17) yields the following:

$$P\left(i \middle| C_n\right) = \text{Prob}\left[V_{in} + \epsilon_{in} \geq V_{jn} + \epsilon_{jn}, \forall j \in C_n\right]. \tag{58.18}$$

Specific random utility models can now be derived by making assumptions about the joint probability distributions of the set of disturbances, $\{\epsilon_{jn}, j \in C_n\}$.

As with gravity models, these models are typically estimated using maximum likelihood techniques. In practice, it is generally assumed that the systematic utilities are linear functions of their parameters. That is,

$$V_{in} = \beta_1 x_{in1} + \beta_2 x_{in2} + \cdots + \beta_G x_{inG} \tag{58.19}$$

where x_{ing} is the gth attribute of alternative i for individual n, and β_{ing} is the "weight" of that attribute. However, as discussed above, this is not a very restrictive assumption.

Probit Models

Suppose that the disturbances are the sum of a large number of unobserved independent components. Then, by the central limit theorem, the disturbances would be normally distributed. The resulting model is called the *probit model.*

For the case of two alternatives, the (binary) probit model is given by

$$P\left(i \middle| C_n\right) = \Phi\left(\frac{V_{in} - V_{jn}}{\sigma}\right) \tag{58.20}$$

where $\Phi(\cdot)$ denotes the cumulative normal distribution function and σ is the standard deviation of the difference in the error terms, $\epsilon_{jn} - \epsilon_{in}$. For more detail see Finney [1971] or Daganzo [1979].

Logit Models

Observe that the probit model above does not have a closed-form solution. That is, the probability is expressed in terms of an integral that must be evaluated numerically. This makes the probit model computationally burdensome. To get around this, a model has been developed which is probitlike but much more convenient. This model is called the *logit model.*

The logit model can be derived by assuming that the disturbances are independently and identically Type-I Extreme Value (i.e., Gumbel) distributed. That is,

$$F\left(\epsilon_{in}\right) = \exp\left[-\exp\left[-\mu\left(\epsilon_{in} - \eta\right)\right]\right] \qquad \forall i, n \tag{58.21}$$

where $F(\epsilon_{in})$ denotes the cumulative distribution function of ϵ_{in}, μ is a positive scale parameter, and η is a location parameter.

With this assumption it is relatively easy to show that

$$P\left(i \middle| C_n\right) = \frac{e^{\mu V_{in}}}{\sum_j e^{\mu V_{jn}}} \tag{58.22}$$

where j represents an arbitrary mode. For a more complete discussion see Domencich and McFadden [1975], McFadden [1976], Train [1984], and Ben-Akiva and Lerman [1985].

It is important to point out that, while widely used, the logit model has one serious limitation. To see this, consider the relative probabilities of two modes, i and k. It follows from (58.22) that

$$\frac{P\left(i\middle|C_n\right)}{P\left(k\middle|C_n\right)}=\frac{e^{\mu V_{in}}\Big/\sum_j e^{\mu V_{jn}}}{e^{\mu V_{kn}}\Big/\sum_j e^{\mu V_{jn}}}=\frac{e^{\mu V_{in}}}{e^{\mu V_{kn}}}=e^{\mu\left(V_{in}-V_{kn}\right)} \tag{58.23}$$

Hence, the ratio of the choice probabilities for i and k is independent of all of the other modes. This property is known as **independence from irrelevant alternatives (IIA)**.

Unfortunately, this property is problematic in some situations. Consider, for example, a situation in which there are two modes, automobile (A) and red bus (R). Assuming that that $V_{An} = V_{Rn}$ it follows from (58.22) that $P(A \mid C_n) = P(R \mid C_n) = 0.50$. Now, suppose a new mode is added, blue bus (B), that is identical to R except for the color of the vehicles. Then, one would still expect that $P(A \mid C_n) = P(\text{Bus} \mid C_n) = 0.50$ and hence that $P(R \mid C_n) = P(B \mid C_n) = 0.25$. However, in fact, it follows from (58.22) that $P(A \mid C_n) = P(R \mid C_n) = P(R \mid C_n) = P(B \mid C_n) = 0.333$. Thus, the logit model would not properly predict the mode choice probabilities in this case. What is the reason? ϵ_{Rn} and ϵ_{Bn} are not independently distributed.

Nested Logit Models

In some situations, an individual's "choice" of mode is actually a series of choices. For example, when choosing between auto, bus, and train the person may also have to choose whether to walk or drive to the bus or train. This can be modeled in one of two ways. On the one hand, the choice set can be thought of as having five alternatives: auto, walk + bus, auto + bus, walk + train, auto + train. On the other hand, this can be viewed as a two-step process in which the person first chooses between auto, bus, and train, and then, if the person chooses bus or train, she must also choose between walk access and auto access.

The reason to use this second approach (i.e., multidimensional choice sets) is that some of the observed and some of the unobserved attributes of elements in the choice set may be equal across subsets of alternatives. Hence, the first approach may violate some of the assumptions of, say, the logit model. To correct for this it is common to use a nested logit model.

To understand the nested logit model, consider a mode and submode choice problem of the kind discussed above. Then, the utility of a particular choice of mode and submode (for a particular individual) is given by

$$U_{ms}=\tilde{V}_m+\tilde{V}_s+\tilde{V}_{ms}+\tilde{\epsilon}_m+\tilde{\epsilon}_s+\tilde{\epsilon}_{ms} \tag{58.24}$$

where $\tilde{V}_m$ is the systematic utility common to all elements of the choice set using mode m, $\tilde{V}_s$ is the systematic utility common to all elements of the choice set using submode s, $\tilde{V}_{ms}$ is the remaining systematic utility specific to the pair (m, s), $\tilde{\epsilon}_m$ is the unobserved utility common to all elements of the choice set using mode m, $\tilde{\epsilon}_s$ is the unobserved utility common to all elements of the choice set using submode s, and $\tilde{\epsilon}_s$ is the remaining unobserved utility specific to the pair (m, s).

Now, assuming that $\tilde{\epsilon}_m$ has zero variance and $\tilde{\epsilon}_s$ and $\tilde{\epsilon}_{ms}$ are independent for all m and s, the terms $\tilde{\epsilon}_{ms}$ are independent and identically Gumbel distributed with scale parameter μ^m, and $\tilde{\epsilon}_s$ is distributed so that $\max_m U_{ms}$ is Gumbel distributed with scale parameter μ^s, then the choice probabilities can be represented as follows:

$$P(s)=\frac{e^{\left(\tilde{V}_s+V_s'\right)\mu^s}}{\sum_t e^{\left(\tilde{V}_t+V_t'\right)\mu^s}} \tag{58.25}$$

where the notation indicating the individual's choice set has been dropped for convenience, t denotes an arbitrary submode, and

$$V_s' = \frac{1}{\mu^m} \ln \sum_m e^{(\tilde{V}_m + \tilde{V}_{ms})\mu^s} \tag{58.26}$$

The conditional probability of choosing mode m given the choice of submode s is then given by

$$P(m|s) = \frac{e^{(\tilde{V}_{ms} + \tilde{V}_m)\mu^m}}{\sum_j e^{(\tilde{V}_{js} + \tilde{V}_j)\mu^m}} \tag{58.27}$$

where j is an arbitrary mode. That is, the conditional probabilities for this nested logit model are defined by a scaled logit model that omits the attributes that vary only across the submodes. Ben-Akiva [1973], Daly and Zachary [1979], Ben-Akiva and Lerman [1985], and Daganzo and Kusnic [1993] provide detailed discussions of these models.

Path Choices

While the shortest distance between any two points on a plane is described by a straight line, it is often impossible to actually travel that way. When using an automobile or bicycle you must, for the most part, use a path that travels along existing roads; when using a bus or train you must use a path that consists of different predefined route segments; even when flying you often must use a path that consists of different flight legs.

In some respects, it is pretty amazing that people are able to make path choices at all, given the enormous number of possible paths that can be used to travel from one point to another. Fortunately, people are able to make these choices and it is possible to model them.

The basic premise which underlies almost all path choice models is that people choose the "best" path available to them (where the "best" may be measured in terms of travel time, travel cost, comfort, etc.). Of course, in general, this assumption may fail to hold. For example, infrequent travelers may not have enough information to choose the best path and may, instead, choose the most obvious path. As another example, in some instances it may be too difficult to even calculate what the actual best path is, as is sometimes the case with complicated transit paths that involve numerous transfers or when a shopper needs to choose the best way to get from home to several destinations and back to home. Nonetheless, this relatively simplistic approach does seem to work fairly well in practice.[3]

Automobile Commuters

The most important thing to capture when modeling the path choices of automobile commuters is congestion. In other words, the path choice of one commuter affects the path choices of all other commuters. Hence, one can imagine that each day commuters choose a particular path, evaluate that path, and the next day choose a new path based on their past experiences. Given that the number of automobile commuters and the characteristics of the network are relatively constant from day to day, such an adjustment process might reasonably be expected to settle down at some point in time. Most models of automobile commuter path choice assume that this process does settle down and, in fact, only consider the final equilibrium point.

These models are typically set on a network comprised of a set of nodes N and a set of arcs (or links) A. Within this context, a path is just a sequence of links that a commuter can travel along from his/her origin to his/her destination. If arc a is a part of path k (connecting r and s) then $\delta_{ak}^{rs} = 1$; otherwise $\delta_{ak}^{rs} = 0$. Most such models assume that the number of people traveling from each origin to each destination by

[3]It is important to note that many behavioral models consider idealized situations in which people make the best possible choice. For example, this is the basic assumption that underlies most of microeconomics. Though this assumption has received a great deal of criticism, as yet nobody has been able to propose as workable an alternative.

automobile is known (i.e., the mode-specific trip table is known) and that each path uses a single link at most once.

The most popular behavioral theory of the path choices of automobile commuters was proposed by Wardrop [1952]. He postulated that, in practice, commuters will behave in such a way that "the journey times on all routes actually used are equal, and less than those which would be experienced by a single vehicle on any unused route." When this situation prevails, Wardrop argued that "no driver can reduce journey time by choosing a new route," and hence that this situation can be thought of as an equilibrium. Mathematically, this definition of a **Wardrop equilibrium** can be expressed as follows:

$$f_k^{rs} > 0 \Rightarrow c_k^{rs} = \min_{j \in \mathcal{K}_{rs}} c_k^{rs} \quad \forall r \in \mathcal{R}, s \in \mathcal{S}, \ k \in \mathcal{K}_{rs} \tag{58.28}$$

where f_k^{rs} denotes the number of people traveling from origin r to destination s on path k, c_k^{rs} denotes the cost on path k (from r to s),[4] $\mathcal{K}_{rs}$ denotes the set of paths connecting r and s, $\mathcal{R}$ denotes the set of all origins, and $\mathcal{S}$ denotes the set of all destinations.

As it turns out, Wardrop was not completely correct in claiming that when (58.28) holds, no driver can reduce his or her travel cost by changing routes. This has led other researchers to define other notions of equilibrium that incorporate this latter idea explicitly. The first such definition was the **user equilibrium** concept proposed by Dafermos and Sparrow [1969] which requires that no portion of the flow on a path can reduce their costs by swapping to another path. A somewhat weaker definition of user equilibrium was proposed by Dafermos [1971] in which no small portion of the users on any path can reduce their travel costs by simultaneously switching to any other path connecting the same OD-pair. An even weaker definition was proposed by Bernstein and Smith [1994] which is closer in spirit to the notion of a Nash equilibrium in which there is no coordination. From a behavioral viewpoint, their definition makes no assertion about potential gains from simultaneous route shifts by any positive portion of the commuters. Rather, it simply asserts that no gains are possible for *arbitrarily* small shifts. A very different equilibrium concept was proposed by Heydecker [1986]. He says that **equilibrated path choices** exist when no portion of the flow on any path, p, can switch to any other path, r, connecting the same OD-pair without making the new cost on r at least as large as the new cost on p. We will ignore such differences here. In most cases of practical interest, the different definitions of user equilibrium and Wardrop equilibrium turn out to be identical.

To simplify the analysis, it is common to assume that commuters are infinitely divisible (i.e., that it makes sense to talk about fractions of commuters on a particular path). It is also common to assume that the cost on link a, which we denote by t_a, is a function only of the number of vehicles on arc a, which we denote by x_a. In this case, the cost functions are said to be separable, and the equilibrium can be found by solving the following nonlinear program:

$$\min \quad \sum_{a \in \mathcal{A}} \int_0^{x_a} t_a(\omega)\, d\omega \tag{58.29}$$

$$\text{s.t.} \quad \sum_{r \in \mathcal{R}} \sum_{s \in \mathcal{S}} \sum_{k \in \mathcal{K}_{rs}} f_k^{rs} \delta_{ak}^{rs} = x_a \qquad \forall a \in \mathcal{A} \tag{58.30}$$

$$\sum_{k \in \mathcal{K}_{rs}} f_k^{rs} = q_{rs} \qquad \forall r \in \mathcal{R}, s \in \mathcal{S} \tag{58.31}$$

$$f_k^{rs} \geq 0 \qquad \forall r \in \mathcal{R}, s \in \mathcal{S}, k \in \mathcal{K}_{rs} \tag{58.32}$$

[4]Though Wardrop [1952] includes only travel time in his definition, it is clear that his ideas can easily be extended to include other costs as well.

where q_{rs} is the number of automobile commuters from r to s. The solution of this nonlinear program is an equilibrium because of the Kuhn-Tucker conditions, which are both necessary and sufficient, are equivalent to the equilibrium conditions in (58.28). This result was first demonstrated by Beckman et al. [1956]. This problem can be solved using a variety of different nonlinear programming algorithms (see, for example, LeBlanc, Morlok, and Pierskalla [1975], and Nguyen [1974, 1978]).

For cases where the arc cost functions are not separable we must instead solve a variational inequality problem in order to find the equilibrium.[5] In particular, letting H denote the set of all vectors $x = (x_a : a \in \mathcal{A})$ and $f = (f_k^{rs} : r \in \mathcal{R}, s \in \mathcal{S}, k \in \mathcal{K}_{rs})$ that satisfy

$$\sum_{r\in\mathcal{R}} \sum_{s\in\mathcal{S}} \sum_{k\in\mathcal{K}_{rs}} f_k^{rs} \delta_{ak}^{rs} = x_a \qquad \forall a \in \mathcal{A} \tag{58.33}$$

$$\sum_{k\in\mathcal{K}_{rs}} f_k^{rs} = q_{rs} \qquad \forall r \in \mathcal{R}, s \in \mathcal{S} \tag{58.34}$$

$$f_k^{rs} \geq 0 \qquad \forall r \in \mathcal{R}, s \in \mathcal{S}, k \in \mathcal{K}_{rs} \tag{58.35}$$

we must find vectors $(\tilde{x}, \tilde{f}) \in H$ that satisfy

$$\sum_{a\in\mathcal{A}} t_a(\bar{x})(x_a - \bar{x}_a) \geq 0 \tag{58.36}$$

for all $(x, f) \in H$. Fortunately, the solution to this variational inequality problem can be obtained in a variety of ways, one of which is to solve a sequence of nonlinear programs related to the one described above (see, for example, Dafermos and Sparrow [1969], Nagurney [1984, 1988], Harker and Pang [1990]).

It is important to note that such equilibria are known to exist and be unique in most cases of practical interest (see Smith [1979] and Dafermos [1980]). It is also important to note that the assumption of perfect information can be relaxed and a stochastic version of the model developed (see, for example, Daganzo and Sheffi [1977], Sheffi and Powell [1982], and Smith [1988]). For a more complete discussion of these models see Friesz [1985], Sheffi [1985], Boyce et al. [1988], or Nagurney [1993].

Transit Travelers

The path choice problem faced by transit travelers is actually quite different from that faced by auto travelers. In particular, transit users must decide (based on a schedule, if one exists) how to best get from their origin to their destination using a group of vehicles traveling along predetermined routes. Of course, they make these choices knowing full-well that almost all aspects of transit service are stochastic (e.g., running times, vehicle arrival times, crowding, etc.).

Early models of transit path choice assumed that travelers essentially choose the path with the minimum expected cost. In the case of a tie (either on the entire path or a portion of the path), travelers are assumed to choose different routes in proportion to their frequency. Models of this kind are discussed by Dial [1967] and le Clercq [1972].

Recently, more attention has been given to how travelers might actually choose between multiple routes that service the same locations (whether they are intermediate points in the path or the actual origin and destination). These models assume that, because of the stochastic nature of vehicle departure and travel times, passengers will probably be willing to use several paths and will actually choose one based on the actual departure times of specific vehicles.

[5]This is not, strictly speaking, true. When the cost functions are nonseparable but symmetric it is still possible to develop a math programming formulation of the equilibrium problem. This is discussed more fully in Dafermos [1971], Abdulaal and LeBlanc [1979], and Smith and Bernstein [1993].

Chriqui and Robillard [1975] assume that travelers will first choose a set of routes they would be willing to use, and then actually choose the first vehicle that arrives which services one of the routes in that set. This model can be formalized as follows. Let n denote the number of routes providing service between two locations, let f_{Wi} denote the probability density function of the waiting times (for the next vehicle) on route i, let $\tilde{F}_{Wi}$ denote the complement of the cumulative distribution function of the waiting times on route i, let $X = (x_1,\ldots, x_n)$ denote the choice vector where $x_i = 1$ if route i is chosen and $x_i = 0$ otherwise, and let t_i denote the expected travel time after boarding a vehicle on route i. Then, following Hickman [1993], the problem of determining the optimal route set is given by

$$\min_{x} \quad \sum_{i=1}^{n} \int_0^{\infty} \left(z+t_i\right)\cdot x_i \cdot fw_i \prod_{j\neq i} \bar{F}w_j\left(z\right)^{x_j} dz \tag{58.37}$$

$$\text{s.t.} \quad \sum_{i=1}^{n} x_i \geq 1 \tag{58.38}$$

$$x_i \in \left\{0, 1\right\} \tag{58.39}$$

In this problem, the expression $x_i f_{Wi} \prod_{j\neq 1} \tilde{F}_{Wj}(z)^{xj}$ denotes the probability that a vehicle on route i will arrive before any other vehicle in the choice set.

The solution technique proposed by Chriqui and Robillard [1975] is not guaranteed to find an optimal solution except when the waiting time distributions and in-vehicle travel times for all routes are identical and when the headways are exponentially distributed. Their heuristic proceeds as follows:

Step 0. Enumerate all of the possible routes. Set $k = 1$.

Step 1. Sort the routes by expected in-vehicle travel "cost" (e.g., time) letting route i denote the ith "cheapest" route.

Step 2. Let the initial guess at the choice set be given by $X^1 = (1, 0,\ldots, 0)$ and let C^1 denote the expected travel cost associated with this choice set.

Step 3. Let the guess at iteration k be given by $X^k = (1_1,\ldots, 1_k, 0_{k+1},\ldots, 0_n)$ where 1_i denotes a 1 in the ith position of the vector X and 0_i denotes a 0 in the ith position of the vector X. Calculate the expected cost of this choice set and denote it by C^k.

Step 4. If $C^k > C^{k-1}$ then STOP (the optimal choice set is given by X^{k-1}). Otherwise GOTO step 3.

This work is discussed and extended by Marguier [1981], Marguier and Ceder [1984], Janson and Ridderstolpe [1992], and Hickman [1993]. Other models of transit path choice are discussed in de Cea et al. [1988], Spiess and Florian [1989], and Nguyen and Pallottino [1988].

Departure-Time Choice

Traditionally, little attention has been given to the modeling of departure-time choice. Hence, this section will briefly discuss some of the approaches to modeling departure-time choice that have been proposed in the theoretical literature but, as yet, have not been widely implemented.

Automobile Commuters

In practice, the departure-time choices of automobile commuters are usually modeled very crudely. That is, the day is normally divided into several periods (e.g., morning peak, midday, evening peak, night) and a trip table is created for each period. Within-period departure-time choices are simply ignored.

The theoretical literature has considered two approaches for modeling within-period departure-time choice. The first approach makes use of the kinds of probabilistic choice models discussed above. These models, however, typically fail to consider congestion effects. The other approach accounts for congestion in a manner that is very similar to the path choice models described above. That is, this approach assumes that each person chooses the best departure-time given the behavior of all other commuters.

To understand this second approach, consider a simple example of the work-to-home commute in which each person chooses a departure time after 5:00 p.m. (denoted by $t = 0$) and before some time $\tilde{t}$ in such a way that his or her cost is minimized given the behavior of all other commuters. Assuming that travel delays are modeled as a deterministic queuing process with service rate $1/\beta$ and the cost of departing at time t is given by

$$C(t) = \beta x(t) + \gamma t \tag{58.40}$$

where $x(t)$ is the size of the queue at time t and $\gamma < 1$ is a penalty for late departure, an equilibrium can be characterized as a departure pattern, h, that satisfies

$$C(t) = C(0) \qquad \forall t \in (0, \bar{t}] \tag{58.41}$$

$$x(0) + \int_0^{\bar{t}} h(w)\,dw = N \tag{58.42}$$

The first condition ensures that the costs are equal for all departure times, while the second ensures that everyone actually departs (where the total number of commuters is denoted by N).

In equilibrium, γ^N people will depart at exactly $t = 0$ (assuming that each individual member of this group will perceive the average cost for the entire group), and over the interval $(0, \beta N)$ the remaining commuters will depart at a rate of $(1 - \gamma)/\beta$.

To see that this is indeed an equilibrium, observe that as long as there are commuters in the queue throughout the period $[0, \tilde{t}]$, the size of the queue at time t is given by

$$x(t) = x(0) + \int_0^{t} h(w)\,dw - 1/\beta t \tag{58.43}$$

Hence, the cost at time t is given by

$$C(t) = \beta\left[x(0) + \int_0^{t} h(w)\,dw - 1/\beta t\right] + \gamma t \tag{58.44}$$

Substituting for $x(0)$ and h yields $C(t) = \beta\gamma N + (1 - \gamma)t - t + \gamma t = \beta\gamma N$ for $t \in (0, \tilde{t})$. And, since $x(0) = \gamma N$ it follows that $C(0) = \beta\gamma N$ and that the flow pattern is, in fact, an equilibrium.

These models are discussed in greater detail by Vickrey [1969], Hendrickson and Kocur [1981], Mahmassani and Herman [1984], Newell [1987], and Arnott et al. [1990a,b]. Stochastic versions are presented by Alfa and Minh [1979], de Palma et al. [1983], and Ben-Akiva et al. [1984].

Transit Travelers

Traditionally, transit models have assumed that (particularly when headways are relatively short) people depart from their homes (i.e., arrive at the transit stop) randomly. In other words, they assume that the interarrival times are exponentially distributed.

There has been some research, however, that has attempted to model departure time choices in more detail. This work is described by Jolliffe and Hutchinson [1975], Turnquist [1978], and Bowman and Turnquist [1981].

Combining the Models

The discussion above treated each of the different models in isolation. However, as mentioned at the outset, many of the decisions being modeled are actually interrelated. Hence, it is common practice to combine these models when they are actually applied.

The most obvious way to combine these models is to apply them sequentially. That is, obtain origin and/or destination totals from a trip generation model, use those totals as inputs to a trip distribution model and obtain a trip table, use the trips by origin-destination pair as inputs to a modal split model, and then assign the mode-specific trips to paths using an assignment model. Unfortunately, however, this process is not as "trouble free" as it might sound. For example, trip distribution models often have travel times as an input. What travel time should you use? Should you use a weighted average across different modes? Perhaps, but you have not yet modeled modal shares. In addition, since you have not yet modeled path choice you do not know what the travel times will be.

This has led many practitioners to apply the models sequentially but to do so iteratively, first guessing at appropriate inputs to the early models and then using the outputs from the later models as inputs in later iterations. Continuing the example above, you estimate travel times for the trip distribution model in the first iteration, then use the resulting trip table and an estimate of mode-specific travel times as an input to a modal split model. Next, you could use the output from the modal split model as an input to a traffic assignment model. Then, you could use the travel costs calculated by the traffic assignment model as inputs to the next iteration's trip distribution model, and so on.

Of course, one is naturally led to ask which approach is better. Unfortunately, there is no conclusive answer. Some people have argued that the simple sequential approach is an accurate predictor of observed behavior. In other words, they argue that the estimates of travel times that people use when choosing where to live and work often turn out to be inconsistent with the travel times that are actually realized. As a result, they are not troubled by the inconsistencies that arise using what is traditionally referred to as the "four-step process" (i.e., first trip generation, then trip distribution, then modal split, and finally traffic assignment).

Others have argued that the number of iterations should depend on the time frame of the analysis. That is, they believe that the iterative approach can be used to describe how these decisions are actually made over time. Hence, by iterating they believe that they can predict how the system will evolve over time.

Still others have argued that, while the iterative approach does not accurately describe how the system will evolve over time, it will eventually converge to the long-run equilibrium that is likely to be realized. That is, they believe that the trajectory of intermediate solutions is meaningless, but that the final solution (i.e., when the outputs across different iterations settle down) is a good predictor of the long-run equilibrium that will actually be realized.

Finally, others have argued that it makes sense to iterate until the outputs converge not because the final answer is likely to be a good predictor (since too many other things will change in the interim), but simply because it is internally consistent. They argue that it is impossible to compare the impacts of different projects otherwise.

Regardless of how you feel about the above debate, one thing is known for certain. There are more efficient ways of solving for the long-run equilibrium than iteratively solving each of the individual models until they converge. In particular, it is possible to solve most combinations of models simultaneously.

As an example, consider the problem of solving the combined mode and route choice problem, assuming that there are two modes (auto and train), that there is one train path for each OD-pair, that the two modes are independent (i.e., that neither node congests the other), that the cost of the train is independent of the number of users of the train, and that the arc cost functions for auto are separable. Then, the combined model can be formulated as the following nonlinear program:

$$\min \quad \sum_{a\in\mathcal{A}} \int_0^{x_a} t_a(\omega)\,d\omega - \sum_{r\in\mathcal{R},\, s\in\mathcal{S}} \int_0^{q_{rs}} \left[\frac{1}{\theta} \ln\left(\frac{\bar{q}_{rs}}{w} - 1 \right) + \hat{u}_{rs} \right] dw \tag{58.45}$$

$$\text{s.t.} \quad \sum_{r\in\mathcal{R}} \sum_{s\in\mathcal{S}} \sum_{k\in\mathcal{K}_{rs}} f_k^{rs}\, \delta_{ak}^{rs} = x_a \qquad \forall a \in \mathcal{A} \tag{58.46}$$

$$\sum_{k\in\mathcal{K}_{rs}} f_k^{rs} = q_{rs} \qquad \forall r \in \mathcal{R},\, s \in \mathcal{S} \tag{58.47}$$

$$0 < q_{rs} < \bar{q}_{rs} \qquad \forall r \in \mathcal{R}, s \in \mathcal{S} \tag{58.48}$$

$$f_k^{rs} \geq 0 \qquad \forall r \in \mathcal{R}, s \in \mathcal{S}, k \in \mathcal{K}_{rs} \tag{58.49}$$

where $\tilde{q}_{rs}$ denotes the total number of travelers on both modes and $\hat{u}_{rs}$ denotes the fixed transit travel cost.

Of course, there are far too many different combinations of the basic models to review them all here. Various different combinations of the traditional "four steps" are discussed by Tomlin [1971], Florian et al. [1975], Evans [1976], Florian [1977], Florian and Nguyen [1978], Sheffi [1985], and Safwat and Magnanti [1988]. There is also a considerable amount of activity currently being devoted to simultaneous models of route and departure-time choice. As these models are quite complicated, in general, they are beyond the scope of this *Handbook*. For auto commuters, see, for example, the deterministic models of Friesz et al. [1989], Smith and Ghali [1990], Bernstein et al. [1993], Friesz et al. [1993], Ran [1993] and the stochastic models developed by Ben-Akiva et al. [1986] and Cascetta [1989]. For transit travel, see the models developed by Hendrickson and Plank [1984] and Sumi et al. [1990].

58.3 Applications and Example Calculations

In this section, several examples are presented and solved. Unfortunately, due to the complexity of some of the models and the ways in which they interact, these examples are not exhaustive.

The Decision to Travel

A number of different trip generation models have been developed over the years. This section contains examples of several.

The first example is a disaggregate regression model estimated by Douglas [1973]:

$$Y = -0.35^* + 0.63^* X_1 + 1.08^* X_2 + 1.88^* X_3 \tag{58.50}$$

where Y denotes the number of trips per household per day, X_1 denotes the number of people per household, X_2 denotes the number of employed people per household, and X_3 denotes the monthly income of the household (in thousands of U.K. pounds). The symbol * indicates that the estimate of the coefficient is significantly different from 0 at the 0.95 confidence level. As one example of how to use this model, observe that $\partial Y/\partial X_1 = 0.63$. Hence, this model says that, other things being equal, an additional unemployed member of the household would make (on average) 0.63 additional trips per day.

The second model is an example of a disaggregate category analysis model developed by Doubleday [1977]:

Type of Person		Total Trip Rate	Regular Trips	Nonregular Trips
Employed males	w/o a car	3.7	2.46	0.55
	with a car	5.7	2.80	1.38
Employed females	w/o a car	4.5	2.20	1.30
	with a car	6.0	2.39	2.13
Homemakers	w/o a car	4.1	—	3.25
	with a car	5.7	—	4.78
Retired persons	w/o a car	2.2	—	1.75
	with a car	4.1	—	3.16

where the numbers in the table are the number of trips per person per day. It should be relatively easy to see how such a model would be used in practice.

The third example is an aggregate regression model estimated by Keefer [1966] for the city of Pittsburgh:

$$Y = 3296.5 + 5.35X_1 + 291.9X_2 - 0.65X_3 - 22.31X_4 \tag{58.51}$$

where Y denotes the total number of automobile trips to shopping centers, X_1 denotes the number of work trips, X_2 denotes the distance of the shopping center from major competitions (in tenths of miles), X_3 denotes the reported travel speed of trip makers (in miles per hour), and X_4 denotes the amount of floor space used for goods other than shopping and convenience goods (in thousands of square feet). The R^2 for this model is 0.920. What distinguishes this model from the disaggregate model above is that it does not focus on the individual household. Instead, it uses aggregate data and estimates the total number of automobile trips to shopping centers.

The final example is an aggregate category analysis model also developed by Keefer [1966]:

Land-Use Category	Square Feet (1000s)	Person Trips	Trips per 1000 sq. ft
Residential	2,744	6,574	2.4
Retail	6,732	54,733	8.1
Services	13,506	70,014	5.2
Wholesale	2,599	3.162	1.2
Manufacturing	1,392	1,335	1.0
Transport	1,394	5,630	4.0
Public buildings	31,344	153,294	4.9

where the numbers in the table are the total number of trips taken. Again, this is an aggregate model because, unlike the earlier category analysis model, it does not focus on the behavior of the individual. Instead, it is based on aggregate data about trip making.

Origin and Destination Choice

This section contains an example of an estimated gravity model and several iterations of an application of the Fratar model.

An Example of the Gravity Model

Putman [1983] presents an interesting example of a gravity model of commuter origin/destination choice. In this model

$$T_{ij} = \frac{L_i^\delta c_{ij}^a \exp(\beta c_{ij})}{\sum_k L_k^\delta c_{ik}^a \exp(\beta c_{ik})} \tag{58.52}$$

where T_{ij} denotes the number of commuting trips from i to j, L_i denotes the size of zone i, c_{ij} is the cost of traveling from i to j, and α, β, and δ are parameters.

He estimated this model for several cities (in slightly different years) and found the following:

City	α	β	δ
Gosford-Wyong, Australia	0.09	–0.03	–2.31
Melbourne, Australia	1.04	–0.06	0.13
Natal, Brazil	0.93	–0.01	0.48
Rio de Janeiro, Brazil	1.08	–0.01	0.60
Monclova-Frontera, Mexico	2.73	–0.14	0.24
Ankara, Turkey	0.64	–0.14	–0.31
Izmit, Turkey	0.90	–0.03	1.05

To see how this type of model would be applied, consider the following two-zone example in which $L_1 = 3000$, $L_2 = 1000$, $c_{11} = 2$, $c_{12} = 10$, $c_{21} = 7$, and $c_{22} = 3$, and suppose that these zones are in Melbourne, Australia. Then, for T_{11}, the numerator of (58.52) is given by $\mathcal{S}_1^{\delta} c_{11}^{\alpha} \exp(\beta c_{11}) = 3000^{0.13} \cdot 2^{1.04} \cdot \exp(0.13 \cdot 2) = 2.83 \cdot 2.06 \cdot 0.89 = 5.16$. Continuing in this manner, the other numerators in (58.52) are given by 17.04 for $i = 1, j = 2$, 12.20 for $i = 2, j = 1$, and 6.43 for $i = 2, j = 2$. It then follows that

$$T_{11} = \frac{5.16}{5.16 + 17.04 + 12.20 + 6.43} = 698 \tag{58.53}$$

$$T_{12} = \frac{17.04}{5.16 + 17.04 + 12.20 + 6.43} = 2302 \tag{58.54}$$

$$T_{21} = \frac{12.20}{5.16 + 17.04 + 12.20 + 6.43} = 655 \tag{58.55}$$

$$T_{22} = \frac{6.43}{5.16 + 17.04 + 12.20 + 6.43} = 345 \tag{58.56}$$

Thus, the model predicts that there will be 698 + 655 = 1353 trips to zone 1, and 2302 + 345 = 2647 trips to zone 2.

In order to understand the sensitivity of this model, it is worth performing these same calculations using the parameters estimated for Ankara, Turkey. In this case, the resulting trip table is given by

$$T_{11} = 1567 \tag{58.57}$$

$$T_{12} = 1433 \tag{58.58}$$

$$T_{21} = 496 \tag{58.59}$$

$$T_{22} = 504 \tag{58.60}$$

An Example of the Fratar Model

Consider the following hypothetical example of the Fratar model in which there are four zones and the original trip table is given by

$$T^0 = \begin{bmatrix} 0.00 & 10.00 & 40.00 & 15.00 \\ 10.00 & 0.00 & 20.00 & 30.00 \\ 40.00 & 20.00 & 0.00 & 10.00 \\ 15.00 & 30.00 & 10.00 & 0.00 \end{bmatrix} \tag{58.61}$$

and the forecasted trip-end totals are given by

$$O = \begin{bmatrix} 130.00 \\ 140.00 \\ 225.00 \\ 90.00 \end{bmatrix} \tag{58.62}$$

In step 1, the production totals are calculated as

$$P^1 = \begin{bmatrix} 65.00 \\ 60.00 \\ 70.00 \\ 55.00 \end{bmatrix} \tag{58.63}$$

Then, in step 2, the factors are calculated as

$$f^1 = \begin{bmatrix} 2.00 \\ 2.33 \\ 3.21 \\ 1.64 \end{bmatrix} \tag{58.64}$$

Next, in step 3, the temporary (asymmetric) trip table is calculated as

$$N^1 = \begin{bmatrix} 0.00 & 17.19 & 94.73 & 18.08 \\ 20.99 & 0.00 & 67.48 & 51.53 \\ 125.85 & 73.41 & 0.00 & 25.74 \\ 20.43 & 47.68 & 21.89 & 0.00 \end{bmatrix} \tag{58.65}$$

Finally, the first iteration is concluded by calculating the symmetric trip table:

$$T^1 = \begin{bmatrix} 0.87 & 0.00 & 19.09 & 110.29 \\ 1.01 & 19.09 & 0.00 & 70.44 \\ 1.10 & 110.29 & 70.44 & 0.00 \\ 0.97 & 19.26 & 49.60 & 23.82 \end{bmatrix} \tag{58.66}$$

In step 1 of the second iteration, the trip-end totals are calculated as

$$P^2 = \begin{bmatrix} 148.64 \\ 139.13 \\ 204.55 \\ 92.68 \end{bmatrix} \tag{58.67}$$

Then in step 2 of iteration 2:

$$f^2 = \begin{bmatrix} 0.87 \\ 1.01 \\ 1.10 \\ 0.97 \end{bmatrix} \tag{58.68}$$

And in step 3 of iteration 2:

$$N^2 = \begin{bmatrix} 0.00 & 15.68 & 99.05 & 15.27 \\ 16.42 & 0.00 & 76.21 & 47.37 \\ 113.95 & 83.73 & 0.00 & 27.32 \\ 16.31 & 48.32 & 25.37 & 0.00 \end{bmatrix} \tag{58.69}$$

And, finally in step 4 of iteration 2:

$$T^2 = \begin{bmatrix} 0.00 & 16.05 & 106.50 & 15.79 \\ 16.05 & 0.00 & 79.97 & 47.85 \\ 106.50 & 79.97 & 0.00 & 26.35 \\ 15.79 & 47.85 & 26.35 & 0.00 \end{bmatrix} \tag{58.70}$$

In step 1 of iteration 3:

$$P^3 = \begin{bmatrix} 138.34 \\ 143.87 \\ 212.81 \\ 89.98 \end{bmatrix} \tag{58.71}$$

And, in step 2 of iteration 3:

$$f^3 = \begin{bmatrix} 0.94 \\ 0.97 \\ 1.06 \\ 1.00 \end{bmatrix} \tag{58.72}$$

Since all of these values are approximately equal to 1 the algorithm terminates at this point.

Mode Choice

Suppose the utility function for individual n is given by

$$V_{jn} = -t_j - \frac{5o_j}{Y_n} \tag{58.73}$$

where t_j is the travel time on mode j, o_j is the out-of-pocket cost on mode j, and Y_n is the income of individual n. Now, consider the following three modes:

Mode	t	o
Drive alone	0.50	2.00
Carpool	0.75	1.00
Bus	1.00	0.75

and consider this person's choices when her income was \$15,000 and not that it is \$30,000.

When her income was \$15,000 the (symmetric) utilities of the three modes were given by –1.17 for driving alone, –1.08 for carpooling, and –1.25 for taking the bus. Now that her income has increased to \$30,000, the utilities have gone to –0.88 for driving alone, –0.92 for carpooling, and –1.13 for taking the bus. (Note: The utilities are negative because commuting itself decreases your overall utility.)

Using a deterministic choice model, one would conclude that this individual would choose the mode with the highest utility (i.e., the lowest disutility). In this case, when she earned \$15,000 she would have carpooled, but now that she earns \$30,000 she drives alone.

On the other hand, using a logit model with $\mu = 1$, the resulting probabilities are given by

$$P(i|C_n) = \frac{e^{V_{in}}}{\sum_j e^{V_{jn}}} \tag{58.74}$$

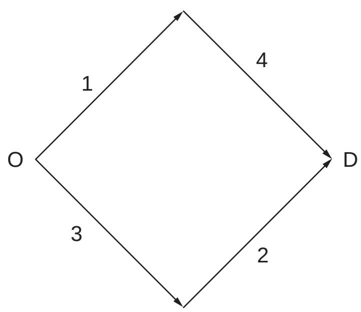

FIGURE 58.1 A four-link network.

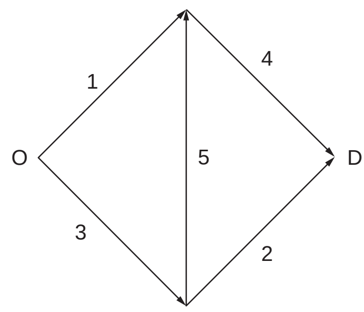

FIGURE 58.2 A five-link network.

Hence, in this choice situation with $Y = 15$:

$$P(\text{Drive alone}) = \frac{0.31}{0.31 + 0.34 + 0.29} = \frac{0.31}{0.94} = 0.33 \tag{58.75}$$

Continuing in this way, one finds that

Mode	$P(i \mid C_n)$ for $Y = 15$	$P(i \mid C_n)$ for $Y = 30$
Drive alone	0.33	0.38
Carpool	0.36	0.34
Bus	0.31	0.28

Roughly speaking, this says that when her income was \$15,000 she drove alone 33% of the time, carpooled 36% of the time, and took the bus 31% of the time. Now, however, she drives alone 38% of the time, carpools 34% of the time, and takes the bus 28% of the time.

Path Choice

This section contains an example of both highway path choice and transit path choice.

Highway Path Choice

A nice way to illustrate equilibrium path choice models is with a famous example called Braess's paradox. Consider the four-link network shown in Fig. 58.1, where $t_1(x_1) = 50 + x_1$, $t_2(x_2) = 50 + x_2$, $t_3(x_3) = 10x_3$, and $t_4(x_4) = 10x_4$. Since these cost functions are separable, the following nonlinear program can be solved to obtain the equilibrium:

$$\min \quad \int_0^{x_1} (50 + \omega)\, d\omega + \int_0^{x_2} (50 + \omega)\, d\omega + \int_0^{x_3} (10\omega)\, d\omega + \int_0^{x_4} (10\omega)\, d\omega \tag{58.76}$$

$$\text{s.t.} \quad f_1 = x_1 \tag{58.77}$$

$$f_2 = x_2 \tag{58.78}$$

$$f_2 = x_3 \tag{58.79}$$

$$f_1 = x_4 \tag{58.80}$$

$$f_1 + f_2 = 6 \tag{58.81}$$

$$f_1 \geq 0 \tag{58.82}$$

$$f_2 \geq 0 \tag{58.83}$$

The solution to this problem is given by $x_1 = 3$, $x_2 = 3$, $x_3 = 3$, $x_4 = 3$. To verify that this is, indeed, an equilibrium, the costs on the two paths can be calculated as follows:

$$c_1^{\text{OD}} = t_1(x_1) + t_4(x_4) = (50+3) + (10 \cdot 3) = 83 \tag{58.84}$$

$$c_2^{\text{OD}} = t_3(x_3) + t_2(x_2) = (10 \cdot 3) + (50+3) = 83 \tag{58.85}$$

The total cost to all commuters is thus $(3 \cdot 83) + (3 \cdot 83) = 498$.

Now, suppose link 5 is added to the network as in Fig. 58.2, where $t_5(x_5) = 10 + x_5$. Then, it follows that the following nonlinear program can be solved to obtain the new equilibrium:

$$\min \quad \int_0^{x_1} (50+\omega)\,d\omega + \int_0^{x_2} (50+\omega)\,d\omega + \int_0^{x_3} (10\omega)\,d\omega + \int_0^{x_4} (10\omega)\,d\omega + \int_0^{x_5} (10\omega)\,d\omega \tag{58.86}$$

$$\text{s.t.} \quad f_1 = x_1 \tag{58.87}$$

$$f_2 = x_2 \tag{58.88}$$

$$f_2 + f_3 = x_3 \tag{58.89}$$

$$f_1 + f_3 = x_4 \tag{58.90}$$

$$f_3 = x_5 \tag{58.91}$$

$$f_1 + f_2 + f_2 = 6 \tag{58.92}$$

$$f_1 \geq 0 \tag{58.93}$$

$$f_2 \geq 0 \tag{58.94}$$

$$f_3 \geq 0 \tag{58.95}$$

The solution to this problem is given by $x_1 = 2$, $x_2 = 2$, $x_3 = 4$, $x_4 = 4$, $x_5 = 2$ (with two commuters using each of the three paths). To verify that this is, indeed, an equilibrium, the costs on the three paths can be calculated as follows:

$$c_1^{\text{OD}} = t_1(x_1) + t_4(x_4) = (50+2) + (10 \cdot 4) = 92 \tag{58.96}$$

$$c_2^{\text{OD}} = t_3(x_3) + t_2(x_2) = (10 \cdot 4) + (50+2) = 92 \tag{58.97}$$

$$c_3^{OD} = t_3(x_3) + t_5(x_5) + t_4(x_4) = (10 \cdot 4) + (10 + 2) + (10 \cdot 4) = 92 \quad (58.98)$$

Now, however, the total cost to all commuters is $(2 \cdot 92) + (2 \cdot 92) + (2 \cdot 92) = 552$.

This example has received a great deal of attention because it illustrates that it is possible to increase total travel costs when you add a link to the network, and this seems counterintuitive. Of course, one is led to ask why people don't simply stop using path 3. The reason is that with 3 people on paths 1 and 2 (and hence with $x_1 = 3$, $x_2 = 3$, $x_3 = 3$, $x_4 = 3$) the cost on path 3 is given by

$$c_3^{OD} = t_3(x_3) + t_5(x_5) + t_4(x_4) = (10 \cdot 3) + (10 + 0) + (10 \cdot 3) = 70 \quad (58.99)$$

and hence people using paths 1 and 2 will want to switch to path 3. And, once they switch, even though their costs will go up they will not want to switch back. To see this, consider the equilibrium with the new link in place, and suppose someone on path 3 switches to path 1. Then, the resulting link volumes are $x_1 = 3$, $x_2 = 2$, $x_3 = 3$, $x_4 = 4$, and $x_5 = 1$. Hence

$$c_1^{OD} = t_1(x_1) + t_4(x_4) = (50 + 3) + (10 \cdot 4) = 93 \quad (58.100)$$

which is higher than the cost of 92 they would experience without switching.

Transit Path Choice

Consider an origin-destination pair that is serviced by four bus routes with the following characteristics:

Route	E[In-Vehicle Time]	E[Headway]	E[Travel Time]
A	20	5	22.5
B	10	30	25
C	30	30	45
D	35	25	47.5

The expected travel times in this table are calculated assuming that passengers arrive at the origin randomly and that the vehicle headways are randomly distributed. The expected waiting time for any particular route is half of the headway.

If one assumes that people simply choose the route with the lowest expected travel time, then it is clear that route A will be chosen. On the other hand, the Chriqui and Robillard [1975] model would predict that both routes A and B would be chosen. Their algorithm proceeds as follows.

In step 1 the routes are sorted based on their expected in-vehicle travel time. Hence, route B will be denoted by 1, route A will be denoted by 2, route C will be denoted by 3, and route D will be denoted by 4.

In step 2, the initial choice set is determined. In this case, $X^1 = (1, 0, 0, 0)$ and $C^1 = 25$.

In step 3, routes are iteratively added to this choice set until the expected travel time increases. So, in the first iteration the choice set is assumed to be $X^2 = (1, 1, 0, 0)$. To calculate the expected travel time for this choice set, observe that (given the above headways) 14 vehicles per hour from this choice set serve the OD-pair. Hence, the expected waiting time (for a randomly arriving passenger) is $4.29/2 = 2.14$ minutes. The expected travel time for this choice set is given by the probability-weighted travel times on the member routes. Hence, the expected travel time is $(2/14) \cdot 10 + (12/14) \cdot 20 = 17.14 + 1.43 = 18.57$ minutes. Thus, the expected total travel time for this choice set, C^2, is 18.57 minutes. Since this is less than C^1, the algorithm continues.

In the second iteration the choice set is assumed to be $X^3 = (1, 1, 1, 0)$. Now, 16 vehicles per hour from this choice set serve the OD-pair. Hence, the expected waiting time is $3.75/2 = 1.875$ minutes. Further, the expected travel time is given by $[(2/16) \cdot 10] + [(12/16) \cdot 20] + [(2/16) \cdot 30] = 1.25 + 15 + 3.75 = 20$ minutes. Thus, the expected total travel time for this choice set, C^3, is 21.875 minutes. Since this is greater than C^2, the algorithm terminates.

Departure-Time Choice

As discussed above, the equilibrium departure pattern for a simple model of departure-time choice can be characterized as $x(0) = \gamma N$, $h(t) = (1 - \gamma)/\beta$ for $t \in (0, \beta N]$. Assuming $N = 10{,}000$, the service rate of the queue is 5000 vehicles/hr (i.e., $\beta = 1/5000$), and the late departure penalty is given by $\gamma = 0.1$, it follows that in equilibrium $x(0) = 1000$, the peak period end at $\tilde{t} = 2$ (i.e., lasts for 2 hours after 5:00 p.m.), and the departure rate during the peak period is 4500 vehicles/hr.

It also follows that the queue at time t is given by

$$x(t) = \gamma N - \frac{\gamma}{\beta} t, \qquad t \in [0, \beta N] \tag{58.101}$$

and hence that the queue is initially 1000 vehicles (at time $t = 0$) and decreases linearly at a rate of 500 vehicles/hr.

Combined Models

In this example, a hypothetical city is thinking about changing the fare on its transit line from \$1.50 to \$3.00 and would like to be able to predict how ridership and congestion levels will change. The network is shown in Fig. 58.3. Node D is the single destination (the central business district) and node O is the single origin (the residential area). The solid line represents the highway link and the dotted line represents the transit link.

The Models

Highway travel times (in-vehicle) will be modeled using the following function recommended by the Bureau of Public Roads (BPR):

$$t_a = t_0 \left[1 + 0.15 \left(\frac{x_a}{k_a} \right)^4 \right] \tag{58.102}$$

where t_0 is the free-flow travel time (in minutes) and k_a is the practical capacity of link a.

Mode choice will be modeled using the following logit model:

$$P(T) = \frac{\exp(V_T)}{\exp(V_A) + \exp(V_T)} \tag{58.103}$$

where $P(T)$ is the probability that a commuter chooses to go to work by transit (train), $P(A)$ is the probability that a commuter chooses to go to work by auto, V_T is the systematic component of the utility of transit, and V_A is the systematic component of the utility of auto.

Path choices obviously do not need to be modeled since there is only one path available to each mode. Hence, all of the commuters that choose a particular mode can simply be assigned to the single path for that mode.

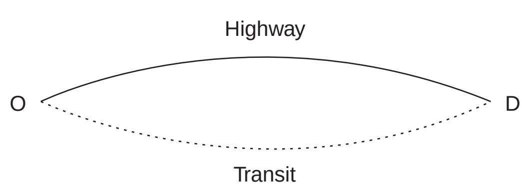

FIGURE 58.3 A multimodal network.

The Data

There are 15,000 people, in total, commuting from node O to node D. Currently (i.e., with a transit fare of $1.50) 12,560 people use transit and 2,440 people use the highway.

The systematic utilities for the logit model have been estimated as

$$V_A = 0.893 - 0.00897 \cdot i_A - 0.0308 \cdot o_A - 0.007 \cdot c_a \tag{58.104}$$

$$V_T = -0.00897 \cdot i_T - 0.0308 \cdot o_T - 0.007 \cdot c_t \tag{58.105}$$

where i is the in-vehicle travel time, o is the out-of-vehicle travel time, and c is the monetary cost per trip on that mode.

For transit, $i = 45$ (in minutes), $o = 10$, and $c = 150$ (cents). For the highway, $o = 5$, $c = 560$ ($0.28 per mile times 20 miles), and, under current conditions, $i = 24.5$.

The practical capacity of the highway is 4,000, and the free-flow speed is 50 mph. Hence, since the highway is 20 miles long, the free-flow time, t_0 is 24 minutes.

Using the Models

It would seem as though it should be relatively easy to use the logit model of mode choice to determine the impact of the fare increase. However, observe that this model requires the auto travel time as input and it is not clear what value should be used. Assuming that the highway will continue to operate at its current level of service, the travel time will be 24.5 minutes. Using this value, one finds that the systematic utilities are given by

$$V_A = 0.893 - 0.00897 \cdot 24.5 - 0.0308 \cdot 5 - 0.007 \cdot 560 = -3.4 \tag{58.106}$$

and

$$V_T = -0.00897 \cdot 45 - 0.0308 \cdot 10 - 0.007 \cdot 300 = -2.8 \tag{58.107}$$

Hence, the choice probabilities are given by $P_A = 0.3569$ and $P_T = 0.6431$. In other words, $0.3569 \cdot 15{,}000 = 5353$ people will use the highway and $0.6431 \cdot 15{,}000 = 9647$ people will use transit after the fare hike.

However, observe that these values are not consistent with the original assumption that the road would operate in near free-flow conditions. In particular, with 5353 highway users the travel time [calculated using (58.102)] will actually be 35.5 minutes, not 24 minutes. Hence, though people may make their initial choices based on free-flow speeds, they are likely to change their behavior in response to their incorrect estimate of the auto travel time.

If one believes that people will keep changing their paths until the travel time that is used as an input to the mode choice model is the same as the travel time that actually results, then it is necessary to solve the two models simultaneously. Doing so in this case, it turns out that $t_a = i_A = 33.38$, $V_A = -3.48$, $V_T = -2.81$, and hence that 5082 people will use auto and 9918 will use transit.

Of course, we could take this one step further. In particular, suppose there was another residential neighborhood, and that residential location choice could be modeled as follows:

$$n_j = N \cdot \frac{A_j \exp(-\theta c_j)}{\sum_i A_i \exp(-\theta c_i)} \tag{58.108}$$

where n_j is the number of commuters that choose to live in zone j, N is the total number of commuters, A_j is the "attractiveness" of the zone j, c_j is the commuting "cost" to the central business district, and θ is the cost sensitivity parameter. Then, it is easy to see that, since the cost of commuting has changed, the number of people living in each zone will also change (at least in the long run). Hence, one might want to simultaneously solve all three models.

Defining Terms

Aggregate models: Aggregate models consider the decisions of a group in total. The groups themselves can be based either on geography or socioeconomic characteristics.

Alternatives: The set of possible decisions that an individual can make.

Choice: The alternative that an individual selects in a given situation.

Disaggregate models: Disaggregate models consider the behavior of individuals (or sometimes households). They essentially consider the choices that individuals make among different alternatives in a given situation.

Equilibrated path choices: Path choices are equilibrated when no portion of the flow on any path, p, can switch to any other path, r, connecting the same OD-pair without making the new cost on r at least as large as the new cost on p.

Independence from irrelevant alternatives (IIA): The property that the ratio of the choice probabilities for i and k is independent of all of the other alternatives (within the context of probabilistic choice models).

Macrostate: The number of trips to and/or from each zone (within the context of a gravity model).

Management/administration: Activities related to the transportation organization itself.

Microstate: The set of such choices made by a group of individuals (within the context of a gravity model).

Modal split: This term is used to refer to both the process of modeling/predicting mode choices and the results of that process.

Operations/control: Activities related to the provision of transportation services when the system is in a stable (or relatively stable) state.

Organizational setting: The organization and administrative rules and practices that distribute decision-making powers and that set limits on the process and on the range of alternatives.

Planning/design: Activities related to changing the way transportation services are provided (i.e., state transitions).

Planning situation: The number of decision makers, the congruity and clarity of values, attitudes and preferences, the degree of trust among decision makers, the ability to forecast, time and other resources available, quality of communications, size and distribution of rewards, and the permanency of relationships.

Resolution: The resolution of a system is defined by how the system of interest is seen in relation to the environment (i.e., all other systems) and its elements which are treated as black boxes.

Societal setting: The laws, regulations, customs, and practices that distribute decision-making powers and that set limits on the process and on the range of alternatives.

System: A set of objects, their attributes, and the relationships between them.

System state: The known properties of the system (within the context of a gravity model).

Telecommuting: Using telecommunications technology (e.g., telephones, FAX machines, modems) to interact with coworkers in lieu of actually traveling to a central location.

Teleshopping: Using telecommunications technology (e.g., telephones, FAX machines, modem) to either acquire information about products or make purchases.

Traffic assignment: The term is used both to describe the process of modeling/predicting path choices and the results of that process.

Transitivity of preferences: Preferences are said to be transitive if whenever A is preferred to B and B is preferred to C it also follows that A is preferred to C.

Transportation planning: Activities related to changing the way transportation services are provided. Typical activities include the generation and evaluation of alternative proposals.

Trip attractions: The number of trips destined for a particular location.

Trip distribution: This term is used both to describe the process of modeling/predicting origin and destination choices and the result of that process.

Trip generation: Determining the number of trips that will originate from and terminate at each zone in the network. Trip generation models attempt to explain/predict the decision to travel.

Trip productions: The number of trips originating from a particular location.
Trip table: The number of trips traveling between each origin-destination pair.
User equilibrium: Several slightly different definitions of user equilibrium exist. The essence of these definitions is that no traveler can reduce his or her travel cost by unilaterally changing paths.
Wardrop equilibrium: A situation in which the cost on all of the paths between an origin and destination actually used are equal, and less than those which would be experienced by a single vehicle on any unused path.

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Further Information

In addition to the references listed above, there are several introductory texts devoted to transportation planning, including *Fundamentals of Transportation Systems Analysis* by M. L. Manheim, *Introduction to Transportation Engineering and Planning* by E. K. Morlok, and *Fundamentals of Transportation Engineering* by C. S. Papacostas.

A variety of journals are also devoted (in whole or in part) to transportation planning, including the *Journal of Transport Economics and Policy, Transportation, Transportation Research,* the *Transportation Research Record,* and *Transportation Science.*