54

Plane Surveying

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54.1 Introduction

The roots of surveying are contained in plane-surveying techniques that have developed since the very first line was measured. Though the fundamental processes haven't changed, the technology used to make the measurements has improved tremendously. The basic methods of distance measurement, angle measurement, determining elevation, etc., are becoming easier, faster, and more accurate.

Even though electronic measurement is becoming commonplace, distances are still being taped. While many transits and optical theodolites are still being used for layout purposes, electronic instruments are rapidly becoming the instruments of choice by the engineer for measurement of angles. Although establishing elevations on the building site is still being performed using levels, the laser is everywhere. It is the responsibility of the engineer in the field to choose the measuring method and the technology that most efficiently meet the accuracy needed.

The requirements of construction layout are more extensive than ever, with the complex designs of today's projects. The engineer must be exact in measurement procedures and must check and double-check to ensure that mistakes are eliminated and errors are reduced to acceptable limits to meet the tolerances required.

This chapter is directed to the engineer who will be working in the field on a construction project. Often that individual is called the *field engineer,* and that term will be used throughout this chapter. The field engineer should be aware of fundamental measuring techniques as well as advances in the technology of measurement.

Although measurement concepts and calculation procedures are introduced and discussed in this chapter, only the basics are presented. It is the responsibility of the field engineer to review other sources for more detailed information to become more competent in these procedures. See the further information and references at the end of this chapter.

54.2 Distance Measurement

Distance may be measured by indirect and direct measurement procedures. Indirect methods include odometers, optical rangefinders, and tacheometry. Direct methods include pacing, taping, and electronic distance measurement.

When approximate distances are appropriate, pacing can be used over short distances and odometers or optical rangefinders can be used over longer or inaccessible distances. These methods can yield accuracies in the range of 1 part in 50 to 1 part in 100 over modest distances. The accuracy of pacing and optical distance measuring methods decreases rapidly as the distance increases. However, these approximate methods can be useful when checking for gross blunders, narrowing search areas in the field, and making preliminary estimates for quantities or future surveying work. Applications where these lower-order accuracy methods can be used to obtain satisfactory distance observations occur regularly in surveying, construction and engineering, forestry, agriculture, and geology.

Tacheometry

Tacheometry uses the relationship between the angle subtended by a short base distance perpendicular to the bisector of the line and the length of the bisecting line. Stadia and subtense bar are two tacheometric methods capable of accuracies in the range of 1 part in 500 to 1 part in 1000.

In the stadia method the angle is fixed by the spacing of the stadia cross hairs (i) on the telescope reticule and the focal length (f) of the telescope. Then the distance on the rod (d) is observed, and the distance is computed by

$$\frac{D}{d} = \frac{f}{i}$$

$$D = d\left(\frac{f}{i}\right)$$

When the telescope is horizontal, as in a level, a horizontal distance is obtained. When the telescope is inclined, as in a transit or theodolite, a slope distance is obtained. Stadia may be applied with level or transit, plane table and alidade, or self-reducing tacheometers.

The subtense bar is a tacheometric method in which the base distance, d, is fixed by the length of the bar and the angle, α, is measured precisely using a theodolite. The horizontal distance to the midpoint of the bar is computed by

$$\tan\frac{\alpha}{2} = \frac{d/2}{D}$$

$$D = \frac{d}{2\tan\left(\frac{\alpha}{2}\right)}$$

Since a horizontal angle is measured, this method yields a horizontal distance and no elevation information is obtained.

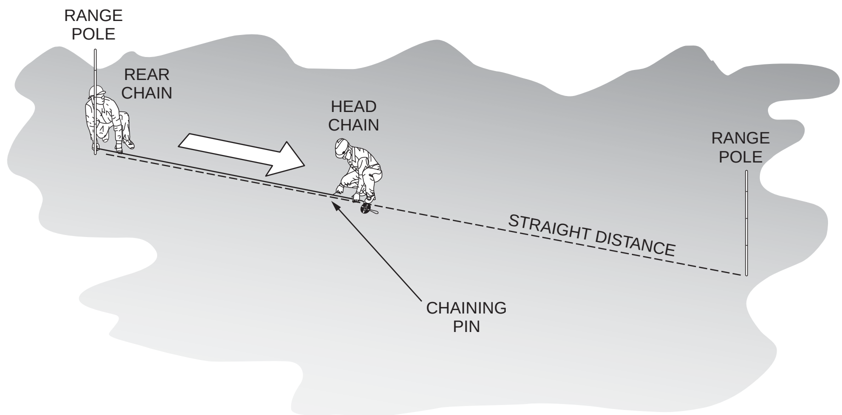

FIGURE 54.1 Taping on level terrain.

Taping

Taping is a direct method of measuring distance in which a tape of known length and graduated at intervals throughout its length is stretched along the line to be measured. Figure 54.1 illustrates taping on level terrain. Low-order accuracy measurements can be obtained using tapes made of cloth or fiberglass material. These tapes are available in a variety of lengths, and they may be graduated in feet, meters, or both. Accurate taped distances are obtained using calibrated metal ribbon tapes made of tempered steel. Tapes made of invar steel for temperature stability are also available. Table 54.1 summarizes the sources of errors and the procedures required to measure distances accurately.

TABLE 51.1 Taping Errors and Procedures

Error	Source	Type	Makes Tape	Importance	Procedure to Eliminate
Tape length	Instrumental	Systematic	Long or short	Direct impact to recorded measurement, always check.	Calibrate tape and apply adjustment.
Temperature	Natural	Systematic or random	Long or short	For a chain standardized at 68°, 0.01 per 15° per 100 ft chain.	Observe temperature of chain, calculate, and apply adjustment.
Tension	Personal	Systematic or random	Long or short	Often not important.	Apply the proper tension. When in doubt, PULL HARD!
Tape not level	Personal or natural	Systematic	Short	Negligible on slopes less than 1%; must be calculated for greater than 1%.	Break chain by using a hand level and plumb bob to determine horizontal; or correct by formula for slope or elevation difference.
Alignment	Personal	Systematic	Short	Minor if less than 1 ft off of line in 100 ft; major if 2 or more ft off of line.	Stay on line; or determine amount off of line and calculate adjustment by formula.
Sag	Natural or personal	Systematic	Short	Large impact on recorded measurement.	Apply proper tension; or calculate adjustment by formula.
Plumbing	Personal	Random	Long or short	Direct impact to recorded measurement.	Avoid plumbing if possible; or plumb at one end of chain only.
Interpolation	Personal	Random	Long or short	Direct impact to recorded measurement.	Check and recheck any measurement that requires estimating a reading.
Improper marking	Personal	Random	Long or short	Direct impact to recorded measurement.	Mark all points so that they are distinct.

TABLE 54.2 Tape Corrections for Systematic Errors

Systematic Error	Correction Formula	0.01-ft Correction Caused by
Tape length	$C_l = \dfrac{L_{\text{tape}} - 100.00}{100.00}$	0.01 per 50-ft tape length
Tension	$C_p = \dfrac{(P - P_0)L}{AE}$	±9 lb
Temperature	$C_t = \alpha(T - T_0)L$	±15°F
Sag	$C_s = \dfrac{-w^2 L_s^3}{24P^2}$	−5 lb in 100-ft tape length
Alignment	$C_a = -\dfrac{h}{2L}$	±1.4 ft

Accuracy in taping requires a **calibrated** tape that has been compared to a length standard under known conditions. A tape calibration report will include the true length of the tape between marked intervals on the tape. The tape is calibrated when it is fully supported throughout its length at a specified pull or tension applied to the tape and at a specified temperature of the tape. The calibration report should also include the cross-sectional area of the tape, the weight of the tape per unit length, the modulus of elasticity for the tape material, and the coefficient of thermal expansion for the tape material. When a distance is observed in the field and the conditions of tape support, alignment, temperature, and tension are recorded, the **systematic errors** affecting the measurement can be corrected and a true distance obtained. The tape correction formulas are summarized in the following example.

Example 54.1

A typical 100-foot surveyor's steel tape is calibrated while supported throughout its length. The calibration report lists the following:

$$\text{True length, 0-ft to 100-ft mark} \quad L_0 = 99.98 \text{ ft}$$

Other length intervals may also be reported (and the tape characteristic constants):

Tension	$P_0 = 25$ lb
Temperature	$T_0 = 68$°F
Cross-sectional area	$A = 0.003$ in.^{2}
Weight	$w = 0.01$ lb/ft
Coefficient of thermal expansion	$\alpha = 0.00000645$/°F
Modulus of elasticity	$E = 29(10^6)$ lb/in.^{2}

Systematic errors in taping, the correction formula, and the change required to cause a 0.01-foot tape correction in a full 100-foot length of the example tape are summarized in Table 54.2. When a distance is measured directly by holding the tape horizontal, the effect of alignment error is present in both the alignment of the tape along the direction of the line to be measured and the vertical alignment necessary to keep the tape truly horizontal.

Normal taping procedures require that the tape be held horizontal for each interval measured. When taping on sloping terrain or raising the tape to clear obstacles, a plumb bob is required to transfer the tape mark to the ground. Steadying the hand-held tape and plumb bob will be the largest source of **random error** in normal taping. Accuracies of 1 part in 2000 to 1 part in 5000 can be expected.

Precise taping procedures eliminate the use of the plumb bob by measuring a slope distance between fixed marks that either are on the ground or are supported on tripods or taping stands. Taping the slope distance, S, will require that the horizontal distance, H, be computed by

$$H = S\cos\alpha = S\sin z$$

where α = **vertical** angle of the tape and z = **zenith** angle of the tape, or by

$$H = \sqrt{S^2 - h^2}$$

where h = difference in elevation between the ends of the tape. If all tape corrections are carefully applied, accuracies of 1 part in 10,000 to 1 part in 20,000 can be obtained.

Electronic Distance Measurement

Distance can be measured electronically if the velocity and travel time of electromagnetic energy propagated along a survey line are determined. Terrestrial electronic distance measurement instruments (EDMIs) measure the travel time by comparing the phase of the outgoing measurement signal to the phase of the signal returning from the remote end of the line. The phase difference is thus a function of the double path travel of the measurement signal. The distance is given by

$$D = \frac{VT}{2}$$

where V = the velocity of electromagnetic energy in the atmosphere and T = the double path travel time determined using the phase difference.

If $\Delta\phi$ is the phase difference observed for the fine or shortest wavelength measurement signal, then the total travel time is found from the equation

$$T = k\left(\frac{1}{f}\right) + \frac{\Delta\phi}{2\pi}\left(\frac{1}{f}\right)$$

where f = the frequency of the measurement signal and k = the integer number of full cycles in the double path distance. The integer k is ambiguous for the fine measurement since the phase difference only determines the fractional part of the last cycle in the double path distance, as illustrated in Fig. 54.2. The value of k can be determined by measuring the phase difference of one or more coarse- or long-wavelength signals to resolve the distance to the nearest full cycle of the fine wavelength.

EDMIs may be classified by type of energy used to carry the measurement signal or by the maximum measurement range of the system. Visible (white) light, infrared light, laser (red) light, and microwaves have been used as carrier energy.

The velocity of electromagnetic energy in the atmosphere is given by the expression

$$V = \frac{c}{n}$$

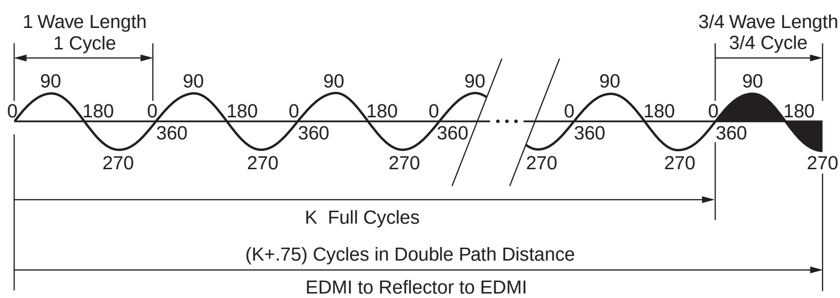

FIGURE 54.2 Phase shift principle of distance measurement.

where c is the velocity in a vacuum, 299,792,158 meters/second, and n is the atmospheric index of refraction for the conditions at the time of observation. The atmospheric index of refraction is a function of the wavelength of the electromagnetic energy propagated and the existing conditions of atmospheric temperature, pressure, and water vapor pressure. For EDMIs using visible, laser, or infrared light carrier wavelengths, the effect of water vapor pressure is negligible, and it is often ignored. For EDMIs using microwave carrier wavelengths, the effect of water vapor pressure is more significant.

Refraction causes a scale error in the observation. The correction is usually expressed in terms of parts per million, ppm, of the distance measured.

$$D_{\text{corrected}} = D + S\left(\frac{D}{10^6}\right)$$

The ppm correction for electro-optical instruments can be expressed in the form

$$S = A + \frac{Bp}{t + 273.2}$$

where p is the atmospheric pressure and t is the atmospheric temperature. The constants A and B are functions of the wavelength of the carrier and the precise frequency used for the highest-resolution measuring signal. The values of A and B can be obtained from the instrument manufacturer. Typically, the ppm value, S, is determined graphically using pressure and temperature read in the field, and the value is entered into the EDMI. On many modern digital instruments, the pressure and temperature readings can be entered directly, and the instrument computes and applies the refraction correction.

The measurement accuracy of an EDMI is expressed as

$$\sigma = \pm(c + s_{\text{ppm}})$$

In this expression, c is a constant that represents the contribution of uncertainty in the offset between the instrument's measurement reference point and the geometric reference point centered over the survey station. The ppm term is a distance-dependent contribution representing the uncertainty caused by measurement frequency drift and atmospheric refraction modeling. An EDMI should be checked periodically to verify that it is operating within its specified error tolerance, σ. A quick check can be done by measuring a line of known length or at least a line that has been measured previously to see if the observed distance changes. When a more rigorous instrument calibration is warranted, use a calibrated base line and follow the procedures recommended in *NOAA Technical Memorandum NOS NGS-10* [Fronczek, 1980]. EDMIs should be sent to the manufacturer for final calibration and adjustment.

54.3 Elevation Measurement

Elevation is measured with respect to a datum surface that is everywhere perpendicular to the direction of gravity. The datum surface most often chosen is called the *geoid*. The geoid is an equipotential surface that closely coincides with **mean sea level.** Elevations measured with respect to the geoid are called orthometric heights. The relationships between the mean sea level geoid, a level surface, and a horizontal line at a point are illustrated in Fig. 54.3.

Benchmark (BM)

A benchmark is best described as a permanent, solid point of known elevation. Benchmarks can be concrete **monuments** with a brass disk in the middle, iron stakes driven into the ground, or railroad spikes driven into a tree, etc.

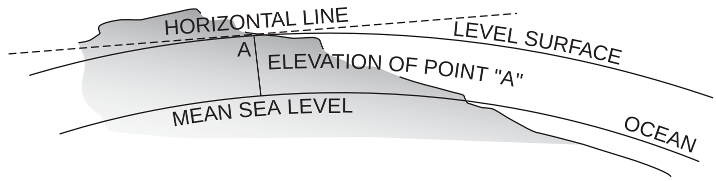

FIGURE 54.3 Elevation datum and level surface.

Turning Point (TP)

A turning point is a point used in the differential leveling process to temporarily transfer the elevation from one setup to the next.

Types of Instruments

Elevation may be measured by several methods. Some of these methods measure elevation directly, whereas some measure the difference in elevation from a reference benchmark to the point to be determined. Many types of instruments have been used for leveling purposes. The instruments have ranged from water troughs and hoses to instruments such as the barometer, dumpy level, automatic level, and laser. The field engineer should realize how each of these can be used to determine elevations for vertical control and construction.

Altimeters

Surveying altimeters are precise aneroid barometers that are graduated in feet or meters. As the altimeter is raised in elevation, the barometer senses the atmospheric pressure drop. The elevation is read directly on the face of the instrument. Although the surveying altimeter may be considered to measure elevation directly, best results are obtained if a difference in elevation is observed by subtracting readings between a base altimeter kept at a point of known elevation and a roving altimeter read at unknown points in the area to be surveyed. The difference in altimeter readings is a better estimate of the difference in elevation because local weather changes, temperature, and humidity that affect altimeter readings are canceled in the subtraction process. By limiting the distance between base and roving altimeters, accuracies of 3 to 5 feet are possible. Other survey configurations utilizing low and high base stations, and/or leap-frogging roving altimeters, can yield good results over large areas.

Level Bubble Instruments

Level bubble instruments include the builder's transit level, the transit, and the dumpy level. Each of these instruments contains a level vial with a bubble that must be centered to be used for leveling. Each instrument consists of three main components: a four-screw leveling head, a level vial attached to the telescope, and a telescope for magnification of the objective.

Instruments that use a level bubble to orient the axes to the direction of gravity depend on the bubble's sensitivity for accuracy. Level bubble sensitivity is defined as the central angle subtended by an arc of one division on the bubble tube. The smaller the angle subtended, the more sensitive the bubble is to dislevelment. A bubble division is typically 2 millimeters long, and bubble sensitivity typically ranges from 60 seconds to 1 second.

Builder's Level. The builder's level is one of the most inexpensive and versatile instruments that is used by field engineers. In addition to being able to perform leveling operations, it can be used to turn angles, and the scope can be tilted for sighting. Many residential builders use this instrument because it serves their purpose of laying out a building. See Fig. 54.4.

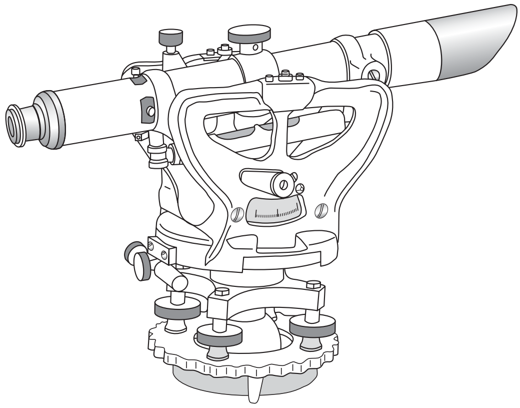

FIGURE 54.4 Builder level.

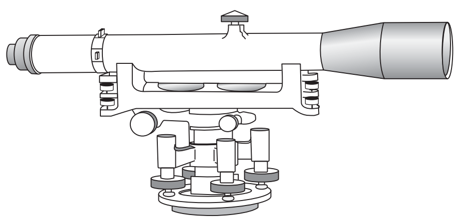

FIGURE 54.5 Engineer's dumpy level.

Transit. Although the primary functions of the transit are for angle measurement and layout, it can also be used for leveling because there is a bubble attached to the telescope. In fact, many construction companies who don't have both an angle measuring instrument and a dumpy or automatic level will do all of their leveling work with a transit. Some people prefer to use the transit for leveling because they are comfortable with its operation. However, the field engineer should be aware that the transit may not be as sensitive and stable as a quality level.

Dumpy Level. The engineer's dumpy level shown in Fig. 54.5 has been the workhorse of leveling instruments for more than 150 years. It has been used extensively for many of the great railroad, canal, bridge, tunnel, building, and harbor projects for the last century and a half. Even with advancements in other leveling instruments such as the automatic level and the laser, the dumpy is still the instrument of choice for a number of persons in construction because of its stability. On any type of project where there is going to be a great deal of vibration — such as pile driving, heavy-equipment usage, or high-rise construction — the dumpy may be the best choice for a leveling instrument.

Automatic Compensator Instruments

Compensator instruments illustrated in Fig. 54.6 were developed about 50 years ago. Although each manufacturer may have developed a unique compensator, all compensators serve the same purpose — maintaining a fixed relationship between the line of sight and the direction of gravity. If the instrument is in adjustment, the line of sight will be maintained as a horizontal line. The operation of a compensator is illustrated in Fig. 54.7. Compensator instruments are extremely fast to set up and level. An experienced person can easily have an automatic level ready for a backsight in less than ten seconds, compared to a minute or more with a bubble-based leveling system. Compensators are available in several styles. Some are constructed

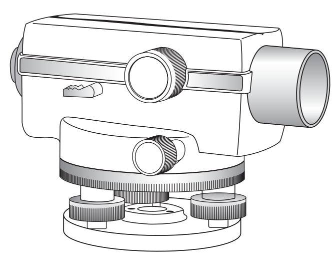

FIGURE 54.6 Automatic level.

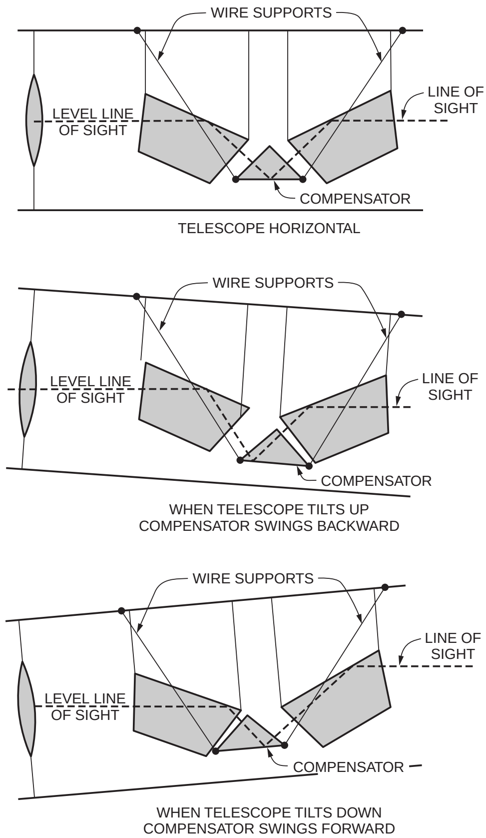

FIGURE 54.7 Leveling the line of sight by compensator.

by suspending a prism on wires. Some have a prism that is contained within a magnetic dampening system. Note that the instrument must be manually leveled to within the working range of the compensator by centering a bull's-eye bubble.

Laser Levels

A laser level uses a laser beam directed at a spinning optical reflector. The reflector is oriented so that the rotating laser beam sweeps out a horizontal reference plane. The level rod is equipped with a sensor to detect the rotating beam. By sliding the detector on the rod, a vertical reading can be obtained. Laser levels are especially useful on construction sites. As the laser beam continuously sweeps out a constant elevation reference plane, anyone with a detector can get a rod reading to set or check grade elevations. Laser levels can be equipped with automatic leveling devices to maintain level orientation. The spinning optics can also be oriented to produce a vertical reference plane.

Digital Levels

Digital levels are electronic levels that can be used to more quickly obtain a rod reading and make the reading process more reliable. The length scale on the level rod is replaced by a bar code. The digital level senses the bar code pattern and compares it to a copy of the code held in its internal memory. By matching the bar code pattern, a rod-reading length can be obtained.

Level Rods

In addition to the leveling instrument, a level rod is required to be able to transfer elevations from one point to another. The level rod is a graduated length scale affixed to a rod and held vertically on a turning

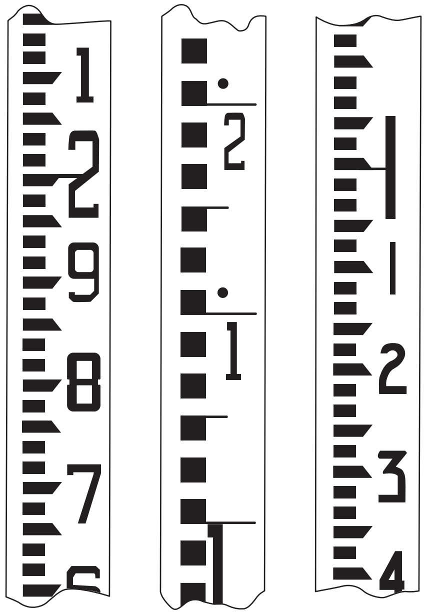

FIGURE 54.8 Typical level rod graduations.

point or benchmark. The scale is read by the person at the instrument. The reading taken is the vertical distance from the point to the line of sight.

Level rods are available in many sizes, shapes, and colors. They are made of wood, fiberglass, metal, or a combination of these materials. There are one-piece rods, two-piece rods, three-piece rods, six-piece rods, etc. Some have a square cross section and others are round or oval. Some are less than 10 feet long, while others are up to 30 feet long. Practically whatever type of rod a field engineer needs is available. Level rods have been named after the cities where they have been manufactured. The Philadelphia rod, for example, is a two-piece rod that can be extended to approximately 13 feet. It is a popular rod used in leveling surveys.

A popular rod that doesn't seem to have a proper name is the telescoping rod. Because a telescoping rod is 25 or 30 feet long, it is the rod of choice for field engineers working on projects where there is a great deal of change in elevation. Great rod lengths increase the elevation difference that can be transferred at one time. It has been argued that these types of rods wear rapidly and therefore aren't as accurate as the more traditional rods. If telescoping rods are well cared for, they are excellent for construction use.

Level rods are graduated in feet, inches, and fractions; feet, tenths, and hundredths; or meters and centimeters. The method of representing units of measurement onto the face of the rod also varies. Typical level rod graduations are shown in Fig. 54.8. The field engineer should, after studying the graduations on the face of the rod, be able to use any rod available. The rod illustrated on the left in Fig. 54.8 shows the markings on a typical "engineer's rod" that is widely used on the construction site. Note that the rod is graduated to the nearest foot with large numbers that are usually painted red. The feet are then graduated to the nearest tenth from 1 to 9. Each tenth is then graduated to the nearest one hundredth, which is the width of the smallest mark on the rod.

Rod Targets

Rod targets are useful devices for several purposes. The rod target can be used as a target by the person looking through the instrument to help locate the rod when visibility conditions are poor. If the rod target has a vernier, it can also be used to obtain a reading on the rod to the nearest thousandth of a foot. In this case the instrument person communicates to the rod person to move the target until it has centered on the horizontal cross hair. The rod person can then read the rod and the vernier to obtain thousandths of a foot. This accuracy is sometimes required on very precise leveling work.

Rod Levels

Rod levels are used to keep the level rod plumb while the reading is being taken. Rod levels are simple devices made of metal or plastic and have a bull's-eye bubble attached. They are held along the edge of the level rod while the level rod is moved until the bubble is centered. If the rod level is in proper adjustment, the rod is plumb. When a rod level is not used, the rod person should slowly rock the rod through the vertical position. The instrument person watches the cross hair appear to move on the rod and records the lowest reading — the point when the rod was vertical.

Turning Point Pin

If a solid natural point is not available during the leveling process to be used as a turning point, then the field engineer will have to create one. This is accomplished by carrying something such as a railroad spike, piece of rebar, wooden stake, or plumb bob that can be inserted solidly into the ground. The rod is placed on top of the solid point while the foresight and backsight readings are taken. These solid points are removed after each backsight to be used the next time a solid turning point is needed.

Fundamental Relationships

All level instruments are designed around the same fundamental relationships and lines. The principal relationships among these lines are described as follows:

- The **axis of the level bubble** (or compensator) should be perpendicular to the vertical axis.
- The **line of sight** should be parallel to the axis of the level bubble.

When these instrument adjustment relationships are true and the instrument is properly set up, the line of sight will sweep out a horizontal plane that is perpendicular to gravity at the instrument location. However, several effects must be considered if the instrument is to be used for differential leveling.

Earth Curvature

The curved shape of the earth means that the level surface through the telescope will depart from the horizontal plane through the telescope as the line of sight proceeds to the horizon. This effect makes actual level rod readings too large by

$$C = 0.0239D^2$$

where D is the sight distance in thousands of feet.

Atmospheric Refraction

The atmosphere refracts the horizontal line of sight downward, making the level rod reading smaller. The typical effect of refraction is equal to about 14% of the effect of earth curvature. Thus, the combined effect of curvature and refraction is approximately

$$(C - r) = 0.0206D^2$$

Instrument Adjustment

If the geometric relationships defined above are not correct in the leveling instrument, the line of sight will slope upward or downward with respect to the horizontal plane through the telescope. The method of testing the line of sight of the level to ensure that it is horizontal is called the *two-peg test*. It requires setting up the level exactly between two points about 200 feet apart and subtracting readings taken on the points to determine the true difference in elevation between them. The instrument is then moved and placed adjacent to one of the points, and rod readings are again taken on the two points. If the line

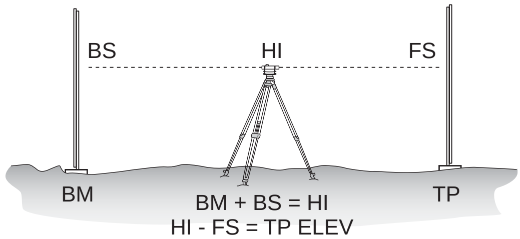

FIGURE 54.9 Differential leveling.

of sight of the instrument is truly horizontal, the difference in elevation obtained from the two setups will be the same. If the line of sight is inclined, the difference in elevation obtained from the two setups will not be equal. Either the instrument must be adjusted, or the slope of the line of sight must be calculated. The slope is expressed as a collimation factor, C, in terms of rod-reading correction per unit sight distance. It may be applied to each sight by

$$\text{Corrected rod reading} = \text{Rod reading} + (C_{\text{Factor}} \cdot D_{\text{Sight}})$$

In ordinary differential leveling discussed next, these effects are canceled in the field procedure by always setting up so that the backsight distance and foresight distance are equal. The errors are canceled in the subtraction process. If long unequal sight distances are used, the rod readings should be corrected for curvature and refraction and for collimation error.

Ordinary Differential Leveling

Determining or establishing elevations is, at times, the most essential activity of the field engineer. Elevations are needed to set slope stakes, grade stakes, footings, anchor bolts, slabs, decks, sidewalks, curbs, etc. Just about everything located on the project requires elevation. Differential leveling is the process used to determine or establish those elevations.

Differential leveling is a very simple process based on the measurement of vertical distances from a horizontal line. Elevations are transferred from one point to another through the process of using a leveling instrument to read a rod held vertically on, first, a point of known elevation and, then, on the point of unknown elevation. Simple addition and subtraction are used to calculate the unknown elevations.

A single-level setup is illustrated in Fig. 54.9. A backsight reading is taken on a rod held on a point of known elevation. That elevation is transferred vertically to the line of sight by reading the rod and then adding the known elevation and the backsight reading. The elevation of the line of sight is the height of instrument, *HI*. By definition, the line of sight is horizontal; therefore, the line of sight elevation can then be transferred down to the unknown elevation point by turning the telescope to the foresight and reading the rod. The elevation of the foresight station is found by subtracting the rod reading from the height of instrument. Note that the difference in elevation from the backsight station to the foresight station is determined by subtracting the foresight rod reading from the backsight rod reading.

A level route consists of several level setups, each one carrying the elevation forward to the next foresight using the differential-leveling method. Figure 54.10 shows a short level route and illustrates the typical format used in the field for differential level notes.

Leveling Closure

Level route **closure** is obtained by taking the last foresight on a benchmark of known elevation. If a second benchmark is not available near the end of the level route, the route should be looped back to the starting benchmark to obtain closure. At the closing benchmark,

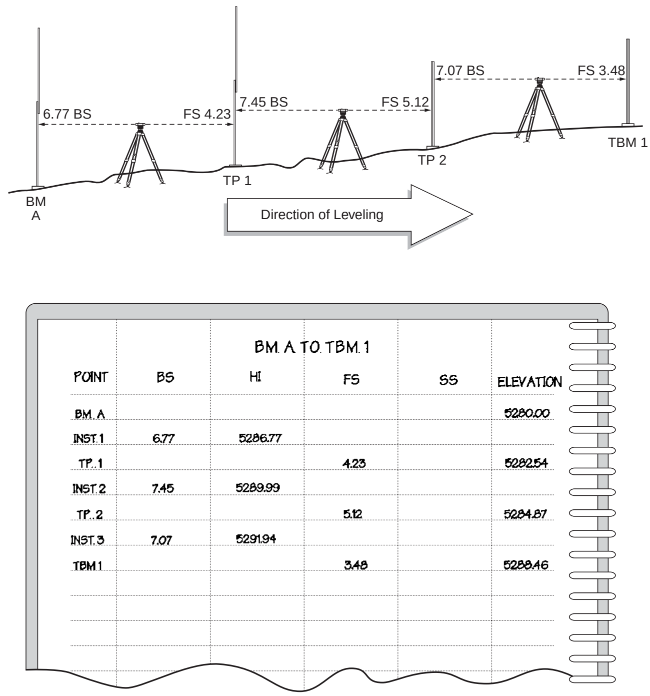

FIGURE 54.10 Level route and field note form.

$$\text{Closure} = \text{Computed elevation} - \text{Known elevation}$$

Since differential leveling is usually performed with approximately equal setup distances between turning points, the level route is adjusted by distributing the closure equally to each setup:

$$\text{Adjustment} = \left(-\frac{\text{Closure}}{n}\right)\text{per setup}$$

where n is the number of setups in the route. When a network of interconnected routes is surveyed, a **least-squares** adjustment is warranted.

Precise Leveling

Precise leveling methods are required for engineering work that requires extreme accuracy. The process requires that special equipment and methods be used. Instruments used in precise leveling are specifically designed to obtain a high degree of accuracy in leveling. Improved optics in the telescope, improved level sensitivity, and carefully calibrated rod scales are all incorporated into the differential-leveling process. Methods have been developed to ensure that mistakes are eliminated and errors are minimized.

Typically when performing precise leveling, a method of leveling called *three-wire leveling* is used. This involves reading the center cross hair as well as the upper and lower "stadia" cross hairs. The basic process of leveling is the same, except that the three cross hair readings are averaged to improve the **precision** of each backsight and foresight value.

Another method that can be used to improve the precision of the level rod reading involves using an optical micrometer on the telescope. The optical micrometer is a rotating parallel-plate prism attached in front of the objective lens of the level. The prism enables the observer to displace the line of sight parallel with itself and set the horizontal cross hair exactly on the nearest rod graduation. The observer

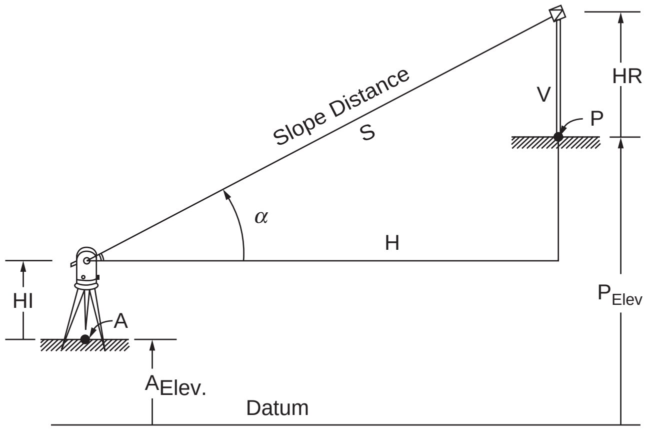

FIGURE 54.11 Trigonometric leveling in plane surveying.

adds the middle cross hair rod reading and the displacement reading on the micrometer to obtain a rod reading precise to the nearest 0.1 millimeter.

When long precise level routes are surveyed, it is necessary to account for the fact that the level surfaces converge as the survey proceeds north. The correction to be applied for convergence of level surfaces at different elevations can be calculated by

$$\text{Correction} = -0.0053 \sin 2\phi H \Delta\phi_{rad}$$

where ϕ is the latitude at the beginning point, H is the elevation at the beginning point, and $\Delta\phi$ is the change in latitude from beginning point to end point in radians.

Trigonometric Leveling

Trigonometric leveling is a method usually applied when a total station is used to measure the slope distance and the vertical angle to a point. This method is illustrated in Fig. 54.11. Assuming the total station is set up on a station of known elevation, the elevation of the unknown station is

$$V = S \sin\alpha$$
$$P_{Elev} = A_{Elev} + HI + V - HR$$

The precision of trigonometric elevations is determined by the uncertainty in the vertical angle measurement and the uncertainty in the atmospheric refraction effects. For long lines the effects of earth curvature and atmospheric refraction must be included.

54.4 Angle Measurement

Horizontal and Vertical Angles

The angular orientation of a line is expressed in terms of horizontal angles and vertical angles. Survey angles illustrated in Fig. 54.12 are defined by specifying the plane that contains the angle, the reference line or plane where the angle starts, the direction of turning, and the terminal line or plane where the angle ends.

Field angles that are measured in the horizontal plane include clockwise angles, counterclockwise angles, and deflection angles. The clockwise angle or "angle to the right," shown in Fig. 54.13, is measured from a backsight line, clockwise in the horizontal plane, to a foresight line. All transits or theodolites

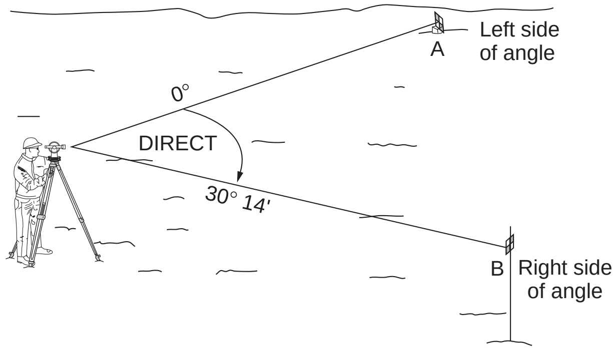

FIGURE 54.12 Horizontal survey angle.

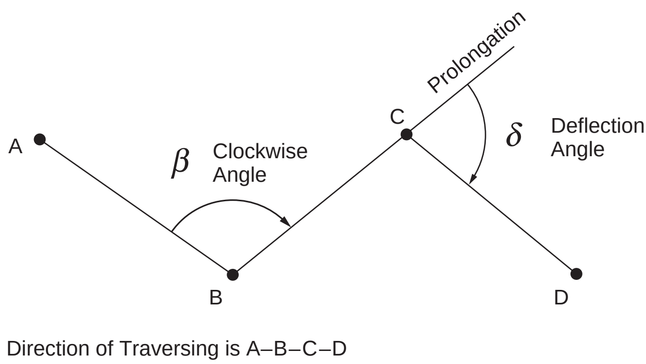

FIGURE 54.13 Horizontal field angles.

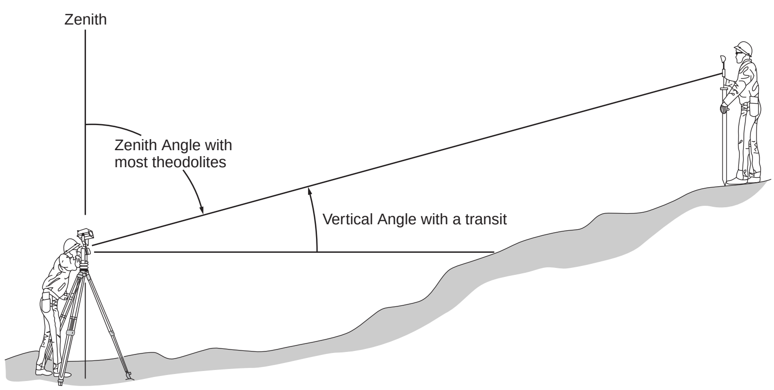

FIGURE 54.14 Vertical field angles.

have horizontal circles marked to read clockwise angles. Transits used to stake out projects typically include counterclockwise markings to facilitate laying out angles to either side of a reference line. A deflection angle, also shown in Fig. 54.13, is measured from the prolongation of the backsight line either left or right to the foresight line. It is commonly used in route centerline surveys.

Vertical angles are measured in the vertical plane as shown in Fig. 54.14. An instrument with a vertical circle graduated to measure vertical angles reads 0° when the instrument is level and the telescope is in the horizontal plane. The vertical angle increases to +90° above the horizontal and to –90° below the horizontal plane. Many instruments today have vertical circles graduated to measure the zenith angle. Zero degrees is at the zenith point directly overhead on the vertical axis. The zenith angle will increase from 0° at the zenith to 90° at the horizon. The zenith angle is greater than 90° if the telescope is pointing below the horizon. The circle graduations continue to 180° at the nadir, to 270° at the horizon with the telescope reversed, and to 360° closing the circle at the zenith.

Direction Angles

Direction angles are horizontal angles from a north–south reference meridian to a survey line. Direction angles that are computed from field angles include bearings and azimuths. A bearing, as shown in Fig. 54.15, is the acute angle from the reference meridian to the survey line. It always includes the quadrant designation. An azimuth, as shown in Fig. 54.16, is the angle from the north end of the reference meridian clockwise to the survey line. It can have a magnitude from 0° to 360°, and a quadrant designation is not necessary.

Several different meridians can be used as a reference for direction. The direction of north at a point can be defined as any of the following:

- Astronomic north
- Geodetic north
- Magnetic north
- Grid north
- Assumed north

The first three types of meridians converge to their respective north poles on the earth. Therefore, they do not form the basis for a Cartesian coordinate system to be used for plane surveying. There is only a small angular difference between astronomic and geodetic north, but astronomic north is typically taken to be synonymous with the term “true north.”

Magnetic north is the direction of the earth’s magnetic field at a point. A declination angle measured from the geodetic meridian to the magnetic meridian defines the relationship between the two systems. However, since the direction of magnetic north changes with time, proper use of magnetic north must account for variation in the magnetic declination angle.

Grid north is used in plane surveying. The grid should be defined on an appropriate map projection, such as the state plane coordinate systems available in each state. Then a defined relationship exists between the geodetic and grid meridians.

Assumed north may be used for preliminary surveys, but it is not recommended for permanent work. If the stations on the ground are lost or destroyed, the basis of direction is lost and the survey cannot be retraced.

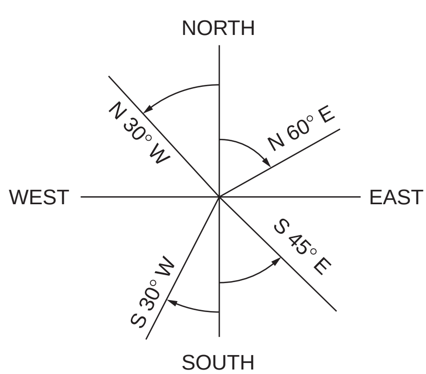

FIGURE 54.15 Bearing direction angles.

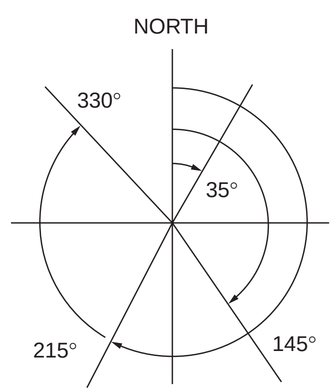

FIGURE 54.16 Azimuth direction angles.

Types of Instruments

There are many types of angle-measuring instruments used in surveying and construction. They include transits, theodolites, digital theodolites, and total stations. Even though the names are different, they look different, and the operation is slightly different, they all are used for the same purpose — angle measurement and layout.

Although there are a number of differences in the construction and features of transits, optical theodolites, and digital theodolites, the major difference is in the method of reading angles. Instruments are often classified according to the smallest interval, or so-called least count, that can be read directly in the instrument. The least count may range from 1 minute for a construction transit to 0.1 seconds for a first-order theodolite. Least counts from 10 seconds to 1 second are common in surveying and engineering instruments. The field engineer should become familiar with the instrument being used on the project.

Transit

The transit was developed to its present form during the 1800s. It has been used on construction projects from railroads across the wild west to the skyscrapers of the modern city. A good, solid, and reliable instrument, the transit is still used by many contractors today. However, optical theodolite technology, and now digital electronic technology, have passed it by. The major companies who manufactured transits have dropped them from their product line. They are slower to use than modern instruments, and they are not as easily adaptable to having an EDMI attached to them. Its major contribution to the field engineer today is that it is a good tool for learning the fundamentals of angle measurement. Its parts are exposed, making it easier to see what is going on in the manipulation of the clamps during the angle measurement process. By understanding the transit, one can easily move on to any other type of angle-measuring equipment.

Repeating Optical Theodolite

The repeating theodolite contains the same upper and lower clamp system as the transit; however, the reading of the angle is different because of the optical-reading capability. *Optical theodolite* is a term that was originally applied in Europe to instruments similar to the transit. However, as instrument technology progressed, *theodolite* became synonymous with a style of instrument that was enclosed, used a magnified optical system to read the angles, had a detachable tribrach with an optical plummet, used a three-screw leveling system, and was more precise than the transit. These features have made it much easier to use than the transit. The better optical theodolites have been "delicate" workhorses since they were introduced. That is, if they are properly cared for, they seem as though they will last forever because of their excellent construction and quality materials. However, they must be handled gently and carefully. A typical optical theodolite may have as many as 20 prisms or lenses as part of the optical angle-reading system. With a sharp bump, these can get out of alignment, which may render the instrument unusable. As with any surveying instrument, the theodolite cannot be exposed to inclement weather because of the optical system.

Scale-Reading Optical Theodolite. The typical scale-reading theodolite has a glass circle with a simple scale that is read directly. The scale is read where it is intersected by the degree readings from the circle. See Fig. 54.17 for an example scale reading. Simply read the degree that shows up in the window and observe where the degree index mark intersects the scale. Both the horizontal circle and the vertical circle are generally observed at the same time.

Micrometer-Reading Optical Theodolite. The micrometer-reading instrument also has a glass circle, but it does not have a scale. An adjustable micrometer is used to precisely read the circle and subdivide the degree intervals into minutes and seconds. See Fig. 54.18 for an example micrometer reading. The operator points the instrument and then uses the micrometer to align the degree index marks. The readings from the degree window and the micrometer window are added together to obtain the angle.

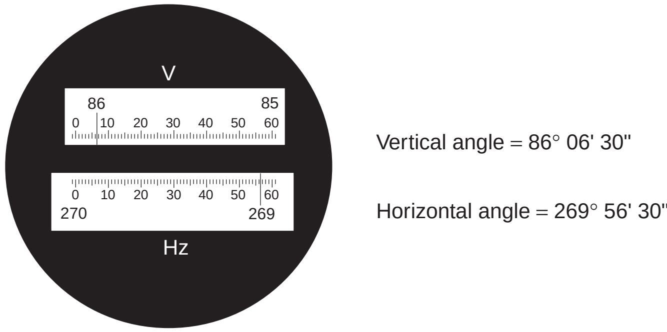

FIGURE 54.17 Scale angle reading display.

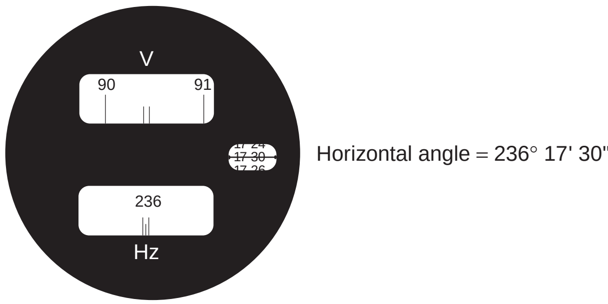

FIGURE 54.18 Micrometer angle reading display.

Directional Optical Theodolite

The directional theodolite is different from other instruments because it does not have a lower motion clamp and tangent screw. Directions (circle readings) are observed and recorded, and then the directions are subtracted to obtain the angle between the backsight and foresight lines of sight. This type of system has generally been used only on the most precise instruments. Zero is usually not set on the instrument. A micrometer is used to optically read the circle. Optical theodolites are excellent instruments, but, just like the transit, they are being surpassed by the technology of electronics.

Digital Theodolite

Digital electronics has recently entered the area of surveying instruments. Angles are no longer read optically, they are displayed on a screen in degree, minute, and second format. The micrometer has been replaced with electronic sensors that determine the angle quickly and precisely. The digital theodolite has the appearance of an optical theodolite in size and overall shape. The telescope, the clamping system, the tribrach, and the optical plummet are the same. Only the angle-measuring and -reading system is different. Digital theodolites are easy to read and fewer reading and recording blunders occur in the field. Because it is electronically based, the digital theodolite is like other electronic equipment — it either works or it doesn't work. If a circuit goes out or the battery is not charged, the instrument is unusable.

Most digital theodolites are designed to be interfaced with top-mounted EDMIs. This essentially has the impact of turning them into what are commonly called *semitotal stations,* which measure distances and angles and can be connected to a data collector for recording measurements. Sighting by both the instrument telescope and the EDMI scope are accomplished separately with this configuration.

Total Stations

The electronic total station is the ultimate in surveying measurement instruments. It is a combination digital theodolite and EDMI that allows the user to measure distances and angles electronically, calculate coordinates of points, and attach an electronic field book to collect and record the data. Since the total station is a combination digital theodolite and EDMI, its cost is quite high compared to a single instrument. However, its capabilities are simply phenomenal in comparison to the way surveyors and engineers had to measure just a few years ago. Total stations with data collectors are especially effective when a large number of points are to be located in the field, as in topographic mapping. The data collector can be used to transfer the points to a computer for final map preparation. Conversely, complex projects can be calculated on a computer in the office and the data uploaded to the data collector. The data collector is then taken to the field and connected to the total station, where hundreds of points can be rapidly established by radial layout methods.

Instrument Components

Although there are differences between the transit, optical theodolite, and electronic theodolite, the construction and operation of all these instruments is basically the same. The three major components of any instrument are illustrated, using a transit, in Fig. 54.19. The upper plate assembly or the alidade, the lower plate assembly or the horizontal circle, and the leveling head are shown.

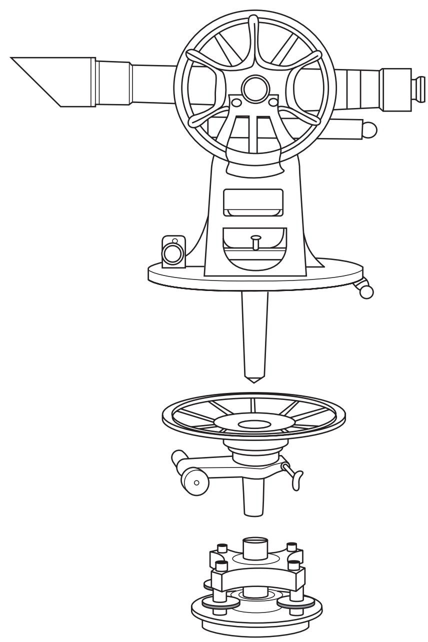

FIGURE 54.19 Transit or theodolite components.

Alidade Assembly

The alidade assembly consists of the telescope, the vertical circle, the vertical clamp and vertical tangent, the standards or structure that holds everything together, the verniers, plate bubbles, telescope bubble, and the upper tangent screw. A spindle at the bottom of the assembly fits down into a hollow spindle on the horizontal circle assembly.

Horizontal Circle Assembly

The horizontal circle assembly comprises the horizontal circle, the upper clamp that clamps the alidade and horizontal circle together, and the hollow spindle that accepts the spindle from the alidade and fits into the leveling head.

Leveling Head

The leveling head is the foundation that attaches the instrument assemblies to the tripod. It consists of leveling screws, the lower clamp that clamps the horizontal circle and the leveling head together, and a threaded bracket for attaching to the tripod.

Fundamental Relationships

All transit or theodolite instruments are designed around the same fundamental relationships and lines. These lines are shown in Fig. 54.20, again illustrated with a standard transit. The principal relationships between these lines are described as follows:

- The axis of the plate level(s) should be perpendicular to the vertical axis.
- The line of sight should be perpendicular to the horizontal axis.
- The horizontal axis should be perpendicular to the vertical axis.
- The vertical circle should read 0° when the instrument is leveled and the telescope is horizontal.

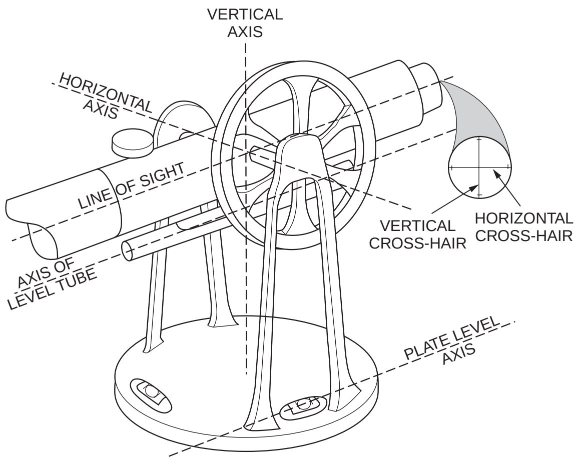

FIGURE 54.20 Principal adjustment relationships.

These four relationships are necessary to measure horizontal and vertical angles correctly. If the instrument will be used as a level, then the following must also be true: The axis of the telescope level should be parallel to the line of sight. Incorrect instrument adjustment in any of the relationships above will be readily apparent during field use of the instrument. Less apparent will be errors in the following relationships:

- The line of sight should be coincident with the telescope optical axis.
- The circles should be mounted concentrically on the axis of rotation.
- The circles should be accurately graduated throughout their circumference.
- The optical plummet should correctly position the instrument vertically over the field station.
- The vertical crosshair should lie in a vertical plane perpendicular to the horizontal axis.

Instrument Operation

The fundamental principle in using any transit or theodolite is the principle of reversion. That is, all operations should be performed in pairs, once with the telescope in the direct position and again with the telescope inverted on the horizontal axis or the reverse position. The correct value is the average of the two observations. Instrument operators should always use the double-centering ability of the instrument to eliminate the instrumental errors listed above. All of the principal instrument adjustment errors — except the vertical axis not being truly vertical — will be compensated for by averaging direct and reversed pairs of observations.

Prolonging a Straight Line

Prolonging a straight line from a backsight station through the instrument station to set a foresight station is a basic instrument operation that illustrates the principle of reversion. Often referred to as double centering, the steps to be performed are outlined as follows:

- Set up and level on the instrument station. With the telescope in the direct position, sight to the backsight station and clamp all horizontal motions.
- Rotate the telescope on the horizontal axis to the reverse position. Sight and set a point P_1 in the foresight direction as illustrated in Fig. 54.21.
- Revolve the instrument on the vertical axis and sight the backsight station again. The telescope will be in the reverse position.

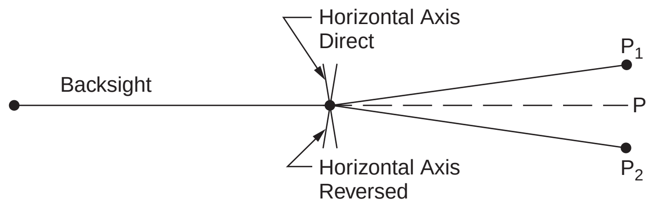

FIGURE 54.21 Double-centering to prolong a straight line.

- Rotate the telescope on the horizontal axis to the direct position. Sight and set a point P_2 in the foresight direction.
- Set the final point, P, on the true extension of the backsight line at the midpoint between P_1 and P_2.

If the stations are on nearly level terrain, the instrument error apparent in the distance between P_1 and P_2 is the line of sight not being perpendicular to the horizontal axis. When an instrument is severely out of adjustment, it is inconvenient to use it in the field. The instrument should be cleaned and adjusted periodically by a qualified instrument service technician.

Horizontal Angles by Repetition

The clamping system is probably the most important feature on an instrument because it determines the angle measurement procedures that can be used with the instrument. Transits and some optical theodolites have two horizontal motion clamps. They are commonly called the *upper motion* and *lower motion*. These types of instruments are called *repeating instruments* since the angle can be repeated and accumulated on the instrument circle. A procedure for measuring angles by repetition is outlined below.

1. Set the horizontal angle to read zero with the upper clamp and upper tangent screws. This is for convenience and any initial angle can be used.
2. Point on the backsight station with the telescope in the direct position using the lower motion. Check the circle reading and record the value in the notes.
3. Loosen the upper clamp (the lower clamp remains fixed), turn the instrument to the foresight station, and point using the upper motion clamp and upper tangent screw.
4. Note the circle reading. An approximate value of the angle can be obtained from this first turning.
5. Loosen the lower clamp, keeping the angle reading on the circle, and repeat steps 2 and 3. The number of repetitions must be an even number with half of the turnings with the telescope in the direct position and half of the turnings with the telescope in the reversed position.
6. With each turning, the circle reading will be incremented by the value of the angle. Thus, the average angle can be determined quickly by dividing the total angle read from the circle by the number of times the angle was measured.

Horizontal Angles by Direction

Many theodolites have only the upper horizontal motion clamp and the fine adjustment screw for pointing. A backsight cannot be made while holding an angle on the circle because there is no lower-motion fine adjustment for pointing. These types of instruments are called *direction instruments* since the circle reading is a clockwise direction angle from an arbitrary orientation of the 0° mark on the circle. A procedure for measuring angles by reading directions is outlined below.

1. Set the horizontal circle to read approximately 0° when pointed toward the backsight point with the telescope in the direct position.
2. Point on the backsight station with the telescope in the direct position using the upper-motion clamp and tangent screw. Read the horizontal circle and record the value in the notes.
3. Loosen the upper clamp and turn the instrument to point on the foresight station using the upper-motion clamp and tangent screw. Read the horizontal circle and record the value in the notes.

4. The clockwise angle is the difference between the foresight reading and the backsight reading.
5. Invert the telescope to the reversed position and repeat steps 2, 3, and 4. The circle readings should be exactly 180˚ from the telescope direct readings if there are no instrument adjustment, pointing, and reading errors. Of course the direct and reversed angles should agree closely and be averaged.
6. The direct and reversed pointings and the average angle constitute one position. Observe several positions and average the results to improve the precision of the final average angle. Advance the horizontal circle approximately between positions in order to distribute the readings around the circle.

54.5 Plane Survey Computations

Plane surveys use a three-dimensional Cartesian coordinate system as shown in Fig. 54.22. The horizontal components of distance, angle, and direction are assumed to be in the plane defined by the *X* and *Y* axes. The vertical components are along the *Z* axis or "up axis" in a vertical plane perpendicular to the horizontal *XY* plane.

Plane surveys referenced to such an orthogonal Cartesian coordinate system ignore the effect of earth curvature, the fact that the actual level surface is perpendicular everywhere to the direction of gravity. Such a computation scheme is suitable only for local project surveys of limited extent. Plane survey computations can be used for horizontal control surveys over a large areal extent if all observations and positions are properly referenced to a grid system using an appropriate map projection. State plane coordinate systems, defined for each state, are an example of this technique.

Traverse

A traverse is an efficient and flexible method of field surveying used to connect points of interest and establish horizontal and/or vertical coordinate reference values for project control. A traverse consists of interconnected straight lines along a traverse route. The straight lines meet at angle points that must be permanently marked by a traverse station monument. The length of each line is measured by field survey and then reduced to the horizontal reference plane. At each traverse station a horizontal angle is measured that will relate the directions of each line to one another in the horizontal plane. The elevation of each traverse station may also be determined if required for the purpose of the survey. Elevation can be

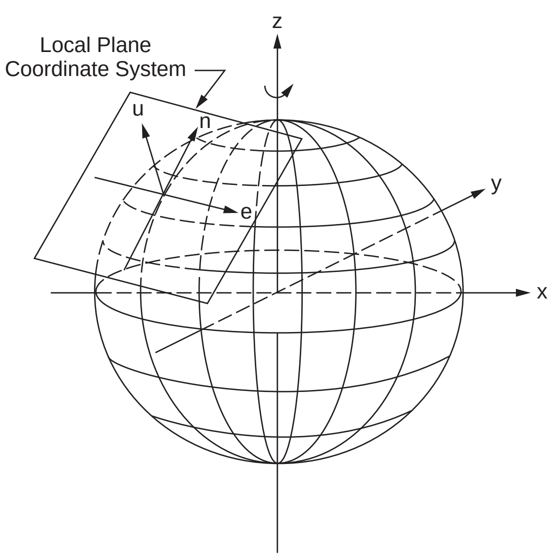

FIGURE 54.22 Local tangent plane coordinate system.

determined by trigonometric leveling as part of the traverse measurements or by differential leveling as a separate survey operation.

Traverses are characterized as open or closed. An open traverse has no check on computed direction or position at the end of the route. It is used only for preliminary and uncontrolled work. Whenever traverse stations are to be used to control subsequent engineering or surveying work, the route should be designed to close on the beginning traverse station or to close on another known point of equal or higher-order accuracy. The closed linear traverse between known control stations is preferred since it can best detect length measurement errors that affect the scale of a survey.

Traverse design is influenced by the purpose of the survey and the terrain through which the survey must progress. The overriding factor in determining where to locate traverse stations is the purpose of the station. In general, a traverse station becomes a reference system control point that is used for one or more of the following purposes:

1. Provide control for mapping by field survey or photogrammetric methods.
2. Provide control for construction layout.
3. Locate property boundaries.
4. Connect lines within the traverse and secondary traverses that may be added to the survey network.

The traverse station must be located so that it is accessible for its intended purpose. Secondary to the purpose of the survey, the traverse station should be located in an area where the monument will be stable and undisturbed for the intended useful life of the station.

The traverse survey route is flexible and generally follows the path of least resistance so that clear lines of sight from traverse station to traverse station are obtained. However, several guidelines should be kept in mind when planning a traverse route. First, avoid lines of sight that pass close to the intervening terrain between stations where atmospheric refraction will seriously degrade measurement accuracy. Second, avoid short lines of sight where setup errors and pointing errors can be the dominant measurement error source. Third, the traverse route should proceed along a generally straight path between terminal stations of a linear closed traverse. A useful rule of thumb is that the deviation of the route, as measured perpendicular to the straight-line path, should not exceed approximately one-third of the straight-line path length.

Accuracy standards for traversing are chosen to match the purpose of the survey. Specifications for the survey process are then developed to meet the required accuracy standards. For control surveys the accuracy standards and specifications developed by the Federal Geodetic Control Committee (FGCC) are often adopted. Surveys may be required to meet **first-**, **second-**, or **third-order accuracy.**

Surveying standards and specifications for surveys intended for other purposes may be available from a variety of sources, such as state statutes or licensing board rules regulating the type of survey, professional societies, or the agency contracting for the survey. For example, standards and specifications developed by ALTA/ACSM are often the requirements adopted for property surveys.

Traverse computations for local plane surveys are outlined in the following conventional procedure:

1. Draw a complete sketch of the traverse.
2. Compute the angular closure. If angle closure is equal to or less than allowable limit, adjust the angles; if angle closure is not acceptable, remeasure angles. The allowable closure is typically specified in the following form:

$$\text{Allowable closure} = c = k\sqrt{n}$$

   where
   - c = allowable error in a series of measurements
   - k = a value specified for the accuracy order of the survey
   - n = number of angles measured
3. Compute the direction (azimuths or bearings) of all lines.

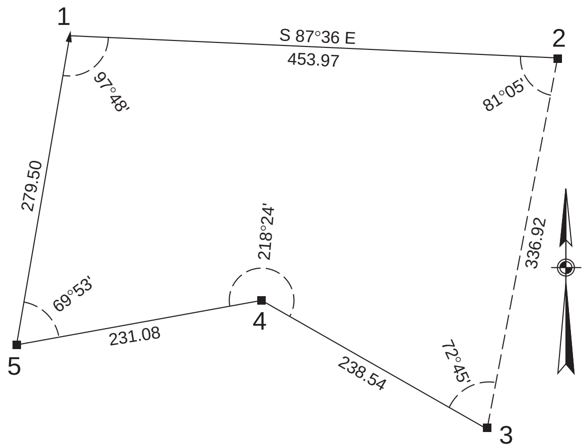

FIGURE 54.23 Sketch of sample traverse.

4. Compute the latitude and departure (relative nothing and relative easting components) for each course. Set up the computations in tabular form.

$$\text{Latitude} = L \cos\alpha$$
$$\text{Departure} = L \sin\alpha$$

5. Compute the traverse misclosure.

$$c = \sqrt{(\text{Latitude error})^2 + (\text{Departure error})^2}$$

6. Compute the traverse precision ratio.

$$\text{Precision} = \frac{\text{Closure error}}{\text{Perimeter}} = \frac{1}{?}$$

7. If the precision ratio is equal to or better than the allowable precision specified for the accuracy order of the survey, then distribute the error of closure throughout the traverse by an appropriate rule or by a least-squares adjustment.
8. Compute the coordinates of the traverse stations using the balanced latitudes and departures.
9. Determine the adjusted traverse course lengths and directions using an inverse computation from the adjusted coordinates or latitudes and departures.
10. Compute the area of a closed loop traverse.

The traverse shown in Fig. 54.23 is used to illustrate conventional plane survey traverse computations. The measured horizontal distance of each line is given, and horizontal angles are measured as interior angles on the closed loop traverse. The orientation of the traverse is defined by the azimuth given for line 1-2. The coordinate system is defined by known coordinates at station of 1000.00N, 500.00E.

1. Draw a complete sketch.
2. Compute the angular closure.

$$\begin{aligned}\text{Field angle closure} &= (97°48' + 81°05' + 72°45' + 218°24' + 69°53') \\ &\quad - (5-2)180° \\ &= 539°55' - 540°00' \\ &= .05'\end{aligned}$$

Assuming the angular error of closure is less than the maximum allowable, apply an equal correction to each angle.

$$\text{Correction per angle} = \frac{05'}{5 \text{ angles}} = 0°01' \text{ to be added to each angle}$$

3. Compute bearings or azimuths. Use the adjusted angles. In this example the azimuth of each line is computed beginning from the given azimuth for line 1-2.
   Azimuth 1 to 2 = given = 92°24′
   Azimuth 2 to 3 = 92°24′ + 180° – 81°06′ = 191°18′
   Azimuth 3 to 4 = 191°18′ + 180° – 72°46′ = 298°32′
   Azimuth 4 to 5 = 298°32′ + 180° – 218°25′ = 260°07′
   Azimuth 5 to 1 = 260°07′ – 180° – 69°54′ = 10°13′
   Azimuth 1 to 2 = 10°13′ + 180° – 97°49′ = 92°24′
   Note that the azimuth of the first line is computed at the end to verify that the computations close on the given azimuth. The adjusted angles and azimuths are shown in Fig. 54.24.
4. Compute the latitude and departure of each course, as shown in Table 54.3.

TABLE 54.3 Computed Latitudes and Departures.

Course	Length (L)	Azimuth α	Latitude $= L \cos \alpha$	Departure $= L \sin \alpha$
1-2	453.97	92°24′	–19.010	453.572
2-3	336.92	191°18′	–330.389	–66.018
3-4	238.54	298°32′	113.943	–209.567
4-5	231.08	260°07′	–39.663	–227.651
5-1	279.50	10°13′	275.068	49.575
Sum =	1540.01	Error =	–0.051	–0.089
		Correction	+0.051	+0.089

5. Compute the traverse misclosure.

$$c = \sqrt{(-0.051)^2 + (-0.089)^2} = 0.102 \text{ ft}$$

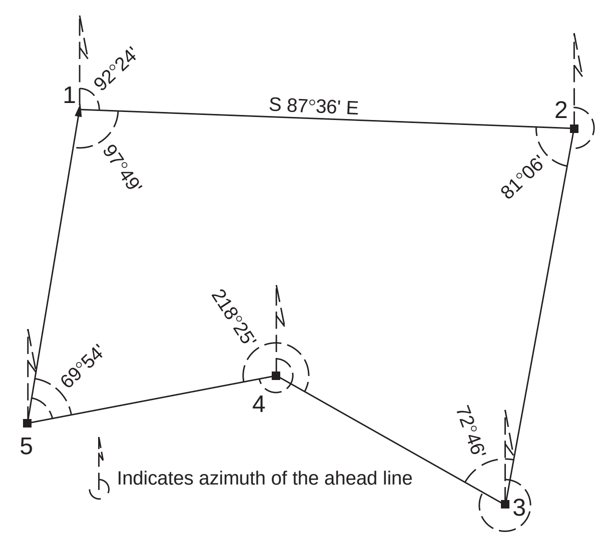

FIGURE 54.24 Adjusted angles and azimuths.

6. Compute the traverse precision.

$$\text{Precision} = \frac{0.102}{1540.01} = \frac{1}{15{,}100}$$

7. Distribute the error of closure. We will assume this precision is sufficient for the particular survey we are carrying out. Adjust the traverse by the compass rule:

$$C_{\text{lat}} = \frac{+\,0.051}{1540.01}(\text{Course length})$$

$$C_{\text{dep}} = \frac{+\,0.089}{1540.01}(\text{Course length})$$

8. Compute the balanced latitudes and departures by applying the corrections, and then calculate coordinates for all of the traverse stations, as shown in Table 54.4.

TABLE 54.4 Compass Rule Adjustment

	Applying Corrections		Balanced		Coordinates	
Station	Latitude	Departure	Latitude	Departure	North (*Y*)	East (*X*)
1					1000.00	500.00
	– 19.010	453.572				
	+.015	+.026	– 18.995	+ 453.598	– 19.00	+ 453.60
2					981.00	953.60
	– 330.389	– 66.018				
	+.011	+.020	– 330.378	– 65.998	– 330.38	– 66.00
3					650.62	887.60
	113.943	– 209.567				
	+.008	+.014	+ 113.951	– 209.553	+ 113.95	– 209.55
4					764.57	678.05
	– 39.663	– 227.651				
	+.008	+.013	– 39.655	– 227.638	– 39.65	– 227.64
5					724.92	450.41
	275.068	49.575				
	+.009	+.016	+ 275.077	+ 49.591	+ 275.08	+ 49.59
1					1000.00	500.00
		Total =	0.000	0.000	Check	Check

9. Adjusted traverse line lengths and directions are computed by an inverse computation using the adjusted latitudes and departures or adjusted coordinates, as shown in Table 54.5.

$$\text{Length} = \sqrt{Dep^2 + Lat^2}$$
$$= \sqrt{(X_j - X_i)^2 + (Y_j - Y_i)^2}$$

$$\text{Azimuth}_{ij} = \tan^{-1}\left[\frac{Dep_{ij}}{Lat_{ij}}\right] = \tan^{-1}\left[\frac{X_j - X_i}{Y_j - Y_i}\right]$$

10. Compute the area. Traverse areas are typically computed using the coordinate method, as shown in Table 54.6.

$$\text{Area} = \frac{\left|\sum_1^n X_i Y_{i-1} - \sum_1^n X_i Y_{i+1}\right|}{2}$$

TABLE 54.5 Adjusted Lengths and Azimuths

Station	Coordinates North (Y)	East (X)	Adjusted Length	Adjusted Azimuth
1	1000.00	500.00		
			454.00	92°23′53″
2	981.00	953.60		
			336.90	191°17′50″
3	650.62	887.60		
			238.53	298°32′12″
4	764.57	678.05		
			231.07	260°07′05″
5	724.92	450.41		
			279.51	10°13′10″
1	1000.00	500.00		

TABLE 54.6 Coordinate Method for Area

Station	Coordinates North (Y)	East (X)	Double Area —	0
1	1000.00	500.00	490,500	
2	981.00	953.60	620,431	953,600
3	650.62	887.60	678,632	870,736
4	764.57	678.05	491,532	441,153
5	724.92	450.41	450,410	344,370
1	1000.00	500.00		362,460
		Total =	2,731,505	2,972,319

The traverse area equals one-half of the difference between the totals of the double-area columns.

$$\text{Traverse area} = \left|\frac{2{,}731{,}505 - 2{,}972{,}319}{2}\right| = 120{,}407 \text{ ft}^2 = 2.764 \text{ acres}$$

Partitioning Land

Partitioning land is a problem that can usually be classified according to one of two types of dividing line — a line of known direction or a line through a known point. A preliminary line is often required that satisfies the given condition. Then the line is translated parallel to itself in the first condition or pivoted about the known point in the second condition to obtain the required area.

As an illustration of these methods, partition the adjusted traverse of the previous section so that 65,000 square feet lie west of a true north line. A preliminary cutoff line bearing true north can be constructed through station 4, as shown in Fig. 54.25. The area west of the preliminary line is 50,620 square feet. The line is translated true east a distance X so that 14,380 square feet will be added to the west parcel. The parcel added is a trapezoid as shown in Fig. 54.25, and the area can be expressed by

$$14{,}380 = 227.97X - \tfrac{1}{2}X(X \tan\theta_1) + \tfrac{1}{2}X(X \tan\theta_2)$$

where $\theta_1 = 2°23′53″$ and $\theta_2 = 28°32′12″$. Rearranging this expression results in a quadratic equation, and the solution for X is found to be 59.22 feet. Then the following distances can be determined for the final cutoff line:

6-7 = 59.27 feet
4-8 = 67.41 feet
7-8 = 257.69 feet

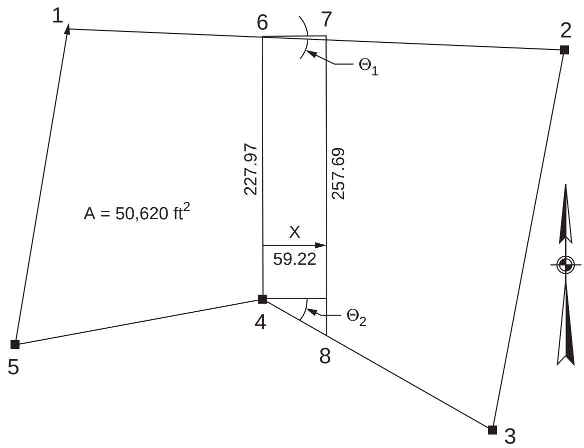

FIGURE 54.25 Partition by sliding a line.

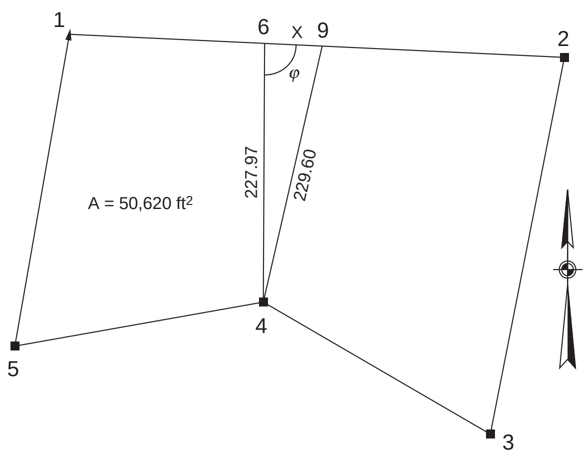

FIGURE 54.26 Partition by pivoting a line.

Next, partition the adjusted traverse so that 55,000 square feet lie west of a line through station 4. The same preliminary cutoff line can be used. The line is pivoted easterly about station 4 so that 4380 square feet is added to the west parcel. The parcel added is a triangle as shown in Fig. 54.26, and the area can be expressed by

$$4380 = \frac{1}{2}227.97X \sin\phi$$

where $\phi = 87°36'07''$. The solution for X is found to be 38.46 feet, and the final cutoff line is

$$4\text{-}9 = 229.60 \text{ feet, } 9°38'05'' \text{ azimuth}$$

54.6 Horizontal Curves

Horizontal curves are used in route projects to provide a smooth transition between straight-line tangent sections. These curves are simple circular curves. The components of a circular curve are illustrated in Fig. 54.27. The design of a circular curve requires that two curve elements be defined and then the

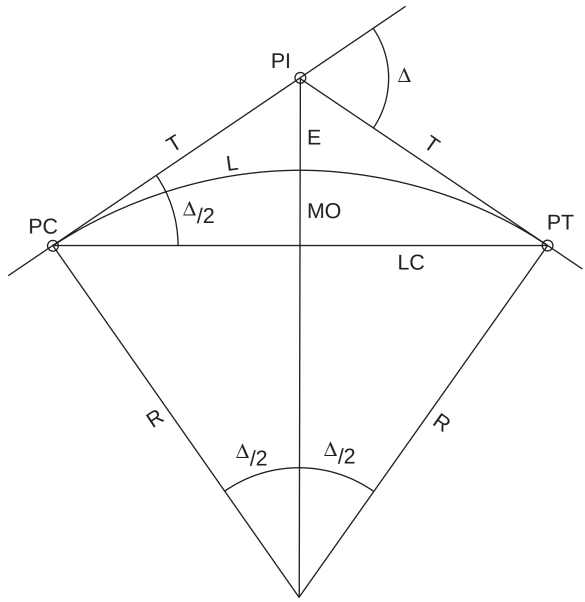

FIGURE 54.27 Circular horizontal curve components.

remaining elements are determined by computation. In the typical case the intersection angle of the tangents is determined by field survey and the curve radius is chosen to meet design specifications such as vehicle speed and minimum sight lengths. The principal relationships between curve elements are

$$T = R\ \tan\frac{\Delta}{2}$$

$$LC = \frac{1}{2}R\ \sin\frac{\Delta}{2}$$

$$E = R\left(\sec\frac{\Delta}{2} - 1\right)$$

$$MO = R\left(1 - \cos\frac{\Delta}{2}\right)$$

$$L = R\,\frac{\pi}{180^\circ}\Delta$$

Important angle relationships used when solving circular curve problems are illustrated in Fig. 54.28.

When a circular curve is staked out in the field, the most common method is to lay off deflection angles at the PC station. Field layout notes are prepared for a theodolite set up at the PC. A backsight is taken along the tangent line to the PI, and foresights are made to specific stations on the curve using the computed deflection angles. If a total station instrument is used the chord distance from the PC to the curve station can be used to set the station. If a tape is used the chord distance measured from the previous station on the curve is intersected with the line of sight to locate the curve station.

The following example illustrates both types of distances used to lay out a horizontal curve. Determine the field information necessary to stake out the horizontal curve shown in Fig. 54.29. First determine the stationing of the PC and PT. The tangent distance is

$$T = R\ \tan\left(\frac{\Delta}{2}\right) = 572.96\ \ \tan 40^\circ 00' = 503.46\ \text{ft}$$

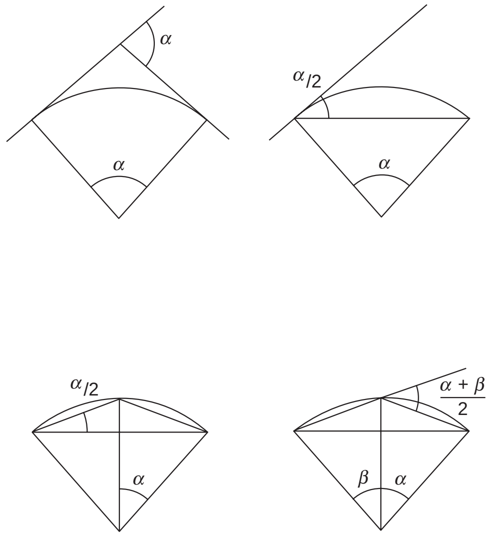

FIGURE 54.28 Angle relationships for circular curves.

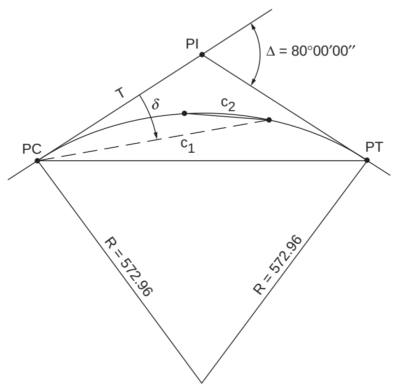

FIGURE 54.29 Horizontal curve layout.

so the PC station is

$$\text{PC sta.} = \text{PI sta.} - T = 100 + 50.00 - 5 + 03.46 = 95 + 46.54 \text{ ft}$$

The arc length of the curve is

$$L = R\left(\frac{\pi}{180°}\right)\delta = 800.00 \text{ ft}$$

so the PT station is

$$\text{PT sta.} = \text{PC sta.} + L = 95 + 46.54 + 8 + 00.00 = 103 + 46.54 \text{ ft}$$

The central angle subtended per station of arc length is

$$D° \text{ per sta} = \frac{\Delta}{L} = \frac{80°00'}{800.00} = 10°00'$$

TABLE 54.7 Deflection Angles and Chords for Layout of a Horizontal Curve

Station	Deflection Angle δ	Arc Length from PC	Chord Length C_1	Chord Length C_2
PC 95 + 46.54				
		53.46	53.44	53.44
96 + 00.00	2°40′23″			
		153.46	153.00	99.87
97 + 00.00	7°40′23″			
		253.46	251.40	99.87
98 + 00.00	12°40′23″			
		353.46	347.88	99.87
99 + 00.00	17°40′23″			
		453.46	441.72	99.87
100 + 00.00	22°40′23″			
		553.46	532.19	99.87
101 + 00.00	27°40′23″			
		653.46	618.62	99.87
102 + 00.00	32°40′23″			
		753.46	700.33	99.87
103 + 00.00	37°40′23″			
		L 800.00	*LC* 736.58	46.53
PT 103 + 46.54	Δ/240°00′00″			

The deflection angle is one-half of the central angle or

$$\delta \text{ per sta} = \frac{D}{2} = 5°00'$$

$$\delta \text{ per ft} = \frac{D}{200} = 0°03'$$

Recall the equation for a chord

$$C = 2R \sin\left(\frac{d}{2}\right)$$

where d is the central angle of the chord. The full stations falling on the curve are determined, and the values in Table 54.7 are computed for an instrument set up on the PC and oriented by a backsight on the PI. Note that C_1 denotes the chord measured from the PC and that C_2 denotes the chord measured from the previous station on the curve.

Alternative methods for staking out a curve include tangent offsets, chord offsets, middle ordinates, and radial staking out from the radius point of the curve or from the PI station.

54.7 Vertical Curves

Vertical curves are used in vertical alignments of route projects to provide a smooth transition between grade lines. These curves are usually equal-tangent parabolic curves. The point of vertical intersection, PVI, of the entrance and exit grade lines always occurs at the midpoint of the length of curve. The length of curve and all station distances are measured in the horizontal plane. Figures 54.30 and 54.31 illustrate the geometry of a sag and crest vertical curve, respectively. Note the following relationships between the curves in Figs. 54.30 and 54.31. The top curve is the elevation curve. The middle curve is the grade curve; it is the derivative of the elevation curve. The bottom curve is the rate of change of grade curve; it is the derivative of the grade curve. The rate of change of grade is always a constant, r, for a parabolic vertical curve.

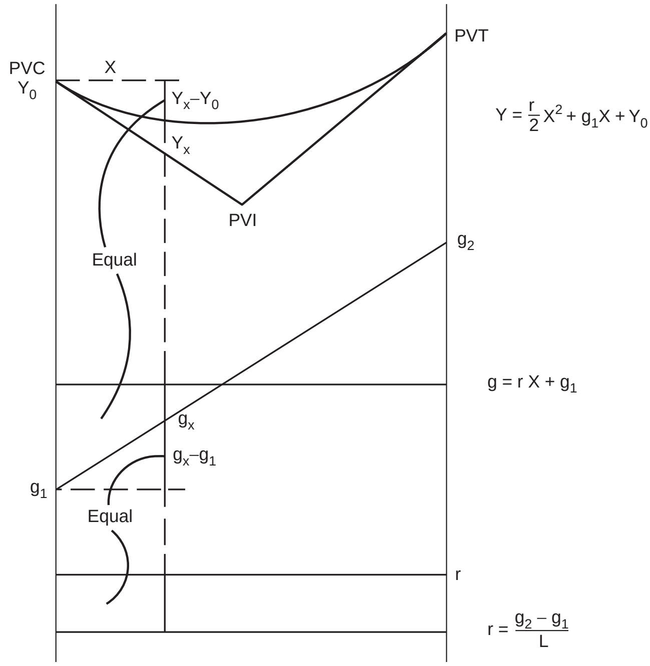

FIGURE 54.30 Sag vertical curve.

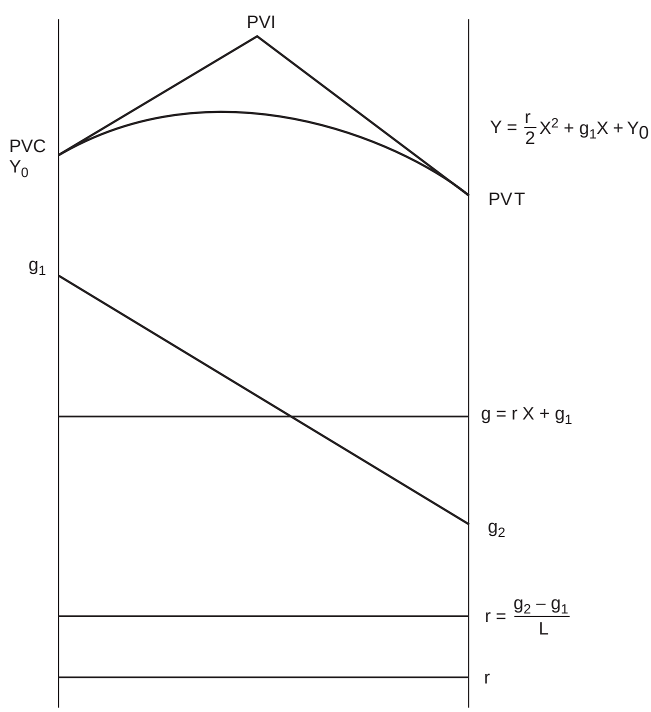

FIGURE 54.31 Crest vertical curve.

Since the curves are related by the derivative/integration operation, the change in an ordinate value on one curve is equal to the area under the next lower curve. Therefore, in Fig. 54.30, the change in elevation on the curve from the PVC to a point at a distance X on the curve is equal to the trapezoidal area under the grade curve

$$Y_x - Y_0 = X\left(\frac{g_x + g_1}{2}\right)$$

The change in grade over the same interval X is the rectangular area under the rate of change of grade curve.

$$g_x - g_1 = Xr$$

These relationships can be used to calculate design values for vertical curves.

For example, suppose it is necessary to find the elevation and station of the high point for the crest curve in Fig. 54.31. Let X equal the distance to the high point that occurs at the point where the grade curve crosses the zero line. The change in grade must be equal to the area under the rate of change of grade curve.

$$g_{\text{high}} = 0$$
$$0 - g_1 = X(-r)$$
$$X = \frac{-g_1}{-r}$$

Note that r is negative for a crest value. Then the station of the high point is

$$P_{\text{high}} = \text{PC} + X$$

The elevation of the high point can be found by evaluating the elevation equation at the known value of X or by calculating the area under the grade curve:

$$Y_x - Y_0 = X\left(\frac{g_1 + 0}{2}\right)$$
$$Y_x = Y_0 + X\left(\frac{g_1}{2}\right)$$

The design of a vertical curve involves choosing the length of the curve that will satisfy design speed considerations, earthwork considerations, and sometimes geometric constraints. As an example of a constrained design, suppose a PVC is located at station 150 + 40.00 and elevation 622.45 feet. The grade of the back tangent is –3.00% and the grade of the forward tangent is –7.00%. It is required that a vertical curve between these tangents must pass through station 152 + 10.00 at elevation 619.05 feet. Since this is a sag vertical curve, refer to Fig. 54.30. The value of r can be expressed as

$$r = \frac{(+7) - (-3)}{L} = +\frac{10}{L}$$

The distance X from the PVC to the known station on the curve is

$$X = (152 + 10.00) - (150 + 40.00) = 1.70000 \text{ stations}$$

The grade at the known station is then found from

$$g_x = (-3) + \left(\frac{10}{L}\right)1.7 = \frac{17}{L} - 3$$

The change in elevation from the PVC to the known station can be set equal to the trapezoidal area under the grade curve.

$$619.05 - 622.45 = 1.70000 \left[\frac{\left(\frac{17}{L} - 3\right) + (-3)}{2} \right]$$

Solving this expression for L, we obtain

$$L = 8.5000 \text{ stations} = 850.00 \text{ ft}$$

The design elevation at each full station along this curve can be evaluated from the parabolic equation. First the station and elevation of the PVI is

$$\text{PVI sta} = \text{PVC sta} + \frac{L}{2} = 150 + 40.00 + 4 + 25.00 = 154 + 65.00 \text{ ft}$$

$$\text{PVI elev} = \text{PVC elev} + g_1 \frac{L}{2} = 622.45 + (-3)(4.25) = 609.70 \text{ ft}$$

and the station and elevation of the PVT is

$$\text{PVT sta} = \text{PVI sta} + \frac{L}{2} = 154 + 65.00 + 4 + 25.00 = 158 + 90.00 \text{ ft}$$

$$\text{PVT elev} = \text{PVI elev} + g_1 \frac{L}{2} = 609.70 + (+7)(4.25) = 639.45 \text{ ft}$$

Then the elevation of any point on the curve is found from

$$Y = \left(\frac{r}{2}\right) X^2 + g_1 X + Y_0 = \left(\frac{10}{17}\right) X^2 - 3X + 622.45$$

A tabular solution for each full station along the curve is given in Table 54.8.

54.8 Volume

The determination of volume is necessary before a project begins, throughout the project, and at the end of the project. In the planning stages, volumes are used to estimate project costs. After the project is started, volumes are determined so the contractor can receive partial payment for work completed. At the end, volumes are calculated to determine final quantities that have been removed or put in place to make final payment. The field engineer is often the person who performs the field measurements and calculations to determine these volumes. Discussed here are the fundamental methods used by field engineers.

General

To compute volumes, field measurements must be made. This typically involves determining the elevations of points in the field by using a systematic approach to collect the needed data. If the project is a roadway, cross-sectioning is used to collect the data that are needed to calculate volume.

If the project is an excavation for a building, borrow-pit leveling will be used to determine elevations of grid points to calculate the volume. Whatever the type of project, the elevation and the location of

TABLE 54.8 Vertical Curve Elevations

Station	X sta	$(r/2)X^2$	g_1X	Elevation Y ft
PVC 150 + 40.00				Y_0 622.45
	0.6000	0.21	– 1.80	
151 + 00.00				620.86
	1.6000	1.50	– 4.80	
152 + 00.00				619.15
	2.6000	3.98	– 7.80	
153 + 00.00				618.83
	3.6000	7.62	– 10.80	
154 + 00.00				619.27
	4.6000	12.45	– 13.80	
155 + 00.00				621.10
	5.6000	18.45	– 16.80	
156 + 00.00				624.10
	6.6000	25.62	– 19.80	
157 + 00.00				628.27
	7.6000	33.98	– 22.80	
158 + 00.00				633.63
	8.5000	42.50	– 25.50	
PVT 158 + 90.00				639.45

points will need to be determined. It is the responsibility of the field engineer to determine the most efficient method of field measurement to collect the data.

However, it should be mentioned that sometimes volumes can be determined using no field measurements at all. In some situations the contractor may be paid for the number of truckloads removed. Keeping track of the number of trucks leaving the site is all that may be necessary. However, this isn't a particularly accurate method since the soil that is removed expands or swells and takes up a larger amount of space than the undisturbed soil. Depending on how the project is bid, it is sometimes accurate enough.

Field Measurements for Volume Computations

Measurements for volume are nothing more than applying basic distance and elevation measurements to determine the locations and elevations of points where the volume is to be determined. It usually is not practical to take the time to collect data everywhere there is a slight change in elevation. Therefore, it must be understood that volume calculations do not give exact answers. Typically, approximations must be made and averages determined. The field engineer will analyze the data and make decisions that result in the best estimate of the volume.

Area

The key to volume calculation is the determination of area. Most volume calculation formulas contain within them the formula for an area, which is simply multiplied by the height to determine the volume. For instance, the area of a circle is pi times the radius squared. The volume of a cylinder is the area of the circle times the height of the cylinder. If an area can be determined, it is generally easy to determine the volume.

Counting Squares

Approximation is possible by plotting the figure to scale on cross-sectional paper and counting the squares. Each square represents x number of square feet. Incomplete squares along the edges of the cross section are visually combined and averaged.

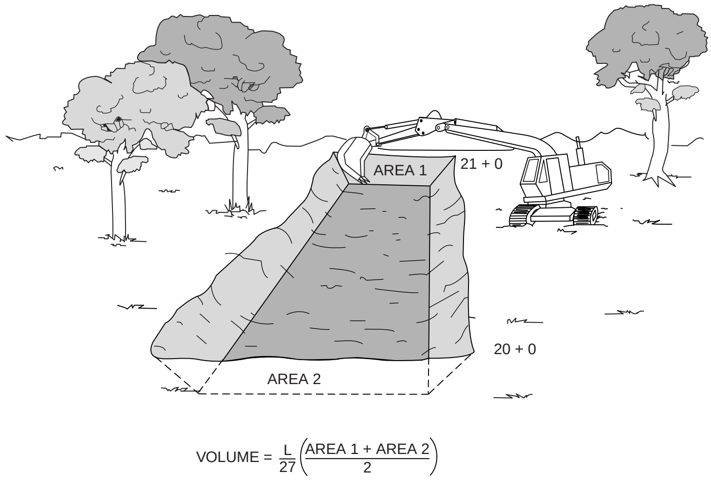

FIGURE 54.32 Average end area method.

Planimeter

The electromechanical digital planimeter is a quick method of determining the area of irregularly shaped figures. The irregular shape is drawn to scale and the planimeter is used to trace the outline of the shape. Inputting a scale factor into the planimeter results in a digital readout of the area.

Geometric Formula

Although a shape at first may seem irregular, it is often possible to break it into smaller regular shapes such as squares, rectangles, triangles, trapezoids, etc., that will allow the use of standard geometric formulas to determine the area. This method may be cumbersome because of all the shapes that may need to be calculated.

Cross-Section Coordinates

If cross-sectional field data are available, use of this data is the recommended method of calculating volume. Once understood, this process is fast and the most accurate way of determining area. Cross section data collected on a project represent elevation and location information for points on the ground. These points can be used as coordinates to determine area.

Volume Computations — Road Construction

In road construction the shape of the ground must be changed to remove the ups and downs of the hills and valleys for the planned roadway. Often mountains of dirt must be moved to create a gentle grade for the roadway. Payment for the removal and placement of dirt is typically on a unit cost basis. That is, the contractor will be paid per cubic yard of soil and will receive a separate price per cubic yard of rock. It can be seen that accurate determination of the volume moved is critical to the owner and to the contractor. Each wants an accurate volume so payment for the work is correct.

For road projects, cross sections of the ground elevations are measured at the beginning of the project, during the project, and at the end of the project. Comparisons between final cross sections and original cross sections are used to determine the volume moved. Areas of the cross sections are most easily determined by using the elevations of the points and their locations from the centerline (coordinates).

The average end area method uses the end areas of adjacent stations along a route and averages them. Refer to Fig. 54.32. This average is then multiplied by the distance between the two end areas to obtain the volume between them. In formula form the process is as follows:

$$\text{Volume} = \frac{L}{27}\left(\frac{\text{Area 1} + \text{Area 2}}{2}\right)$$

where L represents the distance between the cross-sectional end areas being used in the formula, and 27 represents the number of cubic feet in 1 cubic yard. Dividing cubic feet by 27 converts to cubic yards.

Volume Computations — Building Excavation

A method known as *borrow-pit leveling* can be used effectively to determine volume on building projects. A grid is established by the field engineer and elevations on the grid points are determined both before the excavation begins and when the work is complete.

The borrow-pit method uses a grid and the average depth of the excavation to determine the volume. Before the excavating begins the field engineer creates a grid over the entire area where the excavation is to occur. Elevation data are collected at each of the grid points and recorded for future reference. At any time during the excavating, the field engineer can reestablish the grid and determine new elevations for each of the field points. Using the average height formula shown below, the volume of soil removed from each grid area can be determined. Refer to Fig. 54.33. The smaller the grid interval, the more accurate the volume.

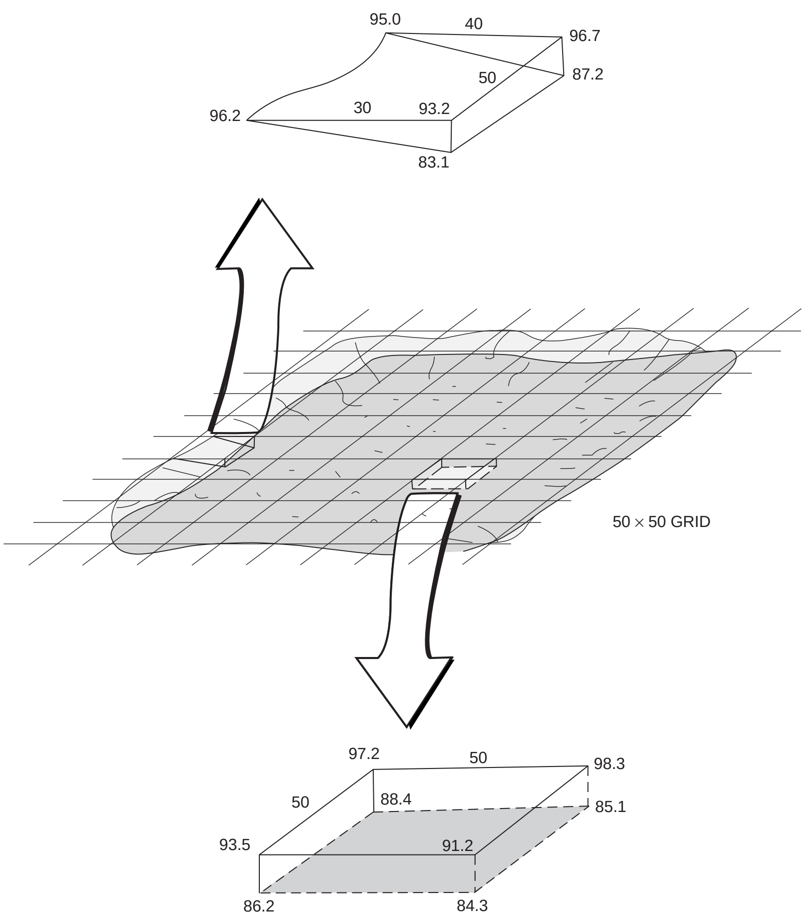

FIGURE 54.33 Borrow-pit method.

Summary

Only two general methods of calculating volumes have been presented here. There are many others that are very specific for the particular situation. For example, when determining volumes along a roadway, there is a constant transition from cut to fill to cut to fill, etc. To more accurately compute the volume, a prismoidal formula is used. The field engineer should check with textbooks that discuss in detail route surveying and earthwork for additional information.

Defining Terms

Accuracy — Refers to the degree of perfection obtained in measurements. It is a measure of the closeness to the true value.

Accuracy (first-order) — The highest accuracy required for engineering projects such as dams, tunnels, and high-speed rail system.

Accuracy (second-order) — The accuracy required for large engineering projects such as highways, interchanges, and short tunnels.

Accuracy (third-order) — The accuracy required for small engineering projects and topographic mapping control.

Accuracy ratio — The ratio of error of closure to the distance measured for one or a series of measurements.

Axis of level bubble — The line tangent to the top inner surface of a spirit level at the center of its graduated scale, and in the plane of the tube and its center of curvature.

Calibration — The process of comparing an instrument or chain with a standard.

Closure — The amount by which a value of a quantity obtained by surveying fails to agree with a value (of the same quantity) determined. It is also called *misclosure* or *error of closure*.

Datum — A reference elevation such as mean sea level or, in the case of some construction projects, a benchmark with elevation 100.00.

Horizontal axis — The axis about which the telescope rotates vertically.

Least squares — A mathematical method for the adjustment of observations based on the theory of probability.

Line of sight — The line extending from an instrument along which distinct objects can be seen. The straight line between two points.

Mean sea level — The average height of the surface of the sea measured over the complete cycle of high and low tides (a period of 18.6 years).

Monument — A physical structure that marks the location of a survey point.

Nadir — The point directly under the observer. The direction that a plumb bob points.

Precision — The closeness of one measurement to another. The degree of refinement in the measuring process. The repeatability of the measuring operation.

Random errors — Errors that are accidental in nature and always exist in all measurements. They follow the laws of probability and are equally high or low.

Refraction — The bending of light rays as they pass through the atmosphere.

Systematic error — Those errors that occur in the same magnitude and the same sign for each measurement of a distance, angle, or elevation. Can be eliminated by mechanical operation of the instrument or by mathematical formula.

Vertical — The direction in which gravity acts.

Zenith — The point directly above a given point on earth.

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Further Information

The material here is intended only as an overview of plane surveying. There are many textbooks dedicated completely to the various aspects of surveying. The authors recommend the following books. For a more complete presentation of surveying theory, consult *Elementary Surveying,* 9th ed., by Wolf and Brinker, HarperCollins, 1992; or *Surveying: Theory and Practice,* 6th ed., by Davis et al., McGraw-Hill, 1981. For illustrated step-by-step descriptions of performing field work, consult *Construction Surveying and Layout,* by Crawford, P.O.B. Publishing, 1994.

To obtain detailed information on the capabilities of various instruments and software, P.O.B. Publishing prepares the trade magazine *P.O.B.,* and American Surveyors Publishing Company prepares the trade magazine *Professional Surveyor.* Each of these publications conducts annual reviews of theodolites, total stations, EDMIs, data collectors, GPSs, and software. These listings allow the reader to keep up-to-date and compare "apples to apples" when analyzing equipment.

Survey control information, software, and many useful technical publications are available from the National Geodetic Survey (NGS). The address is

National Geodetic Survey Division
National Geodetic Information Branch, N/CG17
1315 East-West Highway, Room 9218
Silver Spring, MD 20910-3282