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Composite Steel–Concrete Structures

51.1 Introduction

History • Applications • Case Studies

Composite construction as we know it today was first used in both a building and a bridge in the U.S. over a century ago. The first forms of composite structures incorporated the use of steel and concrete for flexural members, and the issue of longitudinal slip between these elements was soon identified [1].

Composite steel–concrete beams are the earliest form of the composite construction method. In the U.S. a patent by an American engineer was developed for the shear connectors at the top flange of a universal steel section to prevent longitudinal slip. This was the beginning of the development of fully composite systems in steel and concrete.

Concrete-encased steel sections were initially developed in order to overcome the problem of fire resistance and to ensure that the stability of the steel section was maintained throughout loading. The steel section and concrete act compositely to resist axial force and bending moments.

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51.1 Introduction

History

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Concrete-encased steel sections were initially developed in order to overcome the problem of fire resistance and to ensure that the stability of the steel section was maintained throughout loading. The steel section and concrete act compositely to resist axial force and bending moments.
Composite tubular columns were developed because they provided permanent and integral formwork for a compression member and were instrumental in reducing construction times and consequently costs. They reduce the requirement of lateral reinforcement and costly tying, as well as providing easier connection to steel universal beams of a steel-framed structure.

Composite slabs have been introduced recently to consider the increase in strength that can be achieved if the profiled steel sheeting is taken into account in strength calculations. Composite slabs provide permanent and integral reinforcement, which eliminates the need for placing and stripping of plywood and timber formwork.

More recently, composite slab and beam systems have been developed for reinforced concrete framed construction; this provides advantages similar to those attributed to composite slabs for reinforced concrete slab and beam systems. These advantages include reduced construction time due to elimination of formwork, and elimination of excessive amounts of reinforcing steel. This subsequently reduces the span-to-depth ratios of typical beams and also reduces labor costs.

In this chapter, a thorough review is given of research into composite construction, including beams, columns, and profiled composite slabs. Furthermore, design methods are herein summarized for various pertinent failure modes.

Applications

Composite construction has been mainly applied to bridges and multistory buildings, with the more traditional forms of composite beams and composite columns. This section will look at the various applications of composite construction to both bridges and buildings.

Bridges

Composite construction with bridges allows the designer to take full advantage of the steel section in tension by shifting the compression force into the concrete slab in sagging bending. This is made possible through the transfer of longitudinal shear force through traditional headed-stud shear connectors. Headed-stud shear connectors not only provide the transfer of shear force, but also help to assist lateral stability of the section. The top flange of the steel section is essentially fully laterally restrained by the presence of shear connectors at very close spacing, as illustrated in Fig. 51.1.

Buildings

In steel-framed buildings throughout the world, composite floors are essentially the status quo in order to achieve an economic structure. This is for quite a few reasons. First, composite slabs allow reduced construction time by eliminating the need for propping and falsework in the slab-pouring phase. Furthermore, composite beams are economical, as they reduce the structural depth of the floor and thereby increase the available floors in a given building.

FIGURE 51.1 Composite box girders, Hawkesbury River Bridge, Australia.
Other Structures
In addition to bridges and buildings, composite slab and beam systems have seen considerable application in car park structures. Steel and steel–concrete composite construction provide a lighter structure with reduced foundation loads, as shown in Fig. 51.3.

Case Studies
Grosvenor Place, Sydney
Grosvenor Place is considered to be one of the more prestigious office buildings in Sydney, which integrates modern technology within the building fabric to allow office inhabitants great flexibility in the manner in which it is occupied. The structural system of the building consists of an elliptical core with radial steel beams, which span to a perimeter steel frame. These composite beams span up to 15 m and are designed to be composite for strength and serviceability. Furthermore, the beams also take account of semirigidity by a specially designed connection to the elliptical core.
The slabs are designed as one-way slabs, which consist of profiled steel sheeting spanning compositely between steel beams. The steel perimeter columns were designed as steel columns, although they are encased in concrete for fire resistance purposes, and are not designed compositely. The building is shown during construction in Fig. 51.4.

Forrest Place, Perth

Forrest Place is a multistory steel building that was designed with a rectangular concrete core to resist lateral loads and is combined with a perimeter steel frame, which consists of concrete-filled steel box columns. The beams were designed as steel–concrete composite beams, and the slabs are composite, utilizing permanent metal deck formwork. Elements of the building during construction are shown in Fig. 51.5.

Republic Plaza, Singapore

Republic Plaza is one of the tallest buildings in Singapore and thus required an efficient structural system for both gravity and lateral loading. The building consists of an internal reinforced concrete shear core, and beams span to an external perimeter frame, which is actually coupled to the core for the purposes of lateral load resistance. The perimeter frame consists of concrete-filled steel tubes that are designed compositely, as illustrated in Fig. 51.6.

One Raffles Link, Singapore

This is an eight-story building with wide-span column-free space specially tailored for banking and financial sector clients. The composite floor slab is supported by prefabricated cellform beams, which act as main girders, and standard sections as secondary floor beams. The 18-m span girders comprise 1300-mm-deep cellform sections with regularly spaced 900-mm-diameter circular web openings, spaced at 1350-mm centers. The beams are fabricated from 914-deep, 305-wide standard I sections, cut and welded to achieve the desired depth. The cellform beam, as shown in Fig. 51.7, was preferred because it is lightweight and permits the passing of all building services through the beam web. It therefore dispenses with the usual requirement of providing a dedicated services zone beneath the beams. The service cores of the building have been utilized for resisting lateral loads. This design approach allowed the entire structural steel frame to be designed and detailed as pin connected.
51.2 Composite Construction Systems for Buildings

Composite Floor Systems

Composite floor systems typically involve structural steel beams, joists, girders, or trusses made composite via shear connectors, with a concrete floor slab to form an effective T-beam flexural member resisting primarily gravity loads [2]. The versatility of the system results from the inherent strength of the concrete floor component in compression and the tensile strength of the steel member. The main advantages of combining the use of steel and concrete materials for building construction are:

- Steel and concrete may be arranged to produce an ideal combination of strength, with concrete efficient in compression and steel in tension.
- Composite systems are lighter in weight (about 20 to 40% lighter than concrete construction). Because of their light weight, site erection and installation are easier, and thus labor costs can be minimized. Foundation costs can also be reduced.
The construction time is reduced, since casting of additional floors may proceed without having to wait for the previously cast floors to gain strength. The steel decking system provides positive moment reinforcement for the composite floor, requires only small amounts of reinforcement to control cracking, and provides fire resistance.

The construction of composite floors does not require highly skilled labor. The steel decking acts as permanent formwork. Composite beams and slabs can accommodate raceways for electrification, communication, and air distribution systems. The slab serves as a ceiling surface to provide easy attachment of a suspended ceiling.

The composite slab, when fixed in place, can act as an effective in-plane diaphragm, which may provide effective lateral bracing to beams.

Concrete provides corrosion and thermal protection to steel at elevated temperatures. Composite slabs of a 2-h fire rating can be easily achieved for most building requirements.

The floor slab may be constructed by the following methods:

- a flat-soffit reinforced concrete slab (Fig. 51.8(a))
- precast concrete planks with cast in situ concrete topping (Fig. 51.8(b))
- precast concrete slab with in situ grouting at the joints (Fig. 51.8(c))
- a metal steel deck with concrete, either composite or noncomposite (Fig. 51.8(d))

The composite action of the metal deck results from side embossments incorporated into the steel sheet profile. The composite floor system produces a rigid horizontal diaphragm, providing stability to the overall building system, while distributing wind and seismic shears to the lateral load-resisting systems.
Composite Beams and Girders

Steel and concrete composite beams may be formed by shear connectors connecting the concrete floor to the top flange of the steel member. Concrete encasement will provide fire resistance to the steel member. Alternatively, direct sprayed-on cementitious and board-type fireproofing materials may be used economically to replace the concrete insulation on the steel members. The most common arrangement found in composite floor systems is a rolled or built-up steel beam connected to a formed steel deck and concrete slab (Fig. 51.8(d)). The metal deck typically spans unsupported between steel members, while also providing a working platform for concreting work.

Figure 51.9(a) shows a typical building floor plan using composite steel beams. The stress distribution at working loads in a composite section is shown schematically in Fig. 51.9(b). The neutral axis is normally located very near to the top flange of the steel section. Therefore, the top flange is lightly stressed. From a construction point of view, a relatively wide and thick top flange must be provided for proper installation of shear studs and metal decking. However, the increased fabrication costs must be evaluated, which tend to offset the savings from material efficiency.

A number of composite girder forms allow passage of mechanical ducts and related services through the depth of the girder (Fig. 51.10). Successful composite beam design requires the consideration of various serviceability issues, such as long-term (creep) deflections and floor vibrations. Of particular concern is the occupant-induced floor vibrations. The relatively high flexural stiffness of most composite floor framing systems results in relatively low vibration amplitudes, and therefore is effective in reducing perceptibility. Studies have shown that short- to medium-span (6- to 12-m) composite floor beams perform quite well and have rarely been found to transmit annoying vibrations to the occupants. Particular care is required for long-span beams of more than 12 m.
Long-Span Flooring Systems

Long spans impose a burden on the beam design in terms of a larger required flexural stiffness for serviceability design. Besides satisfying serviceability and ultimate strength limit states, the proposed system must also accommodate the incorporation of mechanical services within normal floor zones. Several practical options for long-span construction are available, and they are discussed in the following subsections.

Beams with Web Openings

Standard castellated beams can be fabricated from hot-rolled beams by cutting along a zigzag line through the web. The top and bottom half-beams are then displaced to form castellations (Fig. 51.11). Castellated composite beams can be used effectively for lightly serviced buildings. Although composite action does not increase the strength significantly, it increases the stiffness, and hence reduces deflection and the problem associated with vibration. Castellated beams have limited shear capacity and are best used as long-span secondary beams where loads are low or where concentrated loads can be avoided. Their use may be limited due to the increased fabrication cost and the fact that the standard castellated openings are not big enough to accommodate the large mechanical ductwork common in modern high-rise buildings.

Horizontal stiffeners may be required to strengthen the web opening, and they are welded above and below the opening. The height of the opening should not be more than 70% of the beam depth, and the length should not be more than twice the beam depth. The best location for the opening is in the low shear zone of the beams. This is because the webs do not contribute much to the moment resistance of the beam.
Fabricated Tapered Beams

The economic advantage of fabricated beams is that they can be designed to provide the required moment and shear resistance along the beam span in accordance with the loading pattern along the beam. Several forms of tapered beams are possible. A simply supported beam design with a maximum bending moment at the midspan would require that they all effectively taper to a minimum at both ends (Fig. 51.12), whereas a rigidly connected beam would have a minimum depth toward the midspan. To make the best use of this system, services should be placed toward the smaller depth of the beam cross sections. The spaces created by the tapered web can be used for running services of modest size (Fig. 51.12).

A hybrid girder can be formed with the top flange made of lower strength steel than the steel grade used for the bottom flange. The web plate can be welded to the flanges by double-sided fillet welds. Web stiffeners may be required at the change of section when the taper slope exceeds approximately 6º. Stiffeners are also required to enhance the shear resistance of the web, especially when the web slenderness ratio is too high. Tapered beams are found to be economical for spans up to 20 m.

Haunched Beams

Haunched beams are designed by forming a rigid moment connection between the beams and columns. The haunch connections offer restraints to the beam and help reduce midspan moment and deflection. The beams are designed in a manner similar to that of continuous beams. Considerable economy can be gained in sizing the beams using continuous design, which may lead to a reduction in beam depth up to 30% and deflection up to 50%.

The haunch may be designed to develop the required moment, which is larger than the plastic moment resistance of the beam. In this case, the critical section is shifted to the tip of the haunch. The depth of the haunch is selected based on the required moment at the beam-to-column connections. The length of haunch is typically 5 to 7% of the span length for nonsway frames or 7 to 15% for sway frames. Service ducts can pass below the beams (Fig. 51.13).

Haunched composite beams are usually used in the case where the beams frame directly into the major axis of the columns. This means that the columns must be designed to resist the moment transferred from the beam to the column. Thus a heavier column and more complex connection would be required than would be with a structure designed based on the assumption that the connections are pinned. The
rigid frame action derived from the haunched connections can resist lateral loads due to wind without
the need for vertical bracing. Haunched beams offer higher strength and stiffness during the steel erection
stage, thus making this type of system particularly attractive for long-span construction. However,
haunched connections behave differently under positive and negative moments, as the connection con-
figuration is asymmetrical about the axis of bending.

**Parallel Beam System**

This system consists of two main beams, with secondary beams running over the top of the main beams
(see Fig. 51.14). The main beams are connected to either side of the column. They can be made continuous
over two or more spans supported on stubs and attached to the columns. This will help in reducing the
construction depth and thus avoid the usual beam-to-column connections. The secondary beams are
designed to act compositely with the slab and may also be made to span continuously over the main
beams. The need to cut the secondary beams at every junction is thus avoided. The parallel beam system
is ideally suited for accommodating large service ducts in orthogonal directions (Fig. 51.14). Small savings
in steel weight are expected from the continuous construction because the primary beams are noncom-
posite. However, the main beam can be made composite with the slab by welding beam stubs to the top
flange of the main beam and connecting them to the concrete slab through the use of shear studs (see
Stub Girder System below). The simplicity of connections and ease of fabrication make this long-span
beam option particularly attractive.

**Composite Trusses**

Composite truss systems can be used to accommodate large services. Although the cost of fabrication is
higher in material cost, truss construction can be cost-effective for very long spans when compared with
other structural schemes. One disadvantage of the truss configuration is that fire protection is labor-
intensive, and sprayed protection systems cause a substantial mess to the services that pass through the
web opening (see Fig. 51.15).

**FIGURE 51.14** Parallel composite beam system.

**FIGURE 51.15** Composite truss.
The resistance of a composite truss is governed by: (1) yielding of the bottom chord, (2) crushing of the concrete slab, (3) failure of the shear connectors, (4) buckling of the top chord during construction, (5) buckling of web members, and (6) instability occurring during and after construction. To avoid brittle failures, ductile yielding of the bottom chord is the preferred failure mechanism. Thus the bottom chord should be designed to yield prior to crushing of the concrete slab. The shear connectors should have sufficient capacity to transfer the horizontal shear between the top chord and the slab. During construction, adequate plan bracing should be provided to prevent top chord buckling. When considering composite action, the top steel chord is assumed not to participate in the moment resistance of the truss, since it is located very near to the neutral axis of the composite truss and thus contributes very little to the flexural capacity.

**Stub Girder System**

The stub girder system involves the use of short beam stubs, which are welded to the top flange of a continuous, heavier bottom girder member and connected to the concrete slab through the use of shear studs. Continuous transverse secondary beams and ducts can pass through the openings formed by the beam stub. The natural openings in the stub girder system allow the integration of structural and service zones in two directions (Fig. 51.16), permitting story height reduction, compared with some other structural framing systems.

Ideally, stub girders span about 12 to 15 m, in contrast to the conventional floor beams, which span about 6 to 9 m. The system is therefore very versatile, particularly with respect to secondary framing spans, with beam depths being adjusted to the required structural configuration and mechanical requirements. Overall girder depths vary only slightly, by varying the beam and stub depths. The major disadvantage of the stub girder system is that it requires temporary props at the construction stage, and these props have to remain until the concrete has gained adequate strength for composite action. However, it is possible to introduce an additional steel top chord, such as a T section, which acts in compression to develop the required bending strength during construction. For span lengths greater than 15 m, stub girders become impractical, because the slab design becomes critical.

In the stub girder system, the floor beams are continuous over the main girders and splices at the locations near the points of inflection. The sagging moment regions of the floor beams are usually designed compositely with the deck slab system, to produce savings in structural steel as well as provide stiffness. The floor beams are bolted to the top flange of the steel bottom chord of the stub girder, and two shear studs are usually specified on each floor beam, over the beam–girder connection, for anchorage to the deck slab system. The stub girder may be analyzed as a Vierendeel girder, with the deck slab acting as a compression top chord, the full-length steel girder as a tensile bottom chord, and the steel stubs as vertical web members or shear panels.

**Prestressed Composite Beams**

Prestressing of steel girders is carried out such that the concrete slab remains uncracked under working loads and the steel is utilized fully in terms of stress in the tension zone of the girder.

Prestressing of steel beams can be carried out using a precambering technique, as depicted in Fig. 51.17. First, a steel girder member is prebent (Fig. 51.17(a)); then it is subjected to preloading in the direction against the bending curvature until the required steel strength is reached (Fig. 51.17(b)). Second, the
The lower flange of the steel member, which is under tension, is encased in a reinforced concrete chord (Fig. 51.17(c)). The composite action between the steel beam and the concrete slab is developed by providing adequate shear connectors at the interface. When the concrete gains adequate strength, the steel girder is prestressed by stress-relieving the precompressed tension chord (Fig. 51.17(d)). Further composite action can be achieved by supplementing the girder with in situ or prefabricated reinforced concrete slabs; this will produce a double composite girder (Fig. 51.17(e)).

The main advantage of this system is that the steel girders are encased in concrete on all sides: no corrosion or fire protection is required for the sections. The entire process of precambering and prestressing can be performed and automated in a factory. During construction, the lower concrete chord cast in the works can act as formwork. If the distance between two girders is large, precast planks can be supported by the lower concrete chord, which is used as a permanent formwork.

Prestressing can also be achieved by using tendons that can be attached to the bottom chord of a steel composite truss or the lower flange of a composite girder to enhance the load-carrying capacity and stiffness of long-span structures (Fig. 51.18). This technique is popular for bridge construction in Europe and the U.S., but it is less common for building construction.

**Composite Column Systems**

Composite columns have been used for over 100 years, with steel-encased sections similar to that shown in Fig. 51.19(a) being incorporated in multistory buildings in the United States during the late nineteenth
century [1]. The initial application of composite columns was for fire rating requirements of the steel section [3]. Later developments saw the composite action fully utilized for strength and stability [4,5]. Composite action in columns utilizes the favorable tensile and compressive characteristics of the steel and concrete, respectively. These types of columns are still in use today where steel sections are used as erection columns, with reinforced concrete cast around them as shown in Fig. 51.19(b). One major benefit of this system has been the ability to achieve higher steel percentages than conventional reinforced concrete structures, and the steel erection column allows rapid construction of steel floor systems in steel-framed buildings.

Concrete-filled steel columns, as illustrated in Fig. 51.20, were developed much later during the last century but are still based on the fundamental principle that steel and concrete are most effective in tension and compression, respectively. The major benefits also include constructability issues, whereby the steel section acts as permanent and integral formwork for the concrete. These columns were initially researched during the 1960s, with the use of hot-rolled steel sections filled with concrete considered in Neogi et al. [6] and Knowles and Park [7,8]. These sections, while studied extensively, were essentially expensive, as the steel section itself was designed to be hollow, thus requiring large steel plate thicknesses. This lack of constructional economy has seen the use of concrete-filled steel columns limited in their application throughout the world. Furthermore, restrictive cross section sizes have rendered them unsuitable for application in tall buildings, where demand on axial strength is high.

Japan has been an exception to the rule in regard to the application of concrete-filled steel columns. Widespread use of thick steel tubes or boxes has been invoked to provide confinement for the concrete and thus achieve greater ductility, which is desirable for cyclic loading experienced during an earthquake. The use of concrete-filled steel columns was initially justified after the Great Kanto Earthquake in 1923, when it was found that existing composite structures were relatively undamaged. This has resulted in more than 50% of the building structures of over five stories in Japan being framed with composite steel–concrete columns, as described by Wakabayashi [9].

Recently in Australia, Singapore, and other developed nations, concrete-filled steel columns have experienced a renaissance in their use. The major reasons for this renewed interest are the savings in construction time, which can be achieved with this method. The major benefits include:

- The steel column acts as permanent and integral formwork.
- The steel column provides external reinforcement.
- The steel column supports several levels of construction prior to concrete being pumped.

A comparison of typical costs of column construction has been compiled by Australian consulting engineers, Webb and Peyton [10], and this is summarized in Table 51.1. This reveals the competitive nature of the concrete-filled steel column with or without reinforcement when compared with conventional reinforced concrete columns for buildings over 30 levels. This statistic will be more favorable for concrete-filled steel columns in buildings of over 50 stories, which are becoming common in many densely populated cities throughout the world [2].
A considerable amount of research has been conducted on this form of column construction, and the main objective has been to reduce the steel plate thickness. The optimization of the steel thickness requires a clear understanding of the behavior during all stages of loading. These aspects will be outlined in this chapter, together with reference to international codes, where design guidance can be provided. In particular, attention is made to fundamental aspects that have not yet been implemented in international codes and that often affect the performance of these members in practice.

### 51.3 Material Properties

The principal material properties that need to be considered in composite members include structural steel, concrete, reinforcing steel, and profiled steel sheeting, as well as the properties of the shear connectors, which are generally stud shear connectors. Each of these materials will be discussed, and typical pertinent properties used internationally will be described.

#### Mild Structural Steel

Mild structural steel typical of hot-rolled steel sections exhibits the stress–strain characteristics shown in Fig. 51.21, which shows an elastic region, followed by a plastic plateau, that extends for approximately ten times the yield strain. This is then followed by a strain-hardening region leading to a maximum ultimate stress. The ultimate stress is maintained until the material reaches an ultimate strain, which is sometimes close to 150 times the yield strain, thus exhibiting extremely ductile behavior.

For structural steel of composite sections, the common steel grades as outlined in Eurocode 4 (EC4) [11] are given in Table 51.2. The steel sections may be hot or cold rolled. Nominal values of the yield strength, $f_y$, and the ultimate tensile strength, $f_u$, for structural steel are presented in Table 51.2.

Other material properties related to steel design are:

- Modulus of elasticity, $E$: 210 kN/mm$^2$
- Shear modulus, $G$: $E/[2(1 + \nu_s)]$
- Poisson’s ratio, $\nu_s$: 0.3
- Density, $\rho_s$: 7850 kg/m$^3$

![Idealized stress–strain curve for mild structural steel.](image)

**TABLE 51.1 Comparison of Column Costs**

<table>
<thead>
<tr>
<th>Type of Column</th>
<th>Reinforced Concrete</th>
<th>Concrete with Steel Erection Column</th>
<th>Concrete-Encased Steel Strut</th>
<th>Tube Filled with Reinforced Concrete</th>
<th>Steel Tube Filled with Concrete</th>
<th>Full Steel Column</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relative cost, 10 levels</td>
<td>1.0</td>
<td>1.22</td>
<td>1.53</td>
<td>1.14</td>
<td>1.10</td>
<td>2.27</td>
</tr>
<tr>
<td>Relative cost, 30 levels</td>
<td>1.0</td>
<td>1.13</td>
<td>1.85</td>
<td>1.11</td>
<td>1.02</td>
<td>2.61</td>
</tr>
</tbody>
</table>

High-Strength Steel

The idealized stress–strain curve for high-strength structural steel is shown in Fig. 51.22, which shows an elastic range and a plastic plateau with no significant strain hardening occurring for the material. Typical values of yield strengths for high-strength steel are about 700 MPa.

Unconfined Concrete

In composite structures, concrete can be in an unconfined state of stress in compression generally when used as a slab component of a composite beam. In the modeling of these types of structures, it is important to have a model that represents the concrete stress as a function of strain. The Comite Europeen du Beton (CEB-FIP) [13] model for stress–strain has been used in the past and is shown in Fig. 51.23. Other models exist and can be found in most international codes on concrete structures.

Concrete strengths as defined in Eurocode 4 are based on the characteristic cylinder strength, $f_{ck}$, measured at 28 days. Clause 3.1.2.2 of EC4 also gives the different strength classes and associated cube strengths, as shown in Table 51.3. The classification of concrete, such as C20/25, refers to the cylinder/cube concrete strength at the specified age.

For normal-weight concrete the mean tensile strength, $f_{ctm}$, and the secant modulus of elasticity, $E_{cm}$, for short-term loading are also given in Table 51.3. For lightweight concrete, the secant moduli are obtained by multiplying the $E_{cm}$ value by a factor of $(\rho/2400)^2$, where $\rho$ is the density of lightweight concrete.

Confined Concrete

Concrete in composite structures may be confined in a triaxial state of stress when used in applications such as concrete-filled steel sections. A model to consider this behavior for rectangular or square sections

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**TABLE 51.2** Nominal Values of Strength of Structural Steels to BS EN 10025

<table>
<thead>
<tr>
<th>Nominal Steel Grade</th>
<th>$f_y$ (N/mm²)</th>
<th>$f_u$ (N/mm²)</th>
<th>$f_y$ (N/mm²)</th>
<th>$f_u$ (N/mm²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fe 360</td>
<td>235</td>
<td>360</td>
<td>215</td>
<td>340</td>
</tr>
<tr>
<td>Fe 430</td>
<td>275</td>
<td>430</td>
<td>255</td>
<td>410</td>
</tr>
<tr>
<td>Fe 510</td>
<td>355</td>
<td>510</td>
<td>335</td>
<td>490</td>
</tr>
</tbody>
</table>

The Civil Engineering Handbook, Second Edition has been developed by Tomii [15]; it is illustrated in Fig. 51.24. Other models also exist for concrete-filled steel tubes and will be discussed in relation to some of the existing international standards.

Reinforcing Steel

Reinforcing steel is often used as tensile reinforcement in the hogging moment regions of continuous composite beams, as well as for crack control in the slabs of simply supported composite beams. For

### TABLE 51.3 Properties of Concrete according to EC2-1990

<table>
<thead>
<tr>
<th>Strength Class</th>
<th>C20/25</th>
<th>C25/30</th>
<th>C30/37</th>
<th>C35/45</th>
<th>C40/50</th>
<th>C45/55</th>
<th>C50/60</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_{ck}$ (N/mm²)</td>
<td>20</td>
<td>25</td>
<td>30</td>
<td>35</td>
<td>40</td>
<td>45</td>
<td>50</td>
</tr>
<tr>
<td>$f_{cmt}$ (N/mm²)</td>
<td>2.2</td>
<td>2.6</td>
<td>2.9</td>
<td>3.2</td>
<td>3.5</td>
<td>3.8</td>
<td>4.1</td>
</tr>
<tr>
<td>$E_{cm}$ (N/mm²)</td>
<td>29,000</td>
<td>30,500</td>
<td>32,000</td>
<td>33,500</td>
<td>35,000</td>
<td>36,000</td>
<td>37,000</td>
</tr>
</tbody>
</table>


![Graph showing stress-strain relationship for concrete](image)


![Graph showing model for confined concrete](image)

**FIGURE 51.24** Model for confined concrete. From Tomii, M., in paper presented at 3rd International Conference on Steel–Concrete Composite Structures, ASCCS, Fukuoka, Japan, September 1991.

has been developed by Tomii [15]; it is illustrated in Fig. 51.24. Other models also exist for concrete-filled steel tubes and will be discussed in relation to some of the existing international standards.
continuous composite beams where large rotational capacity is required, ductile reinforcing steel is necessary. Eurocode 4 specifies the types of reinforcing steel that may be used in composite structures. Standardized grades are defined in EN 10080 [16], which is the product standard for reinforcement. Types of reinforcing steel are classified as follows:

- high (class H) or normal (class N) according to ductility characteristics
- plain smooth or ribbed bars according to surface characteristics

Steel grades commonly used in the construction industry are given in Table 51.4.

### Profiled Steel Sheeting

Profiled steel sheeting in composite slabs is often made of cold-formed steel sheeting, which exhibits highly nonlinear stress–strain characteristics, particularly near the proof stress, \( \sigma_p \). The Ramberg–Osgood [18] model is often used to represent the stress–strain characteristics of cold-formed steel. For this model, stress is represented as a function of strain in the form of

\[
\varepsilon = \frac{\sigma}{E} + \varepsilon_p \left( \frac{\sigma}{\sigma_p} \right)^n
\]  

(51.1)

Piecewise linearization is often used in analysis to idealize the stress–strain curves to allow the stress, \( \sigma \), to be uniquely represented as a function of the strain, \( \varepsilon \). However, a proof yield stress is usually used for ultimate strength design.

### Shear Connectors

Shear connectors may exist in quite a few varieties, which include headed shear studs, steel angles, and high-strength friction grip bolts. However, it is the headed shear stud connectors that have seen the greatest application, and these will be outlined herein. In the design of the shear connection in composite structures, the designer is mainly interested in the strength that each stud can transfer in shear. Empirical relationships for the shear resistance of headed shear studs exist in various international codes of practice.

The Australian Standard (AS) AS 2327.1-1996 [19] represents the strength of the shear connectors by the lesser of one of the following two expressions:

\[
f_{vs} = 0.63d_{bs}^2f_{uc}
\]  

(51.2)

\[
f_{vs} = 0.31d_{bs}^2\sqrt{f'_{cc}E_c}
\]  

(51.3)

where \( d_{bs} \) = the diameter of the shank of the stud  
\( f_{uc} \) = the ultimate strength of the material of the stud

### Table 51.4

<table>
<thead>
<tr>
<th>Reinforcing Steel Grades</th>
<th>BS 4449 [17] and BS 4483</th>
<th>BS EN 10080</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f_{sk} ) (N/mm²)</td>
<td>460</td>
<td>500</td>
</tr>
<tr>
<td></td>
<td>250</td>
<td>Not included</td>
</tr>
<tr>
<td>Ductility</td>
<td>Not covered</td>
<td>Classes H and N</td>
</tr>
<tr>
<td>( E_s ) (N/mm²)</td>
<td>210,000</td>
<td>210,000</td>
</tr>
</tbody>
</table>

The characteristic cylinder strength of the concrete

\( E_c = \) the mean value of the secant modulus of the concrete

Equation (51.2) represents the strength of the shear stud if it fails by fracture of the weld collar, whereas Eq. (51.3) represents concrete cone failure surrounding the stud. The design shear resistance of studs in Eurocode 4 for the same failure modes is given by the following:

\[
P_{rd} = 0.8 f_u \left( \pi d^2 / 4 \right) / \gamma_{Mv}
\]

(51.4)

\[
P_{rd} = 0.29 \alpha d^2 \left( f_{ck} E_{cm} \right)^{1/2} / \gamma_{Mv}
\]

(51.5)

where

- \( d = \) the diameter of the shank of the stud
- \( f_u = \) the ultimate strength of the material of the stud
- \( f_{ck} = \) the characteristic cylinder strength of the concrete
- \( E_{cm} = \) the mean value of the secant modulus of the concrete
- \( h = \) the overall height of the stud
- \( \gamma_{Mv} = \) a partial safety factor (taken as 1.25 for the ultimate limit state)
- \( \alpha = 0.2[(h/d) + 1] \) for \( 3 \leq h/d \leq 4 \) and \( = 1.0 \) for \( h/d > 4 \)

51.4 Design Philosophy

Limit States Design

The design philosophy adopted by most international codes throughout the world is one of limit states. The Australian and North American Standards are limit states design or load resistance factor design approaches, whereas the Eurocodes are based on partial safety factor approaches.

In general structural design requirements relate to corresponding limit states, so that the design of a structure that satisfies all the appropriate requirements is termed a limit states design.

Structural design criteria may be determined by the designer, or he or she may use those stated or implied in design codes. The stiffness design criteria are usually related to the serviceability limit state. These may include excessive deflections, vibration, noise transmission, member distortion, etc.

Strength limit states pertain to possible methods of failure or overload and include yielding, buckling, brittle fracture, or fatigue.

The errors and uncertainties involved in the estimation of loads and on the capacity of structures may be accounted for by using appropriate load factors to increase the nominal loads \( (S) \) and capacity reduction factors \( (\phi) \) to reduce the member strength \( (R_u) \). For strength the generic limit states design equation can be represented in the form

\[
S^* \leq \phi R_u
\]

(51.6)

51.5 Composite Slabs

This section will deal with the design of composite slabs in the composite stage. Composite slabs in the noncomposite stage are essentially cold-formed steel structures, and the design of these elements is covered in Chapter 49 “Cold Formed Steel Structures” of this handbook.

Serviceability

Serviceability of composite slabs involves the consideration of the following key issues: deflections, vibrations, and crack control.
Deflections

Deflections of composite slabs are treated very similar to deflections of reinforced concrete slabs. However, this section will reiterate these methods, together with looking at the international standards that already exist in the design of these elements.

In determining the deflections it is important to be able to calculate the effective second moment of area of the composite section. A fully cracked section analysis often overestimates the deformations of a reinforced concrete slab, and subsequently those for a profiled composite slab, for relatively low values of the applied load above the cracking moment of the section. A tension-stiffening model is therefore used here that is related to the transformed cracked and uncracked second moments of area, as well as the ratio of the applied service moment to the cracking moment of the cross section being considered. The model is based on that of Branson [20], except that the uncracked second moment of area \( I_u \) replaces this in the following analysis. The effective second moment of area \( I_{eff} \) is given by

\[
I_{eff} = I_{cr} + (I_u - I_{cr}) \left( \frac{M_{cr}}{M_s} \right)^3
\]

Both BS 5950, Part 4 [21], and ANSI/ASCE 3-91 [22] allow the consideration of a simplified effective second moment of area as

\[
I_{eff} = \frac{I_g + I_{cr}}{2}
\]

where \( I_g \), \( I_u \), and \( I_{cr} \) are the gross, uncracked, and cracked second moments of area, respectively. For determining deflections the transformed section properties are required. In the absence of a more rigorous analysis, the effects of creep may be taken into account by using modular ratios for the calculation of flexural stiffness.

\[
n = \frac{E_a}{E'_c}
\]

where \( E_a \) = the elastic modulus of structural steel; \( E'_c \) = an effective modulus of concrete. \( E'_c = E_{cm} \) for short-term effects; \( E'_c = E_{cm}/3 \) for long-term effects; \( E'_c = E_{cm}/2 \) for other cases.

Vibrations

For long-span composite slabs, which are those types of slabs with deep troughs, it may be necessary to determine the vibrations of the slab and compare these with acceptable vibrations. Where vibration could cause discomfort or damage the response of long-span composite floors should be considered using SCI Publication 076, “Design Guide on the Vibration of Floors” [23].

Crack Control

Crack control requirements are important criteria for composite slabs, particularly when continuous composite slabs are used. Typical crack control requirements are covered by most international reinforced concrete structures codes, and these are also covered in Chapter 50 “Structural Concrete Design” of this handbook.

Strength

Flexural Failure

A rigid plastic assumption is often used to determine the flexural strength of a composite slab. This method assumes the profiled steel sheeting to be at full yield in tension, with the concrete slab assumed to be fully crushed in compression, as shown in Fig. 51.25.
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Assuming the slab is only singly reinforced, the ultimate moment, calculated by summing moments about either the tensile force or compressive force location, is given by

\[ M_u = Ccl = Ts l \]  \hspace{1cm} (51.10)

where \( l \) is the lever arm between the compressive and tensile forces. This method of analysis, based on the rectangular stress block principle, is the method adopted by both the British Standard (BS), BS 5950, Part 4, and the American Standard, ANSI/ASCE 3-91. BS 5950 assumes that the concrete strength is given by 0.4 times the cube strength and the yield strength of the steel is taken. However, this method assumes that there exists a full shear connection between the profiled steel sheeting and the concrete in the tension zone. This is usually the case when a sufficient mechanical and friction bond is developed for the profile in question. Other modes of failure, which may also exist, include longitudinal slip failure and vertical shear failure of the concrete.

**Longitudinal Shear Failure**

When a composite slab exhibits partial shear connection, slip of the sheeting will occur prior to the steel sheeting yielding and the concrete crushing. The strength of the composite slab is thereby governed by the shear bond capacity between the steel sheeting and the concrete. In many countries throughout the world this is based on manufacturer data, and empirical methods of analysis are often applied. The reader is referred to those methods for a more accurate method of analysis.

**Vertical Shear Failure**

Composite slabs may fail by vertical shear, in much the same manner as reinforced concrete slabs. For this failure mode the maximum vertical shear capacity can be evaluated, provided sufficient test data are available. The ultimate vertical shear strength as defined by BS 5950 is given as

\[ V_u = 0.8A \left( m_d \frac{d}{L_v} + k_d \sqrt{f_{ct}} \right) \]  \hspace{1cm} (51.11)

where \( p \) is the ratio of the cross-sectional area of the profile to that of the concrete \( A \) per unit width of slab, \( f_{cu} \) is the cube strength of the concrete, \( d \) is the effective slab depth to the centroid of the profile, and \( L_v \) is the shear span length, taken as one quarter of the slab span. The constants \( m_d \) and \( k_d \) are calculated from the slope and intercept, respectively, of the reduced regression line established from the testing of composite slabs.

A similar approach is existent in the American Standard ANSI/ASCE 3-91, which relies on test data and gives an experimentally determined shear strength as

\[ V_v = b \left( \frac{m_d d}{L_v} + k_d \sqrt{f_{ct}} \right) \]  \hspace{1cm} (51.12)

The approach given in Eurocode 4 to determine the vertical shear resistance of a composite slab is

\[ V_{v,rd} = b_d d \tau_{ld} k_d (1.2 + 40p) \]  \hspace{1cm} (51.13)
where \( b_v \) = the mean width of the concrete ribs
\( \tau_{bd} \) = the basic shear strength to be taken as 0.25\( f_{ck} / \gamma_c \) (\( f_{ck} \) is the lower 5 percentile confidence limit characteristic strength)
\( \rho = A_p / b_{df} \) \(< 0.02 \) (\( A_p \) is the effective area of the steel sheet in tension)
\( k_v = (1.6 - d_p) = 1.0 \), with \( d_p \) expressed in meters

**Ductility**

Ductility of composite slabs is also a very important consideration, although it appears that many of the existent international codes throughout the world do not have an inherent ductility clause, which is reflected in the design of reinforced concrete slabs. The codes investigated include BS 5950, EC4, and ANSI/ASCE. It is suggested that in the absence of current recommendations for ductility that the following consideration be given for the design of simply supported composite slabs, which limits the depth of the neutral axis so that

\[
d_n \leq 0.4d
\]

This is the ductility requirement for reinforced concrete slabs used in the Australian Standard for concrete structures, AS 3600 [24], and is thus also assumed to be applicable for composite slabs to ensure adequate ductility.

**51.6 Simply Supported Beams**

Simply supported composite steel–concrete beams are the original form of composite construction developed early in the 1900s. This section will consider the design of simply supported composite beams for serviceability, strength, and ductility. This section will mainly concentrate on the behavior of the beam in the composite stage, as the behavior of beams in the noncomposite stage is essentially the behavior of a steel beam, which is covered in Chapter 50 “Structural Concrete Design” of this handbook.

**Serviceability**

Deflections of simply supported composite beams need to incorporate the effects of both creep and shrinkage, in addition to the loading effects. These time-dependent effects are taken into account by generally transforming the concrete slab to an equivalent area of steel using a modular ratio. The modular ratio should include the effects of the disparate elastic moduli, as well as the effects of creep of concrete. Now, since the concrete is in the compression zone of simply supported composite beams in sagging bending, the concrete is considered to be fully effective; however, the effects of shear lag need to be determined using an effective breadth relationship. Effective widths from various international codes are included below

**AS 2327.1-1996**

The effective width, \( b_c \), of the concrete flange for positive bending in AS 2327.1-1996 for a beam in a regular floor system is determined as the minimum of the following

\[
b_c = \min \left( \frac{L_{cf}}{4}, b, b_{sf} + 16D_c \right)
\]

where \( L_{cf} \) is the effective span of the composite beam, \( b \) is the width between steel beams, \( b_{sf} \) is the width of the steel flange, and \( D_c \) is the depth of the concrete slab. For the determination of deflections in AS 2327.1-1996, the modular ratio is determined for immediate deflections using the value \((E_s/E_c)\), while for long term deflections, a modular ratio of 3 is suggested. The effective second moments of area of
composite beams for immediate and long-term deflections, respectively, are calculated in AS 2327.1-1996 as

\[
I_{eti} = I_i + 0.6(1 - \beta_m)(I_s - I_i)
\]

\[
I_{etl} = I_d + 0.6(1 - \beta_m)(I_s - I_d)
\]

where \(I_i\) and \(I_d\) are the transformed second moments of area of a composite beam under immediate and long-term loads, respectively; \(\beta_m\) is the level of shear connection, and \(I_s\) is the second moment of area of the steel section alone.

**BS 5950, Part 3 [25]**

The effective width of the concrete flange for a typical internal beam in this code should not be taken as greater than one quarter of the distance between points of contraflexure. The imposed load deflections in each span should be based on the loads applied to the span and the support moments for that span, modified as recommended to allow for pattern loading and shakedown. Provided that the steel beam is uniform without any haunches, the properties of the gross uncracked composite section should be used throughout. (This includes the use of a modular ratio to account for long-term effects.)

**Long-Term Effects (Creep and Shrinkage)**

Simplified methods for determining the cross section properties in BS 5950, Part 3, involve the use of a modular ratio. An effective modular ratio is expressed as

\[
\alpha_e = \alpha_s + \rho_i (\alpha_l - \alpha_s)
\]

where \(\alpha_l\) = the modular ratio for long-term loading

\(\alpha_s\) = the modular ratio for short-term loading

\(\rho_i\) = the proportion of the total loading, which is long term

**Deflection Due to Partial Shear Connection**

The increased deflection under serviceability loads arising from partial shear connection should be determined from the following expressions:

For propped construction,

\[
\delta = \delta_s + 0.5 \left(1 - \frac{N_a}{N_p}\right) (\delta_c - \delta_s)
\]

For unpropped construction,

\[
\delta = \delta_s + 0.3 \left(1 - \frac{N_a}{N_p}\right) (\delta_c - \delta_s)
\]

where \(\delta_s\) = the deflection of the steel beam acting alone

\(\delta_c\) = the deflection of a composite beam with a full shear connection for the same loading

\(N_a\) = the actual number of shear connectors provided

\(N_p\) = the number of shear connectors for full composite action

**Vibrations**

Where vibration could cause discomfort or damage the response of long-span composite floors should be considered using SCI Publication 076, “Design Guide on the Vibration of Floors” [23].
Eurocode 4

The deflection calculation provisions of Eurocode 4-1994 are given herein. For an internal beam, the effective width of the concrete flange for a typical internal beam in this code should not be taken as greater than one quarter of the distance between points of contraflexure.

Long-Term Effects

In the absence of a more rigorous analysis, the effects of creep may be taken into account by using modular ratios, as given in Section 3.1.4.2, for the calculation of flexural stiffness.

\[ n = \frac{E_a}{E'_c} \]  \hspace{1cm} (51.21)

where \( E_a \) is the elastic modulus of structural steel and \( E'_c \) is an effective modulus of concrete. \( E'_c = E_{cm} \) for short-term effects, \( E'_c = E_{cm}/3 \) for long-term effects, and \( E'_c = E_{cm}/2 \) for other cases.

Deflections of Beams

Deflections of the steel beam shall be calculated in accordance with EC3. Deflections of the composite beam shall be calculated using elastic analysis with corrections. Shear lag can be ignored for deflection calculations, except where \( b > L/8 \), then shear lag is included by determining the effective width of the flange according to Section 4.2.2.1. The effects of incomplete interaction may be ignored in spans or cantilevers where critical cross sections are either class 3 or 4. The effects of incomplete interaction may be ignored in unpropped construction, provided that shear connectors are designed according to Chapter 6: the shear connection ratio is greater than 0.50 or the forces on the shear connectors do not exceed 0.7\( P_{rk} \), or in the case of a ribbed slab with ribs transverse to the beam, the height of the ribs does not exceed 80 mm. If these conditions are violated but \( N/N_f = 0.4 \), then in lieu of testing or accurate analysis, the increased deflection arising from incomplete interaction may be determined from the following equations:

For propped construction,

\[ \frac{\delta}{\delta_c} = 1 + 0.5 \left( 1 - \frac{N}{N_f} \right) \left( \frac{\delta}{\delta_c} - 1 \right) \]  \hspace{1cm} (51.22)

For unpropped construction,

\[ \frac{\delta}{\delta_c} = 1 + 0.3 \left( 1 - \frac{N}{N_f} \right) \left( \frac{\delta}{\delta_c} - 1 \right) \]  \hspace{1cm} (51.23)

where \( \delta_c \) is the deflection for the composite beam with complete interaction

\( N/N_f \) is the degree of shear connection

Strength

The flexural strength of simply supported steel–concrete composite beams in sagging bending is determined using a rigid plastic method of analysis, where the concrete slab is assumed to be fully crushed in compression and the steel beam is assumed to be fully yielded in tension and compression, depending on the location of the plastic neutral axis, as well as the strength of the longitudinal shear connection. The following cases thus may exist.
Plastic Neutral Axis in the Concrete Slab (Full Shear Connection, $\beta = 1.0$)

When the concrete slab is stronger than the steel beams, the plastic neutral axis will lie within the concrete slab. For the case when the plastic neutral axis lies within the concrete slab, the ultimate flexural strength is determined from a simple couple, as shown in Fig. 51.26.

$$M_u = TL = CL$$  \hspace{1cm} (51.24)

Plastic Neutral Axis in the Steel Beam (Full Shear Connection, $\beta = 1.0$)

When the steel beam is stronger than the concrete slab, the plastic neutral axis for the beam with a full shear connection will lie within the steel beam. For this case it is more convenient to sum the moments about the centroid of the tension force, as illustrated in Fig. 51.27.

$$M_u = C_sL_s = C_sL_s$$  \hspace{1cm} (51.25)

Partial Shear Connection ($\beta < 1.0$)

For the case of partial shear connection of composite beams, the shear connection is the weakest element. Again, summing the moments on a convenient point on the cross section will yield the ultimate flexural moment of the beam, as illustrated in Fig. 51.28.

$$M_u = C_sL_s = C_sL_s$$  \hspace{1cm} (51.26)
Existing International Standards

Existing international standards that deal with the flexural strength of composite beams include the American Institute of Steel Construction Load and Resistance Design Specification (AISC-LRFD), Australian Standards (AS 2327.1-1996), British Standards (BS 5950, Part 3), and Eurocode 4-1994. While some of these standards have a closed-form solution for the flexural strength determination, it is best left in a more general form, in terms of stress blocks, as shown in Figs. 51.26 to 51.28, and for individuals to refer to the individual regional standards to determine the strength equations and apply the relevant load and capacity reduction factors. The most general manner in which to assess the flexural strength of a composite beam is as

\[ M^* \leq \phi M_u \]  

(51.27)

Influence of Shear on Flexural Strength

The influence of shear on the ultimate flexural strength of steel–concrete composite beams can be significant when the relative ratio of applied shear force to shear strength is high. Most of the design methods for this failure mode are based on test data, and an appropriate interaction equation is used by various international standards, such as the Australian and British Standards. The Australian Standard (AS 2327.1-1996) uses a linear relationship for reduction, which is largely based on the steel standard, whereas the British Standard uses a quadratic expression, based on test data. Both of these relationships are shown in Fig. 51.29.

AS 2327.1-1996:

\[ M_{u,v} = M_u - (M_u - M_{u,f})(2\gamma - 1) \]  

(51.28)

BS 5950, Part 3:

\[ M_{u,v} = M_u - (M_u - M_{u,f})(2\gamma - 1)^2 \]  

(51.29)

where \( \gamma \) = the ratio of shear force to shear strength

\( M_{u,v} \) = the ultimate flexural strength incorporating shear

\( M_u \) = the ultimate flexural strength with zero shear

\( M_{u,f} \) = the ultimate flexural strength of the beam considering the flanges only

FIGURE 51.29 Influence of shear on ultimate flexural strength of composite beams.
**Ductility**

None of the existing international codes have a ductility clause; however, it has been suggested by Rotter and Ansourian [26] that in order to achieve a plastic hinge, strain hardening in the bottom flange must develop, and from a treatment of the mechanics of the problem, they found that ductility can be guaranteed in a simply supported beam if a ductility parameter, $\chi$, is greater than 1. The ductility parameter is determined as

\[
\chi = \frac{0.85 f_c b_e \varepsilon_u (h_{conc} + h_{steel})}{A_{steel} f_y (\varepsilon_u + \varepsilon_{st})}
\]  

(51.30)

where 
- $f_c$ = the characteristic cylinder strength
- $b_e$ = the effective slab width
- $\varepsilon_u$ = the ultimate strain of concrete
- $h_{conc}$ = the depth of the slab
- $h_{steel}$ = the depth of the steel beam
- $A_{steel}$ = the cross-sectional area of the steel section
- $f_y$ = the yield strength of the steel
- $\varepsilon_{st}$ = the strain to cause strain hardening of the section

**51.7 Continuous Beams**

The design of continuous composite beams requires only an augmentation of the behavior of design of simply supported composite beams. In particular, the salient point in regard to serviceability and strength needs to take into account that composite beams in hogging bending behave completely differently than beams subjected to sagging bending. This is because in hogging bending the concrete slab is subjected to tension, and this will significantly affect both the stiffness and strength of the cross sections in hogging moment regions.

**Serviceability**

When considering serviceability effects in continuous composite beams, one must include the effects of cracking in the negative moment regions, as well as the effects of creep and shrinkage associated with long-term loading. Since continuous beams are indeterminate, it is difficult to develop a general approach that is amenable for design that reflects the exact behavior. Existing code methods provide a good basis for simplifying the problem to account for the indeterminacy, as well as the nonuniform flexural rigidity, that exists along the length of a beam. These methods will be outlined herein.

**BS 5950, Part 3**

**Calculation of Moments**

The calculation of moments for supports can be determined using the following two methods.

*Pattern Loading and Shakedown* — The support moments required for these cases should be based on an elastic analysis using the properties of the gross uncracked section throughout.

*Simplified Method* — The moments in continuous composite beams for serviceability may be determined using the coefficients below, provided that the following conditions are satisfied: the steel beam should be of uniform section with no haunches; the steel beam should be of the same section in each span; loading should be uniformly distributed; live loads should not exceed 2.5 times the dead load; no span should be less than 75% of the longest span; end spans should not exceed 115% of the length of adjacent spans; and there should not be any cantilevers.
Support moments can then be taken as:

- two-span beam: \[ \frac{wL^2}{8} \] (51.31)
- first support in a multispan beam: \[ \frac{wL^2}{10} \] (51.32)
- other internal supports: \[ \frac{wL^2}{14} \] (51.33)

\( w \) is the unfactored uniformly distributed load on the span \( L \). Where the spans on each side of a support differ, the mean value of \( w \) is adopted.

**Calculation of Deflections**

For continuous beams under uniform load or symmetric point loads, the deflection \( \delta_c \) at midspan may be determined from the expression:

\[
\delta_c = \delta_o \left( 1 - 0.6 \frac{(M_1 + M_2)}{M_o} \right) 
\] (51.34)

where
- \( \delta_o \) = the deflection of a simply supported beam for the same loading
- \( M_o \) = the maximum moment in the simply supported beam
- \( M_1 \) and \( M_2 \) = the moments at the adjacent supports (modified as appropriate)

**Partial Shear Connection**

The increased deflection under serviceability loads arising from a partial shear connection should be determined from the following expressions:

For propped construction,

\[
\delta = \delta_c + 0.5 \left( 1 - \frac{N_a}{N_p} \right) (\delta_s - \delta_c) 
\] (51.35)

For unpropped construction,

\[
\delta = \delta_c + 0.3 \left( 1 - \frac{N_a}{N_p} \right) (\delta_s - \delta_c) 
\] (51.36)

where
- \( \delta_c \) = the deflection of the steel beam acting alone
- \( \delta_s \) = the deflection of a composite beam with a full shear connection for the same loading
- \( N_a \) = the actual number of shear connectors provided
- \( N_p \) = the number of shear connectors for a complete interaction
Cracking
Reference is made to BS 8110 [27]. Floors in car park structures are alluded to as being of importance, and additional reinforcement should be provided to avoid these over support regions. No consideration for increased deflections is made due to cracking.

Vibrations
Where vibration could cause discomfort or damage the response of long-span composite floors should be considered using SCI Publication 076, “Design Guide on the Vibration of Floors.”

Eurocode 4
Design of continuous composite beams for serviceability in EC4 is covered in Chapter 5, which is on serviceability. Furthermore, relevant sections for internal forces and moments in continuous composite beams are covered in Section 4.5. For stiffness calculations, modular ratios are considered in Section 3.1.4.2.

Scope
This chapter of the code covers the following limit states of deflection control and crack control. Other limit states such as vibration may be important but are not covered in Eurocode 4.

Assumptions
Calculation of stresses and deformations at the serviceability limit state shall take into account shear lag; incomplete interaction; cracking; tension stiffening of concrete in hogging moment regions; creep and shrinkage of concrete; yielding of steel, if any, when unpropped; and yielding of reinforcement in hogging moment regions.

Long-Term Effects
In the absence of a more rigorous analysis, the effects of creep may be taken into account by using modular ratios, as given in Section 3.1.4.2, for the calculation of flexural stiffness.

\[ n = \frac{E_a}{E'_c} \]  
(51.37)

where \( E_a \) is the elastic modulus of structural steel and \( E'_c \) is an effective modulus of concrete. \( E'_c = E_{cm} \) for short-term effects, \( E'_c = E_{cm}/3 \) for long-term effects, and \( E'_c = E_{cm}/2 \) for other cases.

Deformations
The effect of cracking of concrete in hogging moment regions may be taken into account by adopting one of the following methods of analysis.

Hogging moments and top-fiber concrete stresses, \( \sigma_{ct} \), are determined at each internal support using the flexural stiffnesses \( E_aI_t \). For each support at which \( \sigma_{ct} \) exceeds 0.15\( f_{ck} \), the stiffness should be reduced to the value \( E_aI_2 \) over 15% of the length of the span on each side of the support. A new distribution of bending moments is then determined by reanalyzing the beam. At every support where stiffnesses \( E_aI_2 \) are used for a particular loading they should be used for all other loadings, as shown in Fig. 51.31.

For beams with classes 1–3, where \( \sigma_{ct} \) exceeds 0.15\( f_{ck} \), the bending moment at the support is multiplied by a reduction factor \( f_1 \) and corresponding increases are made to the bending moments in adjacent spans, as shown in Fig. 51.32. Curve A should be used when loading on all spans is equal and the lengths of all
spans do not differ by more than 25%. Otherwise, the approximate lower bound \( f_1 = 0.60 \) should be used (i.e., line B).

In unpropped beams account may be taken of the influence of local yielding over a support by multiplying the bending moments at the support by:

- \( f_2 = 0.5 \) if \( f_y \) is reached before the concrete slab has hardened.
- \( f_2 = 0.7 \) if \( f_y \) is caused by the loading after the concrete has hardened.

### Cracking

Some important points to note about cracking in EC4, which are covered in Section 5.3, include: design crack widths should be agreed with the client; minimum reinforcement requirements are specified; maximum steel stresses are specified for bar sizes and required crackwidths; and elastic global analysis is used to ascertain internal forces and moments.

### Strength

In the hogging moment region, the moment resistance of the composite beam section depends on the tensile resistance of the steel reinforcement and the compression resistance of the steel beam section. Partial shear connection also exists; however, it depends on the steel reinforcement strength in tension, rather than the concrete slab in compression. The moment resistance depends on the location of the plastic neutral axis as follows.

#### Plastic Neutral Axis in the Concrete Slab (Full Shear Connection, \( \beta = 1.0 \))

When the steel reinforcing is stronger than the steel beam, the plastic neutral axis will lie within the concrete slab. For the case when the plastic neutral axis lies within the concrete slab, the ultimate flexural strength is determined from a simple couple, as shown in Fig. 51.33.

\[
M_u = TL = CL
\]  

(51.38)

FIGURE 51.32  Reduction factor for bending moment at supports.

FIGURE 51.33  Ultimate flexural moment, plastic neutral axis in concrete slab (\( \beta = 1.0 \)).
Plastic Neutral Axis in the Steel Beam (Full Shear Connection, $\beta = 1.0$)

When the steel beam is stronger than the reinforcing steel, the plastic neutral axis for the beam with full shear connection will lie within the steel beam. For this case it is more convenient to sum the moments about the centroid of the compression force, as illustrated in Fig. 51.34.

$$M_u = T_s L_s + T_r L_r$$ (51.39)

Partial Shear Connection ($\beta < 1.0$)

Although not generally allowed by international codes of practice, partial shear connection in the negative moment region may need to be considered for special cases. For the case of partial shear connection of composite beams in hogging bending, the shear connection is the weakest. Again, summing the moments on a convenient point on the cross section will yield the ultimate flexural moment of the beam.

$$M_u = T_s L_s + T_r L_r$$ (51.40)

Ductility

The assumption of a plastic collapse mechanism in continuous composite beams will generally be dependent on the formation of hinges at both the sagging and hogging regions. Oehlers and Bradford [3] have shown that when the ductility parameter $\chi > 1.6$, a plastic mechanism will be formed. The ductility parameter is determined as

$$\chi = \frac{0.85 f_y b_t \varepsilon_u (h_{concrete} + h_{steel})}{A_{steel} f_y (\varepsilon_u + \varepsilon_h)}$$ (51.41)

51.8 Composite Columns

The design of composite columns requires the consideration of both short-column and slender-column behavior. In addition, bending moments, which may occur about either axis due to imperfections and applied loading, must be considered. This section will consider the existing codes for the design of composite columns. The most comprehensive method is the Eurocode 4 approach, followed by the AISC-LRFD [28] approach. An Australian approach incorporating AS 3600 and AS 4100 [29] will also be considered herein.
Eurocode 4

Resistance of the Cross Section to Compression

The plastic resistance to compression of a composite cross section represents the maximum load that can be applied to a short composite column. It is important to recognize that concrete-filled circular hollow sections exhibit enhanced resistance due to the triaxial containment effects. Fully or partially concrete-encased steel sections and concrete-filled rectangular sections do not achieve such enhancement. Hence these two categories are dealt with separately in EC4.

Encased Steel Sections and Concrete-Filled Rectangular Hollow Sections

The plastic resistance of an encased steel section or a concrete-filled rectangular or square hollow section is given by the sum of the resistances of the components as follows:

\[ N_{pl,Rd} = A_a f_y / \gamma_a + A_c f_{ck} / \gamma_c + A_s f_{sk} / \gamma_s \]  

(51.42)

where  
- \( A_a, A_c, \) and \( A_s \) = the cross-sectional areas of the structural section, the concrete, and the reinforcing steel, respectively  
- \( f_y, f_{ck}, \) and \( f_{sk} \) = the yield strength of the steel section, the characteristic compressive strength of the concrete, and the yield strength of the reinforcing steel, respectively  
- \( \gamma_a, \gamma_c, \) and \( \gamma_s \) = the partial safety factors at the ultimate limit state (\( \gamma_a = 1.10, \gamma_c = 1.5, \) and \( \gamma_s = 1.15 \))  
- \( a_c \) = the strength coefficient for concrete, 1.0 for concrete-filled hollow sections and 0.85 for fully and partially concrete-encased steel sections.

For ease of expression, \( f_y / \gamma_a, f_{ck} / \gamma_c, \) and \( f_{sk} / \gamma_s \) are presented as the design strengths of the respective materials, such as \( f_{yd}, f_{cd}, \) and \( f_{sd} \). Equation (51.42) can therefore be rewritten as follows:

\[ N_{pl,Rd} = A_a f_{yd} + A_c f_{cd} + A_s f_{sd} \]  

(51.43)

An important design parameter, \( \delta \), the steel contribution ratio, is defined as follows:

\[ \delta = \frac{A_a f_{yd}}{N_{pl,Rd}} \]  

(51.44)

The column is classified as composite if the steel contribution ratio falls within the range of 0.2 ≤ \( \delta \) ≤ 0.9. If \( \delta \) is less than 0.2, the column shall be designed as a reinforced concrete column; otherwise, if \( \delta \) is greater than 0.9, the column shall be designed as a bare steel column.

Concrete-Filled Circular Hollow Sections

For concrete-filled circular hollow sections, the load-bearing capacity of the concrete can be increased due to the confinement effect from the surrounding tube. This effect is shown in Fig. 51.36. When a concrete-filled circular section is subjected to compression, causing the Poisson’s expansion of the concrete to exceed that of steel, the concrete is triaxially confined by the axial forces associated with the development of hoop tension in the steel section. The development of these hoop tensile forces in the tube, combined with the compressive axial forces in the steel shell, lowers the effective plastic resistance of the steel section in accordance with the von Mises failure criteria. However, the increase in concrete strength over the normal unconfined cylinder strength often more than offsets any reduction in the resistance of the steel. The net effect is that such columns show an enhanced strength.

The plastic resistance of the cross section of a concrete-filled circular hollow section is given by:

\[ N_{pl,Rd} = A_a \eta_2 f_y / \gamma_a + A_c f_{ck} \left( 1 + \eta_1 (t/d) f_y / f_{ck} \right) + A_s f_{sk} / \gamma_s \]  

(51.45)
The concrete enhancement effect is included in the bracket term associated with the concrete component term in Eq. (51.45). The hoop stress effect is reflected by the $\eta_2$ factor in the steel component term. Other parameters given in Eq. (51.45) are defined as follows: $t$ is the thickness of the circular hollow section,

$$\eta_i = \eta_{10} \left(1 - \frac{10e}{d}\right)$$

$$\eta_2 = \eta_{20} + \left(1 - \eta_{20}\right) \frac{10e}{d}$$

A linear interpolation is carried out for load eccentricity, $e \leq d/10$, with the basic values $\eta_{10}$ and $\eta_{20}$, which depend on the relative slenderness $\lambda$:

$$\eta_{10} = 4.9 - 18.5\lambda + 17\lambda^2 \quad \geq 0.0$$

$$\eta_{20} = 0.25(3 + 2\lambda) \quad \leq 1.0$$

No reinforcement is necessary for concrete infilled sections; however, if the contribution from reinforcement is to be considered in the load-bearing capacity, the ratio of reinforcement should fall within the range of $0.3\% \leq A_r/A_c \leq 4\%$. Additional reinforcement may be necessary for fire resistance design, but shall not be taken into account if the ratio exceeds 4%.

The effects of triaxial containment tend to diminish as the column length increases. Consequently, this effect may only be considered up to a relative slenderness of $\lambda \leq 0.5$. For most practical columns the value of $\lambda$ of 0.5 corresponds to a length-to-diameter ratio ($l/d$) of approximately 12. In addition, the eccentricity of the normal force, $e$, may not exceed the value $d/10$, $d$ being the outer diameter of the circular hollow steel section. If the eccentricity, $e$, exceeds the value $d/10$, or if the nondimensional slenderness $\lambda$ exceeds the value 0.5, then $\eta_1 = 0$ and $\eta_2 = 1.0$ must be applied, and Eq. (51.45) reduces to Eq. (51.42).

The eccentricity, $e$, is defined by:

$$e = \frac{M_{\text{max},Sd}}{N_{Sd}}$$
where $M_{\text{max,}SD}$ = the maximum design moment from first-order analysis
$N_{SD}$ = the design axial force

Table 51.5 gives the basic values $\eta_{10}$ and $\eta_{20}$ for different values of $\lambda$. It should be noted that the evaluation of $N_{pl,Rd}$ for concrete-filled circular hollow sections always starts with Clause 4.8.3.3(1), with unity material factors to give $\lambda$, and thus it is not an iteration process.

From a numerical study carried out by Bergmann et al. [30], the application of Eqs. (51.45) to (51.49) and its relation to Eq. (51.43) is shown in Table 51.6. The table gives the respective increase in the axial resistance to axial loads for different ratios of $d/t$, $f_y/f_{ck}$, and selected values for $e/d$ and $\lambda$ due to confinement.

### Table 51.5
Basic Values $\eta_{10}$ and $\eta_{20}$ to Allow for the Effect of Triaxial Confinement in Concrete-Filled Circular Hollow Sections

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>0.0</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
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</thead>
<tbody>
<tr>
<td>$\eta_{10}$</td>
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<td>0.88</td>
<td>0.22</td>
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<td>$\eta_{20}$</td>
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<td>0.85</td>
<td>0.90</td>
<td>0.95</td>
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</table>

### Table 51.6
Increase in Resistance to Axial Loads for Different Ratios of $d/t$, $f_y/f_{ck}$, and Selected Values for $e/d$ and $\lambda$ Due to Confinement

<table>
<thead>
<tr>
<th>$\lambda$</th>
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<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
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<td>$d/t$</td>
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<td>60</td>
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<td></td>
<td></td>
<td></td>
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<tr>
<td>$e/d$</td>
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<td>10</td>
<td>15</td>
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<tr>
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</table>

Source: Bergmann, R. et al., CIDECT, Verlag TÜV Rheinland, Germany, 1995.
strength of the column caused by the confinement effect for certain ratios of steel-to-concrete strengths, selected values for $\lambda$, and certain ratios of $e/d$ and $d/t$. For the calculation, the longitudinal reinforcement is assumed to be 4%, with a yield strength of 500 N/mm². It must be recognized that for higher slenderness and larger eccentricities, the advantage of the confinement effect is very low. Similarly, for higher diameter-to-thickness ratios of the circular hollow section, and smaller steel-to-concrete-strength ratios, the confinement effect decreases. Therefore, in the calculation of axial strength for the column, a significant increase in strength due to the confinement effect is obtained only when the values of $l$ are less than 0.2 and the eccentricity ratios $e/d$ are less than 0.05.

**Local Buckling**
Both Eqs. (51.43) and (51.45) are valid, provided that local buckling in the steel sections does not occur. To prevent premature local buckling, the plate slenderness ratios of the steel section in compression must satisfy the following limits:

- $d/t \leq 90\varepsilon^2$ for concrete-filled circular hollow sections.
- $h/t \leq 52\varepsilon$ for concrete-filled rectangular hollow sections.
- $b/t \leq 44\varepsilon$ for partially encased I sections.

where $d$ = the outer diameter of the circular hollow section with thickness, $t$
$h$ = the depth of the rectangular hollow section with thickness, $t$
$b$ = the breadth of the I section with a flange thickness, $t_f$
$\varepsilon = \sqrt{235/f_y}$
$f_y$ = the yield strength of the steel section (N/mm²).

Table 51.7 shows the limit values for the plate slenderness ratio for steel sections in class 2, which have limited rotation capacity. In such cases, plastic analysis, which considers moment redistribution due to the formation of plastic hinges, is not allowed.

For fully encased steel sections, no verification for local buckling is necessary. However, the concrete cover to the flange of a fully encased steel section should not be less than 40 mm or less than one sixth of the breadth, $b$, of the flange. The cover to reinforcement should be in accordance with Clause 4.1.3.3 of EC2-1990.

**Effective Elastic Flexural Stiffness**
Elastic flexural stiffness of a composite column is required in order to define the elastic buckling load, which is defined as

$$N_{el} = \pi^2 (EI)_e / l^2$$ (51.51)

The term $(EI)_e$ is the effective elastic flexural stiffness of the composite column and $l$ is the buckling length of the column.

The buckling length may be determined using EC3 [31] by considering the end conditions due to the restraining effects from the adjoining members.

Special consideration of the effective elastic flexural stiffness of the composite column is necessary, as the flexural stiffness may decrease with time, due to creep and shrinkage of concrete. The design rules for the evaluation of the effective elastic flexural stiffness of composite columns under short-term and long-term loading are described in the following two sections.
Short-Term Loading — The effective elastic flexural stiffness \((EI)_e\) is obtained by adding up the flexural stiffnesses of the individual components of the cross section:

\[
(EI)_e = E_a I_a + 0.8E_{cd} I_c + E_s I_s
\]  
\[E_{cd} = E_{cm}/1.35\]  

where \(I_a, I_c,\) and \(I_s\) = the second moments of area for the steel section, concrete (assumed uncracked), and reinforcement about the axis of bending, respectively

\(E_a, E_s\) = the moduli of elasticity of the steel section and the reinforcement, respectively

\(0.8E_{cd}I_c\) = the effective stiffness of the concrete

\(E_{cm}\) = the secant modulus of elasticity of concrete

The simplified design method of EC4 has been developed with a secant stiffness modulus of the concrete of 600\(f_{ck}\). In order to have a similar basis like EC2, the secant modulus of the concrete \(E_{cm}\) was chosen as the reference value. The transformation led to the factor 0.8 in Eq. (51.52). This factor, as well as the safety factor 1.35 in Eq. (51.53), may be considered as the effect of cracking of concrete under moment action due to the second-order effects. So, if this method is used for a test evaluation of composite columns, which is typically done without any safety factor, the safety factor for the stiffness should be taken into account subsequently, i.e., the predicted member capacity should be calculated using \((0.8E_{cm}/1.35)\). In addition, the value of 1.35 should not be changed, even if different safety factors are used in the country of application.

Long-Term Loading — For slender columns under long-term loading, the creep and shrinkage of concrete will cause a reduction in the effective elastic flexural stiffness of the composite column, thereby reducing the buckling resistance. However, this effect is significant only for slender columns. As a simple rule, the effect of long-term loading should be considered if the buckling length-to-depth ratio of a composite column exceeds 15.

If the eccentricity of loading, as defined in Eq. (51.50), is more than twice the cross section dimension, the effect on the bending moment distribution caused by increased deflections due to creep and shrinkage of concrete will be very small. Consequently, it may be neglected, and no provision for long-term loading is necessary. Moreover, no provision is necessary if the nondimensional slenderness, \(\lambda\), of the composite column is less than the limiting values given in Table 51.8.

Otherwise, the effect of creep and shrinkage of concrete should be allowed for by employing the modulus of elasticity of the concrete, \(E_c\), instead of \(E_{cd}\) in Eq. (51.54), which is defined as follows:

\[
E_c = E_{cd} \left[1 - \left(0.5N_{G,SD}/N_{Sd}\right)\right]
\]  

where \(N_{Sd}\) = the design axial load

\(N_{G,SD}\) = the part of the design load permanently acting on the column

Table 51.8 also allows the effect of long-term loading to be ignored for concrete-filled hollow sections with \(\lambda \leq 2.0\), provided that \(d\) is greater than 0.6 for braced (nonsway) columns and 0.75 for unbraced (sway) columns.

<table>
<thead>
<tr>
<th>Type of Cross Section</th>
<th>Braced Nonsway Systems</th>
<th>Unbraced and Sway Systems</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concrete encased</td>
<td>&lt;0.8</td>
<td>&lt;0.5</td>
</tr>
<tr>
<td>Concrete filled</td>
<td>(&lt;0.8/(1 – \delta)^a&gt;</td>
<td>(&lt;0.5/(1 – \delta)^a&gt;</td>
</tr>
</tbody>
</table>

\(\delta\) is the steel contribution ratio as defined in Eq. (51.44).
The plastic resistance to compression of a composite cross section, $N_{pl}$, represents the maximum load that can be applied to a short column. For slender columns with low elastic critical loads, overall buckling may be critical.

In a typical buckling curve for a column, as shown in Fig. 51.37(a), the horizontal line represents $N_{pl}$, while the curve represents $N_{cr}$, which is a function of the column slenderness. They are the boundaries to the compressive resistance of the column.

In Fig. 51.37(b), the buckling resistance of a column may be expressed as a proportion $\chi$ of the plastic resistance to compression, $N_{pl}$, thereby nondimensionalizing the vertical axis of Fig. 51.37(b). The horizontal axis may be nondimensionalized similarly by $N_{cr}$, as shown in Fig. 51.37(b).

By incorporating the effects of both residual stresses and geometric imperfections, the multiple column curves may be drawn on this basis, as shown in Fig. 51.37(c). They form the basis of column buckling design for both steel and composite columns.

In order to determine the compressive resistance of a column with the European buckling curves, the nondimensional slenderness of the column should be first evaluated as follows:

$$\lambda = \sqrt{\frac{N_{pl,R}}{N_{cr}}}$$  \hspace{1cm} (51.55)

where $N_{pl,R}$ = the plastic resistance of the cross section to compression, according to Eq. (51.42), with $\gamma_s = \gamma_t = \gamma_e = 1.0$

$N_{cr}$ = the elastic critical buckling load, as defined in Eq. (51.51)

Once the nondimensional slenderness of a composite column is established, the buckling resistance to compression of the column may be evaluated.

**FIGURE 51.37** Slender composite column behavior: (a) idealized column buckling $N_{Rd}/N_{pl,Rd}$ curve, (b) nondimensionalized buckling curve, (c) European buckling curves according to EC3-1992.

**Resistance of Members to Axial Compression**

The plastic resistance to compression of a composite cross section, $N_{pl}$, represents the maximum load that can be applied to a short column. For slender columns with low elastic critical loads, overall buckling may be critical.

In a typical buckling curve for a column, as shown in Fig. 51.37(a), the horizontal line represents $N_{pl}$, while the curve represents $N_{cr}$, which is a function of the column slenderness. They are the boundaries to the compressive resistance of the column.

In Fig. 51.37(b), the buckling resistance of a column may be expressed as a proportion $\chi$ of the plastic resistance to compression, $N_{pl}$, thereby nondimensionalizing the vertical axis of Fig. 51.37(b). The horizontal axis may be nondimensionalized similarly by $N_{cr}$, as shown in Fig. 51.37(b).

By incorporating the effects of both residual stresses and geometric imperfections, the multiple column curves may be drawn on this basis, as shown in Fig. 51.37(c). They form the basis of column buckling design for both steel and composite columns.

In order to determine the compressive resistance of a column with the European buckling curves, the nondimensional slenderness of the column should be first evaluated as follows:

$$\lambda = \sqrt{\frac{N_{pl,R}}{N_{cr}}}$$  \hspace{1cm} (51.55)

where $N_{pl,R}$ = the plastic resistance of the cross section to compression, according to Eq. (51.42), with $\gamma_s = \gamma_t = \gamma_e = 1.0$

$N_{cr}$ = the elastic critical buckling load, as defined in Eq. (51.51)

Once the nondimensional slenderness of a composite column is established, the buckling resistance to compression of the column may be evaluated.
A composite column has sufficient compression resistance if, for both axes of bending,

$$N_{sd} \leq \chi N_{pl,Rd}$$ (51.56)

where $N_{sd}$ = the design applied load to be resisted

$N_{pl,Rd}$ = the cross section resistance in accordance with Eq. (51.42) or (51.45)

$\chi$ = the reduction coefficient due to column buckling and is a function of the nondimensional slenderness of the composite column

The European buckling curves illustrated in Fig. 51.37(c) may be used for composite columns. They are selected according to type of steel section and the axis of bending:

- curve $a$ for concrete-filled hollow sections
- curve $b$ for fully or partially concrete-encased I sections buckling about the strong axis of the steel sections
- curve $c$ for fully or partially concrete-encased I sections buckling about the weak axis of the steel sections

These curves can also be calculated based on the following equation:

$$\chi = 1 - \frac{1}{\left[ \phi + \left( \phi^2 + \lambda^2 \right)^{1/2} \right]} \leq 1.0$$ (51.57)

where

$$\phi = 0.5 \left[ 1 + \alpha (\lambda - 0.2) + \lambda^2 \right]$$ (51.58)

The factor $\alpha$ is used to allow for different levels of imperfections in the columns. It is important to note that the second-order moment due to imperfection, or the imperfection moment, has been incorporated in the method through different buckling curves; no additional consideration is necessary. The various values of $\alpha$ are in Table 51.9.

### Combined Compression and Uniaxial Bending

The design for a composite column under combined compression and bending is carried out in stages as follows:

- The composite column is isolated from the framework, and the end moments that result from the analysis of the system as a whole are taken to act on the column under consideration. Internal moments and forces within the column length are determined from the structural consideration of end moments and axial and transverse loads.

- For each axis of symmetry, the buckling resistance to compression should be checked with the relevant nondimensional slenderness of the composite column.

- In the presence of an applied moment about one particular axis, e.g., the $y$-$y$ axis, the moment resistance of the composite cross section should be checked with the relevant nondimensional slenderness of the composite column, i.e., $l_y$ instead of $l_z$, although $l_z$ may be larger than and thus more critical than $l_y$.

- For slender columns, both long-term loading and second-order effects should be included.

In the simplified method of EC4, imperfections within the column length need not be considered, as they are taken into account in the determination of the buckling resistance of the columns.
In EC4, the resistance of the composite column to combined compression and bending is determined with the help of an interaction curve. Unlike the interaction diagram for a bare steel section, where the moment resistance undergoes a continuous reduction with increase in axial load, the moment resistance for very short composite columns may be increased by the presence of an axial load. This is because the prestressing effect of an axial load may in certain circumstances prevent cracking, and so make the concrete more effective in resisting moment. Figure 51.38 represents the nondimensional interaction curve for compression and uniaxial bending for a composite cross section.

Such a curve for short composite columns can be determined by considering different positions of the neutral axis over the whole cross section and determining the internal action effects from the resulting stress blocks. This approach can only be carried out by computer analysis.

With the simplified method of EC4, it is possible to calculate by hand four or five points (ACDB and E) of the interaction curve. The interaction curve may be replaced by the polygon ACDB(E) through these points (see Fig. 51.39). The method is applicable to the design of columns with cross sections that are symmetrical about both principal axes.

The simplified design method in EC4 uses the following method to determine points A to E.

**Figure 51.40** shows the stress distributions in the cross section of a concrete-filled rectangular hollow section at each point (A, B, C, D, and E) of the interaction curve. It is important to note that:
• Point A marks the plastic resistance of the cross section to compression:

\[ N_A = N_{pl,Rd} \]
\[ M_A = 0 \]

• Point B corresponds to the plastic moment capacity of the cross section:

\[ N_B = 0 \]
\[ M_B = M_{pl,Rd} \]

• Point C, the compressive and moment resistances of the column, are given as follows:

\[ N_C = N_{pm,Rd} = A_c f_{cd} \]
\[ M_C = M_{pl,Rd} \]
The expressions may be obtained by combining the stress distributions of the cross section at points B and C. The compression area of the concrete at point B is equal to the tension area of the concrete at point C. The moment resistance at point C is equal to that at point B, since the stress resultants from the additionally compressed parts nullify each other in the central region of the cross section. However, these additionally compressed regions create an internal axial force, which is equal to the plastic resistance to compression of the concrete alone, $N_{pm.Rd}$.

At point D, the plastic neutral axis coincides with the centroidal axis of the cross section and the resulting axial force is half of that at point C:

$$N_D = \frac{N_{pm.Rd}}{2}$$

$$M_C = M_{max.Rd}$$

Point E is midway between points A and C. It is often required for highly nonlinear interaction curves, in order to achieve better approximation. In general, it is not needed for concrete-encased I sections subject to moments about the major axis, or if the design axial force does not exceed $N_{pm.Rd}$. For concrete-filled hollow sections, the use of point E will give more economical design, although much calculation effort is required. For simplicity, point E may be omitted in design.

Analysis of Bending Moments Due to Second-Order Effects

Under the action of a design axial load $N_{sd}$ on a column with an initial imperfection $e_o$, as shown in Fig. 51.41, there will be a maximum internal moment of $N_{sd}(e_o)$. It is important to note that this second-order moment does not need to be considered separately, as its effect on the buckling resistance of the composite column is already accounted for in the European buckling curves, as shown in Fig. 51.37(c).

However, in addition to axial forces, a composite column may be also subjected to end moments as a consequence of transverse loads acting on it, or because the composite column is part of a frame. The moments and displacements obtained initially are referred to as first-order values. For slender columns, the first-order displacements may be significant, and additional or second-order bending moments may be induced under the actions of applied loads. As a simple rule, the second-order effects should be considered if the buckling length-to-depth ratio of a composite column exceeds 15.

The second-order effects in an isolated nonsway column may be allowed for by multiplying the maximum first-order bending moment $M_{max.Sd}$ with a correction factor $k$, which is defined as

$$k = \frac{\beta}{1 - \left(\frac{N_{sd}}{N_{cr.L}}\right)} \geq 1.0$$

(51.59)
where \( N_{Sd} \) = the design axial load
\( N_{cr.L} \) = the elastic critical load of the composite column based on the system length, \( L \)
\( \beta \) = an equivalent moment factor

For columns with transverse loading within the column length, the value for \( b \) should be taken as 1.0. For pure end moments, \( \beta \) is determined as follows:

\[
\beta = 0.66 + 0.44 \, r \geq 0.44 \quad (51.60)
\]

**Resistance of Members under Combined Compression and Uniaxial Bending**

The principle for checking sections under compression and uniaxial bending is shown graphically in Fig. 51.39 or 51.42. In this method an initial imperfection has been incorporated, so that any additional consideration of geometrical imperfection is unnecessary in the calculations of moments within the column length.

The axial resistance of the composite column in the absence of moment is given by \( \chi N_{pl.Rd} \) (refer to Eq. (51.46)). Therefore, at the level \( \chi = N_{rd}/N_{pl.Rd} \), no additional bending moment can be applied to the column. The corresponding value for bending \( \mu_k \) of the cross section is therefore the moment for imperfection of the column, and the influence of this imperfection is assumed to decrease linearly to the value \( \chi_n \). For an axial load ratio less than \( \chi_n \), the effect of imperfections is neglected.

It is important to recognize that the value \( \chi_n \) accounts for the fact that the influences of the imperfections and bending moment do not always act together unfavorably. For columns with end moments, \( \chi_n \) may be obtained as follows:

\[
\chi_n = \chi(1-r)/4 \quad (51.61)
\]

If transverse loads occur within the column height, \( r \) is taken as unity and \( \chi_n \) is thus equal to zero. With a design axial load of \( N_{Sd} \), the axial load ratio \( \chi_d \) is defined as

\[
\chi_d = N_{Sd}/N_{pl.Rd} \quad (51.62)
\]

The horizontal distance from the interaction curve, \( \mu \), defines the ultimate moment resistance that is still available, having taken account of the influence of second-order effects in the column.

EC4 considers that the design is adequate when the following condition is satisfied:

\[
M_{Sd} \leq 0.9 \, \mu M_{pl.Rd} \quad (51.63)
\]
where \( M_{sd} \) = the design bending moment, which may be factored to allow for second-order effects, if any

\( m = \) the moment resistance ratio obtained from the interaction curve

\( M_{pl.Rd} = \) the plastic moment resistance of the composite cross section

The interaction curve has been determined without considering the strain limitations in the concrete. Hence the moments, including second-order effects if necessary, are calculated using the effective elastic flexural stiffness \( (EI)_e \) and taking into account the entire uncracked concrete area of the cross section (i.e., concrete is uncracked). Consequently, a reduction factor of 0.9 is applied to the moment resistance in Eq. (51.63) to allow for the simplifications in the approach.

In certain regions of the interaction curve, the moment resistance ratio is allowed to be greater than the unity in the presence of an axial load. This is due to the fact that in the presence of an axial load, the amount of concrete in tension and thus area of cracked section is reduced, and more concrete is included in the evaluation of the moment resistance. However, if the bending moment and applied load are independent of each other, the value of \( m \) must be limited to 1.0.

For concrete-filled hollow sections, inclusion of point E in the interaction curve, as shown in Fig. 51.42, will give more economical design, especially for columns under a high axial load and low end moments.

**Combined Compression and Biaxial Bending**

For the design of a composite column under combined compression and biaxial bending, the axial resistance of the column in the presence of the bending moment for each axis must be evaluated separately. In general, it will be obvious which axis is more critical. If not, checks have to be carried out for compression and uniaxial bending for each axis separately. Imperfections should be considered only for the plane in which failure is expected to occur.

After finding the moment resistance ratios \( \mu_y \) and \( \mu_z \) for both axes, the interactions of the moments are checked using the moment interaction curve shown in Fig. 51.43. This linear interaction curve is cut off at 0.9\( \mu_y \) and 0.9\( \mu_z \). The design moments \( M_{y,sd} \) and \( M_{z,sd} \), related to the respective plastic moment resistances, must lie within the moment interaction curve.

EC4 considers the check adequate when all of the following conditions are satisfied:
In columns with a different distribution of moments in both of the main axes, the determination of the position of the critical combination of moments is often very difficult. For the purpose of simplification, the maximum moments of the bending axes may be used in Eq. (51.66).

**AISC-LRFD**

The concept of applying AISC-LRFD column design methodology to composite columns by the use of modified properties was first presented by Furlong [32]. Modified yield stress $F_{my}$, modified modulus of elasticity $E_m$, and modified radius of gyration $r_m$ were incorporated into an allowable stress design procedure that was published by Task Group 20 of the Structural Stability Research Council [33].

**Axially Loaded Column**

When a column is under axial compression, concrete spalls and fails when longitudinal strain reaches about 0.18 to 0.20%. Cross section strength $P_o$ is the sum of axial load capacities of the materials that make up the cross section. Thus, for steel that yields at strains no greater than 0.2%,

$$P_o = A_s F_y + A_r F_{yr} + 0.85 A_c f'_c$$  \hspace{1cm} (51.67)

where
- $A_s$ = the area of structural shape in the cross section
- $A_r$ = the area of longitudinal reinforcement in the cross section
- $A_c$ = the concrete in the cross section
- $F_y$ = the yield strength of the structural shape steel
- $F_{yr}$ = the yield strength of the longitudinal reinforcement
- $f'_c$ = the strength of concrete from standard cylinder tests

The design strength of composite columns is determined from the same equations as those applicable to bare steel columns, except that the formulas are entered with modified properties $F_{my}$, $E_m$, and $r_m$. The axial design strength is computed as

$$
\phi_c P_n = 0.85 A_s F_{cr}
$$  \hspace{1cm} (51.68)

where $F_{cr}$ is the critical stress of the column given by Eqs. (51.69) and (51.70). Both equations include the estimate effects of residual stresses and initial of-out-straightness of the members. The factor 0.877 in Eq. (51.70) accounts for the effect of member initial out-of-straightness.

$$
F_{cr} = \left(0.658 \lambda_c^2\right) F_{my} \quad \text{for} \quad \lambda_c \leq 1.5
$$  \hspace{1cm} (51.69)

and

$$
F_{cr} = \frac{0.877}{\lambda_c^2} F_{my} \quad \text{for} \quad \lambda_c > 1.5
$$  \hspace{1cm} (51.70)
where $F_{my} = \text{modified yield stress}$ and $\lambda_c = (KL/r_m \pi)/(F_{my}/E_m)$, in which $E_m$ is the modified modulus of elasticity, $r_m$ is the modified radius of gyration about the axis of buckling, $K$ is the effective length factor, and $L$ is the laterally unbraced length of a member.

The modified properties $F_{my}$, $E_m$, and $r_m$ account for the effects of concrete and longitudinal reinforcing bars. The modified radius of gyration $r_m$ is the radius of gyration of the steel section, and it shall not be less than 0.3 times the overall thickness of the composite cross section in the plane of buckling. The modified values $F_{my}$ and $E_m$ are given by the following equations:

$$F_{my} = F_y + \frac{c_1 F_{yr} A_y}{A_s} + \frac{c_2 f'_c A_y}{A_s}$$  \hspace{1cm} \text{(51.71)}

and

$$E_m = E + \frac{c_3 E_c A_y}{A_s}$$ \hspace{1cm} \text{(51.72)}

where $F_y$ = the yield strength of structural steel, $\leq 60$ ksi (414 MPa)
$F_{yr}$ = the yield strength of longitudinal reinforcement, $\leq 60$ ksi (414 MPa)
$E$ = the modulus of elasticity of steel
$E_c$ = the modulus of elasticity of concrete
$c_1$, $c_2$, and $c_3$ = the numerical coefficients listed in Table 51.10

Table 51.10 - Numerical Coefficients for Design of Composite Columns

<table>
<thead>
<tr>
<th>Composite Column Type</th>
<th>Numerical Coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concrete-filled pipe and tubing</td>
<td>$c_1$</td>
</tr>
<tr>
<td>Concrete-encased shapes</td>
<td>1.0</td>
</tr>
<tr>
<td></td>
<td>0.7</td>
</tr>
</tbody>
</table>

Coefficients $c_1$, $c_2$, and $c_3$ are higher for filled composite columns than for encased composite columns. With the steel encasement always available to provide lateral confinement to concrete in filled composite columns, there is no uncertainty that the contained concrete will reach at least as much strength as that reached by concrete in unconfined standard concrete cylinders used in determining $f'_c$. In contrast, there is less uncertainty that an unconfined concrete encasement can attain stress as high as 0.85$f'_c$. If the unconfined concrete fails to reach 0.85$f'_c$, the longitudinal reinforcement it stabilizes may not reach its yield stress, $F_{yr}$, either. The values of $c_1$ and $c_2$ for encased composite columns are 70% of the values for filled composite columns, reflecting the higher degree of uncertainty.

To account for the uncertainty regarding the contribution of concrete to the buckling strength of a composite column, Eq. (51.72) includes the numerical coefficient $c_3$, which is equal to 0.4 for filled composite columns and 0.2 for encased composite columns. These coefficients are consistent with values recommended in the ACI building code for flexural stiffness, $EI$, in estimates of inelastic buckling loads.

Concrete loses stiffness at strains near 0.2% and may not be fully effective for stabilizing steel at strains higher than 0.2%, which translates into steel–stress values of about 60 ksi (414 MPa). The yield stresses of structural steel ($F_y$) and reinforcing bars ($F_{yr}$) used in calculating the strength of composite columns should not exceed 60 ksi. It is further recommended that the concrete strength $f'_c$ be limited to 10 ksi (69 MPa) and smaller, since only very few tests are available for composite columns with $f'_c$ in excess of 10 ksi. A lower limit of $f'_c = 2.5$ ksi (17 MPa) is recommended in order to encourage a degree of quality control commensurate with this readily available and familiar grade of structural concrete.

**Flexural Strength**

The nominal flexural strength, $M_{nu}$, of a column cross section may be determined from the plastic state of stress or from an analysis of flexural strength at the ultimate state of strain. For simplicity, the
commentary in the AISC-LRFD (Section C-14) offers an approximate equation for moment capacity of doubly symmetric sections. The sum of flexural capacities for component parts includes the plastic moment capacity of the steel shape, an estimate of the yield moment of reinforcement, and the moment capacity for which compression concrete is considered reinforced at middepth by longitudinal bars and the web of the steel shape.

\[ M_u = Z F_y + \frac{1}{3} \left( h_2 - 2 C_r \right) A_y F_y + \left( \frac{h_2}{2} - \frac{A_w F_y}{1.7 f'_c h_1} \right) A_w F_y \]  \hspace{1cm} (51.73)

where  
- \( A_w \) = the web area of steel shape plus any longitudinal bars at the center of the section  
- \( Z \) = the plastic section modulus of the steel shape  
- \( h_1 \) = the concrete width perpendicular to the plane of bending  
- \( h_2 \) = the concrete thickness in the plane of bending  
- \( c_r \) = the thickness of concrete cover from the center of bar to the edge of the section in the plane of bending

**Combined Axial Compression and Moments**

For composite columns symmetrical about the plane of bending, the interaction of compression and flexure should be limited by the formulas in the following AISC-LRFD equations:

\[
\frac{P_u}{\phi_c P_n} + \frac{8}{9} \left( \frac{M_{ux}}{\phi_c M_{ux}} + \frac{M_{uy}}{\phi_c M_{uy}} \right) \leq 1 \quad \text{for} \quad P_u \geq 0.2 \phi_c P_n \tag{51.74}
\]

and

\[
\frac{P_u}{2 \phi_c P_n} + \frac{M_{ux}}{\phi_b M_{ux}} + \frac{M_{uy}}{\phi_b M_{uy}} \leq 1 \quad \text{for} \quad P_u < 0.2 \phi_c P_n \tag{51.75}
\]

where  
- \( P_u \) = the required compressive strength  
- \( P_n \) = the nominal compressive strength  
- \( M_u \) = the required flexural strength  
- \( M_n \) = the nominal flexural strength  
- \( \phi_c \) = the resistance factor for compression, 0.85  
- \( \phi_b \) = the resistance factor for flexure, 0.90

and subscripts \( x \) and \( y \) denote the major and minor axes, respectively.

Second-order effects may be considered in the determination of \( M_u \) for use in Eqs. (51.74) and (51.75). The simplicity of AISC-LRFD results in a conservative design. The supporting comparisons with the beam-column test, given in the commentary of AISC-LRFD, included 48 concrete-filled pipes or tubing and 44 concrete-encased steel shapes (see Galambos and Chapuis [34]). The overall mean test-to-prediction ratio was 1.23, and the coefficient of variation was 0.21.

**Australian Standards AS 3600 and AS 4100**

**Strength of Short Columns**

The design of a concrete-filled steel column in Australia can be undertaken using a combination of the Australian Standards for concrete and steel structures. Thus the ultimate axial force of a column can be represented as

\[ N_u = N_{uc} + N_{us} \]  \hspace{1cm} (51.76)
The Australian Standard for concrete structures, AS 3600, will not allow confinement, as it does not treat the behavior of concrete-filled steel columns directly. The concrete contribution to strength can be determined using Eq. (51.77), where

\[ N_{uc} = 0.85A f_c \]  

(51.77)

The Australian Standard for steel structures, AS 4100, suggests a set of slenderness limits that do not allow for the beneficial effect of local buckling. Slenderness limits for inelastic local buckling are as low as \( b/t = 30 \) for heavily welded sections. However, this standard allows one to use a rational local buckling method to determine the post-local buckling strength. The steel strength can therefore be determined from Eq. (51.78), where

\[ N_{uc} = A_s f_y \]  

(51.78)

If one combines the concrete and steel strengths, \( N_{uc} \) and \( N_{us} \), respectively, from the AS 3600 and AS 4100 analysis, the resulting ultimate axial strength can be written as

\[ N_u = 0.85A f_c + A_s f_y \]  

(51.79)

Therefore, while the beneficial effect of the concrete is taken into account for post-local buckling, the effect of concrete confinement is ignored; the initial slenderness limits are too stringent for use in concrete-filled steel columns, which are economical in steel construction.

**Local Buckling**

If a composite section has a thin-walled steel section, which is able to buckle locally, then a reduction for the strength of the steel section must be made. The local buckling load and strength of a concrete-filled steel section are significantly higher than those of a hollow steel section, as shown in Fig. 51.45.

The local buckling stress can be determined using Eq. (51.80), where \( k \) is determined from a rational local buckling analysis and is given in Table 51.11 for various boundary and fill conditions.

\[ \sigma_{ol} = \frac{k\pi^2 E}{12(1-v^2)\left(\frac{b}{t}\right)^2} \]  

(51.80)

**Post-Local Buckling**

Post-local buckling is often predicted using the effective width concept. Effective width models for hot-rolled and fabricated sections have been modified to incorporate residual stresses and initial imperfections by Bradford [35] and Bradford et al. [36], and were adopted in the Australian Standard AS 4100-1990.
This model is based on the Winter formula, which is also present in steel codes in the U.S. and Europe and is of the form

\[ a = \text{parameter used to account for residual stresses and initial geometric imperfections.} \]

This parameter varies, depending on the type of section and its method of fabrication. Furthermore, the type of boundary condition also affects the determination of \( a \). Values for this parameter, including all these factors, are summarized in Table 51.12, which was calibrated for steel structures in the Australian Standard AS 4100-1990 [29] and reported by Bradford et al. [36]. These parameters were thus used in calibrating the model with the post-local buckling test results determined by Uy [37] for concrete-filled sections. The results are illustrated in Figs. (51.46) and (51.47).
Slender composite columns can be analyzed using either the concrete approach, AS 3600, or the steel approach, AS 4100. The approach used by the Eurocode 4 is a useful manner in which to distinguish between the two methods.

**Concrete Approach, AS 3600**

Slender columns are analyzed in the Australian Concrete Structures Code, AS 3600, using a strength interaction diagram and a loading line. The loading line is used to account for nonlinearities and second-order effects.

**Steel Approach, AS 4100**

The approach of the Australian Standard for steel columns is essentially the same as the approach of EC3 and EC4, which use the column curves with different levels of imperfections and residual stresses. The
Composite Steel–Concrete Structures

The method relies on determining a critical buckling load, $N_c$, where the critical buckling load depends on both the member slenderness and the level of imperfections and residual stresses:

$$N_c = a_c N_s$$  (51.83)

where $a_c$ is the coefficient, which depends on both member slenderness and method of manufacture. $N_s$ is determined based on the section capacity, which can account for local buckling. However, Vrcelj and Uy [38] have developed a more comprehensive method to consider local buckling for concrete-filled steel sections, and to use this method, $N_s$ in Eq. (51.83) is calculated as the member squash load.

**Effects of Local Buckling**

The slender column buckling load, $N_{clb}$, which incorporates local buckling, can be represented in the form of Eq. (51.84) in terms of $N_c$, the column buckling load, which ignores the effects of local buckling.

$$N_{clb} = \alpha_{lb} N_c$$  (51.84)

where $\alpha_{lb}$ is the interaction coefficient to account for local buckling and is in the range

$$0 \leq \alpha_{lb} \leq 1.0$$  (51.85)

and is calculated as

$$\alpha_{lb} = \frac{100 - p_i}{100}$$  (51.86)
where the percentage reduction, $p_r$, is given as

$$p_r = \left( \frac{N_r - N_{clb}}{N_r} \right) \times 100$$

(51.87)

By determining $N_r$ using an existing standard, one can include the effects of local buckling by determining $\alpha_{lb}$. The global buckling load $N_{clb}$ can be determined using Eq. (51.84). The reduction factors may be determined using Fig. 51.48.

### 51.9 Lateral Load Resisting Systems

This section discusses four classical lateral load resisting systems for multistory building frames. They are the (1) core braced system, (2) moment–truss system, (3) outrigger and belt system, and (4) tube system. Multistory buildings that utilize cantilever action will have higher efficiencies, but the overall structural efficiency depends on the height-to-width ratio. Interactive systems involving a moment frame and vertical truss or core are effective up to 40 stories and represent most building forms for tall structures. Outrigger and belt trusses help to further enhance the lateral stiffness by engaging the exterior frames with the core braces to develop cantilever actions. Exterior framed tube systems with closely spaced exterior columns connected by deep girders mobilize the three-dimensional action to resist lateral and torsional forces. Bundled tubes improve the efficiency of exterior frame tubes by providing internal stiffening to the exterior tube. Finally, by providing diagonal braces to the exterior framework, a superframe is formed and can be used for ultratall megastructures. Further details on the comparison of various framing schemes are reported in Liew et al. [39].

### Core Braced Systems

This type of structural system relies entirely on the internal core for lateral load resistance. The basic concept is to provide an internal shear wall core to resist the lateral forces (Fig. 51.49). The surrounding
Steel framing is designed to carry a gravity load only if simple framing is adopted. Otherwise, a rigid framing surrounding the core will enhance the overall lateral force resistance of the structure. The steel beams can be simply connected to the core walls by using a typical corbel detail, by bearing in a wall pocket, or by using a shear plate embedded in the core wall through studs. If rigid connection is required, the steel beams should be rigidly connected to the steel columns embedded in the core wall. Rigid framing surrounding the cores is particularly useful in high seismic areas and for very tall buildings that tend to attract stronger wind loads. They act as moment frames and provide resistance to some part of the lateral loads by engaging the core walls in the building.

The core generally provides all torsional and flexural rigidity and strength, with no participation from the steel system. Conceptually, the core system should be treated as a cantilever wall system with punched openings for access. The floor framing should be arranged in such a way that it distributes enough gravity load to the core walls so that their design is controlled by compressive stresses, even under wind loads. The geometric location of the core should be selected so as to minimize eccentricities for lateral load. The core walls need to have adequate torsional resistance for possible asymmetry of the core system, where the center of the resultant shear load is acting at an eccentricity from the center of the lateral force resistance.

A simple cantilever model should be adequate to analyze a core wall structure. However, if the structural form is a tube with openings for access, it may be necessary to perform a more accurate analysis to include the effect of openings. The walls can be analyzed by a finite element analysis using thin-walled plate elements. An analysis of this type may also be required to evaluate torsional stresses when the vertical profile of the core wall assembly is asymmetrical.

The concrete core walls can be constructed using slip-form techniques, where the core walls could be advanced several floors (typically four to six stories) ahead of the exterior steel framing. The core wall system represents an efficient type of structural system up to a certain height premium because of its
cantilever action. However, when it is used alone, the massiveness of the wall structure increases with height, thereby inhabiting the free planning of interior spaces, especially in the core. The space occupied by the shear wall leads to loss of overall floor area efficiency, compared to the tube system, which could otherwise be used.

In commercial buildings where floor space is valuable, the large area taken up by a concrete column can be reduced by the use of an embedded steel column to resist the extreme loads encountered in tall buildings. Sometimes, particularly at the bottom open floors of a high-rise structure, where large open lobbies or atriums are utilized as part of the architectural design, a heavy embedded steel section as part of a composite column is necessary to resist high load and because of the large unbraced length. A heavy steel section in a composite column is often utilized where the column size is restricted architecturally and where reinforcing steel percentages would otherwise exceed the code’s maximum allowed values for the design of reinforced concrete columns.

**Moment–Truss Systems**

Vertical shear trusses located around the inner core in one or both directions can be combined with perimeter moment-resisting frames in the facade of a building to form an efficient structure for lateral load resistance. An example of a building consisting of moment frames with shear trusses located at the center is shown in Fig. 51.50(a). For the vertical trusses arranged in the North–South direction, either the K or X form of bracing is acceptable, since access to lift shafts is not required. However, K trusses

![Diagram of Moment Frame with Internal Braced Trusses](image1)

**FIGURE 51.50** (a) Moment frame with internal braced trusses. (b) Moment frame with internal core walls.
are often preferred, because in the case of the X or single-brace form, bracing the influence of gravity loads is rather significant. In the East–West direction, only knee bracing is effective in resisting lateral load.

In some cases internal bracing can be provided using concrete shear walls, as shown in Fig. 51.50(b). The internal core walls substitute the steel trusses in K, X, or single-brace form, which may interfere with openings that provide access to, for example, elevators.

The interaction of shear frames and vertical trusses produces a combination of two deflection curves with the effect of more efficient stiffness. These moment frame–truss interacting systems are considered to be the most economical steel systems for buildings up to 40 stories. Figure 51.51 compares the sway characteristic of a 20-story steel frame subjected to the same lateral forces, but with different structural schemes, namely, (1) unbraced moment frame, (2) simple truss frame, and (3) moment–truss frame. The simple truss frame helps to control lateral drift at the lower stories, but the overall frame drift increases toward the top of the frame. The moment frame, on the other hand, shows an opposite characteristic for side sway, compared with the simple braced frame. The combination of the moment frame and the truss frame provides overall improvement in reducing frame drift; the benefit becomes more pronounced toward the top of the frame. The braced truss is restrained by the moment frame at the upper part of the building, while at the lower part, the moment frame is restrained by the truss frame. This is because the slope of frame sway displacement is relatively smaller than that of the truss at the top, while the proportion is reversed at the bottom. The interacting forces between the truss frame and moment frame, as shown in Fig. 51.52, enhance the combined moment–truss frame stiffness to a level larger than the summation of individual moment frame and truss stiffness.

**Outrigger and Belt Truss Systems**

Another significant improvement of lateral stiffness can be obtained if the vertical truss and the perimeter shear frame are connected on one or more levels by a system of outrigger and belt trusses. Figure 51.53 shows a typical example of such a system. The outrigger truss leads the wind forces of the core truss to...
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The exterior columns, providing cantilever behavior of the total frame system. The belt truss in the facade improves the cantilever participation of the exterior frame and creates a three-dimensional frame behavior.

Figure 51.54 shows a schematic diagram that demonstrates the sway characteristic of the overall building under lateral load. Deflection is significantly reduced by the introduction of the outrigger–belt trusses. Two kinds of stiffening effects can be observed: one is related to the participation of the external columns, together with the internal core, to act in a cantilever mode; the other is related to the stiffening of the external facade frame by the belt truss to act as a three-dimensional tube. The overall stiffness can be increased up to 25%, compared to the shear truss and frame system without such outrigger–belt trusses.

The efficiency of this system is related to the number of trussed levels and the depth of the truss. In some cases the outrigger and belt trusses have a depth of two or more floors. They are located in service floors where there are no requirements for wide open spaces.

Frame Tube Systems

Figure 51.55 shows a typical frame tube system, which consists of a frame tube at the exterior of the building and gravity steel framing at the interior. The framed tube is constructed from large columns
FIGURE 51.54 Improvement of lateral system using outrigger and belt trusses.

FIGURE 51.55 Composite tube system.
placed at close centers connected by deep beams, creating a punched-wall appearance. The exterior frame tube structure resists all lateral loads of wind or earthquake, whereas the simple steel framing in the interior resists only its share of gravity loads. The behavior of the exterior frame tube is similar to that of a hollow perforated tube. The overturning moment under the action of lateral load is resisted by compression and tension of the leeward and windward columns. The shear is resisted by bending of the columns and beams at the two sides of the building parallel to the direction of the lateral load.

Deepening on the shear rigidity of the frame tube, there may exist a shear lag across the windward and leeward sides of the tube. As a result, not all flange columns resist the same amount of axial force. An approximate approach is to assume an equivalent column model, as shown in Fig. 51.56. In the calculation of the lateral deflection of the frame tube it is assumed that only the equivalent flange columns on the windward and leeward sides of the tube and the web frames would contribute to the moment of inertia of the tube.

The use of an exterior framed tube has two distinct advantages: (1) it develops high rigidity and strength for torsional and lateral load resistance, since the structural components are effectively placed at the exterior of the building, forming a three-dimensional closed section; and (2) the massiveness of frame tube system eliminates potential uplift difficulties and produces better dynamic behavior. The use of simple steel framing in the interior has the advantage of flexibility and enables rapid construction.

Composite columns are commonly used in the perimeter of the building where the closely spaced columns are rigidly connected by deep spandrel beams to form a three-dimensional cantilever tube. The exterior frame tube significantly enhances the structural efficiency in resisting lateral loads and thus reduces the shear wall requirements. However, in cases where a higher magnitude of lateral stiffness is required (such as for very tall buildings), internal wall cores and interior columns with floor framing can be added to transform the system into a tube-in-tube system. The concrete core may be strategically located to recapture elevator space and to provide transmission of mechanical ducts from shafts and mechanical rooms.

**Steel–Concrete Composite Systems**

Steel–concrete composite construction has gained wide acceptance as an alternative to pure steel and pure concrete construction. Composite building systems can be broadly categorized into two forms: one utilizes the core braced system by means of interior shear walls, and the other utilizes exterior framing to form a tube for lateral load resistance. Combining these two structural forms will enable taller buildings to be constructed.
For composite frames resisting gravity load only, the beam-to-column connections behave as they do when pinned before the placement of concrete. During construction, the beam is designed to resist concrete dead load and the construction load (to be treated as a temporary live load). At the composite stage, the composite strength and stiffness of the beam should be utilized to resist the full design loads. For simple frames consisting of bare steel columns and composite beams, there is now sufficient knowledge available for the designer to use composite action in the structural element, as well as the semirigid composite joints, to increase design choices, leading to more economical solutions. Practical design guidelines for semicontinuous composite braced frames are given in Liew et al. [40]. Deflection equations are derived, and vibration studies were conducted.

Figure 51.57 shows two typical beam-to-column connections: one using a flushed end plate bolted to the column flange, and the other using a bottom angle with double web cleats. Composite action in the joint is developed based on the tensile forces in the rebars that act with the balancing compression forces transmitted by the lower portion of the steel section that bears against the column flange to form a couple. Properly designed and detailed composite connections are capable of providing moment resistance up to the hogging resistance of the connecting members.

In designing the connections, slab reinforcements placed within a horizontal distance of six times the slab depth are assumed to be effective in resisting the hogging moment. Reinforcement steels that fall outside this width should not be considered in calculating the resisting moment of the connection (see Fig. 51.58). The connections to edge columns should be carefully detailed to ensure adequate anchorage of rebars. Otherwise, they shall be designed and detailed as simply supported. In braced frames a moment connection to the exterior column will increase the moments in the column, resulting in an increase of column size. Although the moment connections restrain the column from buckling by reducing the effective length, this is generally not adequate to offset the strength required to resist this moment.

For an unbraced frame subjected to gravity and lateral loads, the beam is typically bent in double curvature with hogging moment at one end of the beam and sagging moment at the other. The concrete is assumed to be ineffective in tension; therefore, only the steel beam stiffness on the hogging moment region and the composite stiffness on the sagging moment region can be utilized for frame action. The frame analysis can be performed with variable moments of inertia for the beams, and the second-order effect can be included in the advanced analysis [41].

If semirigid composite joints are used in unbraced frames, the flexibility of the connections will contribute to additional drift over that of a fully rigid frame. In general, semirigid connections do not require the column size to be increased significantly over an equivalent rigid frame. This is because the design of frames with semirigid composite joints takes advantage of the additional stiffness in the beams provided by the composite action. The increase in beam stiffness would partially offset the additional flexibility introduced by the semirigid connections.
Methods for an accurate modeling of effective stiffness of composite members in unbraced frames, including second-order effects, are reported in Liew and Uy [42]. Advanced analysis for modeling beam-to-column semirigid connections, steel frames, and concrete core wall interaction is reported in Liew [43]. Further research is required to assess the performance of various types of composite connections used in building structures with mixed systems.

**Notation**

- $A_c$: Area of concrete
- $A_r$: Area of reinforcement
- $A_s$: Area of steel section
- $A_w$: Web area of steel section
- $B_1, B_2$: Factors used in determining $M_n$ for combined bending and axial forces when elastic, first-order analysis is employed
- $b$: Breadth of steel section
- $b_c$: Overall breadth of composite column
- $c_1, c_2, c_3$: Numerical coefficients
- $c_r$: Thickness of concrete cover
- $D_o$: Outer diameter of circular hollow steel section
- $d$: Depth of steel section


$E_a$  Modulus of elasticity of steel section  
$E_c$  Modulus of elasticity of concrete for long term  
$E_{cd}$  Modulus of elasticity of concrete for short term  
$E_{cm}$  Secant modulus of the concrete  
$E_e$  Modulus of elasticity of reinforcement  
$e$  Eccentricity of applied loading  
$e_o$  Initial imperfection  
$(EI)_e$  Effective elastic flexural stiffness of a composite cross section  
$F$  Force in the element  
$F_v$  Average shear force  
$f_{cr}$  Characteristic strength of concrete due to confinement effect  
$f_{cd}$  Design strength of concrete  
$f_{ck}$  Characteristic cylinder strength of concrete  
$f_{cu}$  Characteristic cube strength of concrete  
$f_{ed}$  Design strength of reinforcement  
$f_{ek}$  Nominal yield strength of reinforcement  
$f_u$  Ultimate strength of the shear connector  
$f_y$  Characteristic strength of reinforcement  
$f_{sd}$  Design strength of steel section  
$G_a$  Shear modulus of steel  
$h_c$  Overall width of composite column  
$h_1$  Concrete width perpendicular to plane of bending  
$h_2$  Concrete thickness in plane of bending  
$I_a$  Second moment of area of steel section  
$I_c$  Second moment of concrete  
$I_s$  Second moment of area reinforcement  
$L$  Length of column  
$l_e$  Effective length of column  
$M_{Sd}$  Design moment applied to steel section  
$M_c$  Moment capacity of composite section  
$M_{cs,Sd}$  Design moment applied to concrete and reinforcement  
$M_{fr}$  Required flexural strength in member due to lateral frame transition  
$M_{r}$  Required flexural strength of column  
$M_{rd}$  Required flexural strength in member, assuming there is no lateral transiton of frame  
$M_{pl,c,Rd}$  Plastic moment resistance of concrete and reinforcement  
$M_{pl,Rd}$  Plastic moment resistance of the composite cross section  
$M_f$  Plastic moment capacity of steel beam alone  
$M_{sd}$  Design moment applied to composite column  
$M_p$  Moment capacity of column  
$N_{a,Rd}$  Resistance to compression of steel section  
$N_{a,Sd}$  Design axial load applied to steel section  
$N_{cr}$  Elastic buckling load of column  
$N_{cz,L}$  Elastic critical load of composite column based on system length  
$N_{cz,Sd}$  Design axial load applied to concrete and reinforcement  
$N_{G,Sd}$  Part of the design load acting permanently on column  
$N_{pl,R}$  Plastic resistance of composite column  
$N_{pl,Rd}$  Resistance to compression of composite cross section  
$N_{sd}$  Design axial force of column  
$N_a$  Squash load of column  
$P_{Rd}$  Design resistance of headed-stud connector  
$p_y$  Design strength of structural steel (N/mm$^2$)  
$r_m$  Modified radius of gyration of steel shape, pipe, or tubing in composite columns  
$r_y$  Radius of gyration of a member about its minor axis
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References

4. Faber, O., Savings to be effected by the more rational design of cased stanchions as a result of recent full size tests, Struct. Eng., 88–109, 1956.

\[ S_x \] Plastic section modulus
\[ t_f \] Flange thickness
\[ t_w \] Web thickness
\[ t_{wd} \] Reduced web thickness
\[ V_{pl, Rd} \] Shear resistance of steel section
\[ V_{sd} \] Design shear force resisted by steel section
\[ W_p \] Plastic section modulus
\[ Z_x \] Elastic section modulus
\[ \alpha \] Imperfection factor
\[ \alpha_c \] Strength coefficient of concrete
\[ \beta \] Equivalent moment factor
\[ \chi \] Reduction factor for buckling
\[ \chi_{pm} \] Axial resistance ratio due to concrete
\[ \delta \] Steel contribution ratio
\[ \epsilon \] Constant \((275/p_c)^{1/2}\)
\[ \gamma_s \] Partial factor of safety for steel section
\[ \gamma_c \] Partial factor of safety for concrete
\[ \gamma_s \] Partial factor of safety for reinforcement
\[ \lambda \] Slenderness of column
\[ \mu \] Moment resistance ratio or coefficient of friction
\[ \mu_1, \mu_2 \] Coefficients used for evaluating confinement effect
\[ \mu_{10}, \mu_{20} \] Coefficients used for evaluating confinement effect
\[ \rho \] Density of concrete

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16. EN 10080, Steel for the Reinforcement of Concrete, draft.
34. Galambos, T.V. and Chapuis, J., LRFD Criteria for Composite Columns and Beam Columns, revised draft, Washington University, Department of Civil Engineering, St. Louis, MO, 1980.

Further Information