35
Sediment Transport in Open Channels

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35.1 Introduction

The erosion, deposition, and transport of sediment by water arise in a variety of situations with engineering implications. Erosion must be considered in the design of stable channels or the design for local scour around bridge piers. Resuspension of possibly contaminated bottom sediments have consequences for water quality. Deposition is often undesirable since it may hinder the operation, or shorten the working life, of hydraulic structures or navigational channels. Sediment traps are specifically designed to promote the deposition of suspended material to minimize their downstream impact, e.g., on cooling water inlet works, or in water treatment plants. A large literature exists on approaches to problems involving sediment transport; the following can only introduce the basic concepts in summary fashion. It is oriented primarily to applications in steady uniform flows in a sand-bed channel; problems involving flow nonuniformity, unsteadiness, and gravel-beds, are only briefly mentioned and coastal processes are treated in the section on coastal engineering. Cohesive sediments, for which physico-chemical attractive forces may lead to the aggregation of particles, are not considered at all. The finer fractions (clays and silts, see Section 35.2) that are susceptible to aggregation are found more in estuarial and coastal shelf regions rather than in streams. A recent review of problems in dealing with cohesive sediments is given by Mehta et al. (1989 a, b).

35.2 The Characteristics of Sediment

Density, Size, and Shape

The density of sediment depends on its composition. Typical sediments in alluvial water bodies consist mainly of quartz, the specific gravity of which can be taken as $s = 2.65$. The specific weight is therefore $\gamma_s = 165.4$ lb/ft,^{3} or 26.0 kN/m.^{3} In many formulae, the effective specific weight, which includes the effect of buoyancy, is used, i.e., $(s–1)\gamma$, where γ is the specific weight of water.

The exact shape of a sediment particle is not spherical, and so a compact specification of its geometry or size is not feasible. Two practical measures of grain size are: (i) the *sedimentation or aerodynamic diameter* — the diameter of the sphere of the same material with the same fall velocity, w_s, (see below for definition) under the same conditions, and (ii) the *sieve diameter* — the length of a side of the square sieve opening through which the particle will just pass. Because size determination is most often performed with sieves, the available data for sediment size usually refer to the sieve diameter, which is taken to be the geometric mean of the adjacent sieve meshes, i.e., the mesh size through which the particle has passed, and the mesh size at which the particle is retained. The sedimentation diameter is related empirically to the sieve diameter by means of a *shape factor*, S.F., which increases from 0 to 1 as the particle becomes more spherical (for a well-worn sand, S.F. ≈ 0.7).

Size Distribution

Naturally occurring sediment samples exhibit a range of grain diameters. A characteristic diameter, d_a, may be defined in terms of the percent, a, by weight of the sample that is smaller than d_a. Thus, for a sample with $d_{84} = 0.35$ mm, 84% by weight of the sample is less than 0.35 mm in diameter. The *median* size is denoted as d_{50}. Frequently, the grain size distribution is assumed to be *lognormally* distributed, and a geometric mean diameter and standard deviation are defined as $d_g = \sqrt{d_{16}d_{84}}$, and $\sigma_g = \sqrt{d_{84}/d_{16}}$. For a lognormal distribution, $d_{50} = d_g$, and the arithmetic mean diameter, $d_m = d_g e^{0.5\ln(2\sigma_g)}$. Similarly, d_a can be determined from d_g and σ_g from the relation, $d_a = d_g\sigma_g^{Z_a}$, where Z_a is the standard normal variate corresponding to the value of a. For example, if $a = 65\%$, $d_g = 0.35$ mm, and $\sigma_g = 1.7$, then $Z_a = 0.39$, and so $d_{65} = (0.35 \text{ mm})(1.7)^{0.39} = 0.43$ mm. In natural sand-bed streams, σ_g typically ranges between 1.4 and 2, but in gravel-bed streams, it may attain values greater than 4. Qualitative discussions of sediment size may be based on a standard sediment grade scale terminology established by the American Geophysical Union. A simplified grade scale divides the size range into cobbles and boulders ($d > 64$ mm), gravels ($2 \text{ mm} < d < 64$ mm), sands ($0.06 \text{ mm} < d < 2$ mm), silts ($0.004 \text{ mm} < d < 0.06$ mm), and clays ($d < 0.004$ mm).

Fall (or Settling) Velocity

The *terminal velocity* of a particle falling alone through a stagnant fluid of infinite extent is called its fall or settling velocity, w_s. The standard drag curve for a spherical particle provides a relationship between d and w_s (see chapter on Fundamentals of Hydraulics). For non-spherical sand particles *in water*, the fall velocity at various temperatures can be determined from Fig. 35.1 if the sieve diameter and S.F. are known or can be assumed (note the different fall velocity scales). As an example, for a geometric sieve diameter of 0.3 mm and a shape factor, S.F. = 0.7, the fall velocity in water at 10°C is determined as ≈ 3.6 cm/s. In a horizontally flowing turbulent suspension, the actual mean fall velocity of a given particle may be influenced by neighboring particles (hindered settling) and by turbulent fluctuations.

Angle of Repose

The *angle of repose* of a sediment particle is important in describing the initiation of its motion and hence sediment erosion of an inclined surface, such as a stream bank. It is defined as the angle, θ, at which the particle is just in equilibrium with respect to sliding due to gravitational forces. It will vary

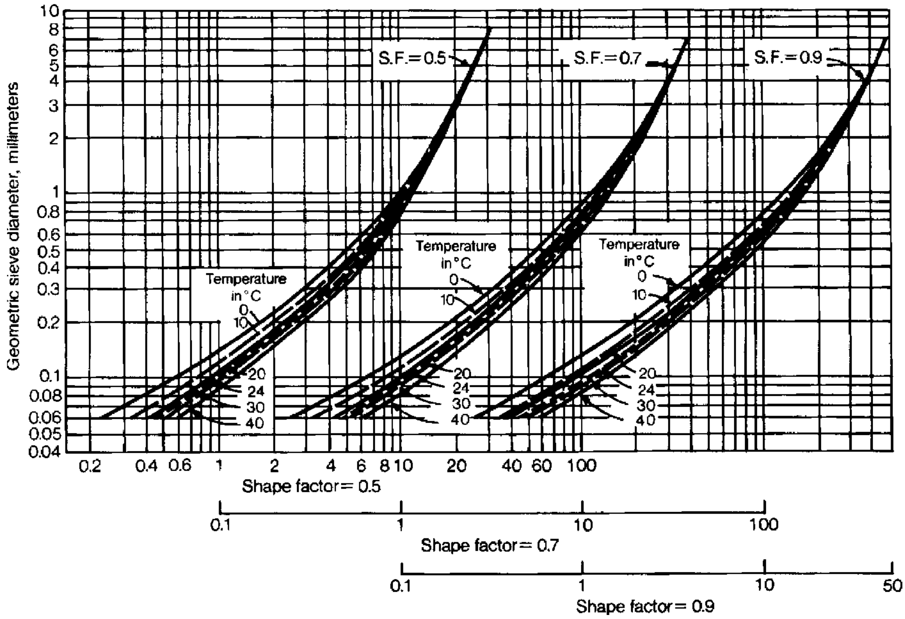

FIGURE 35.1 Relationship between fall velocity, sand-grain diameter, and shape factor (taken from Vanoni, 1975).

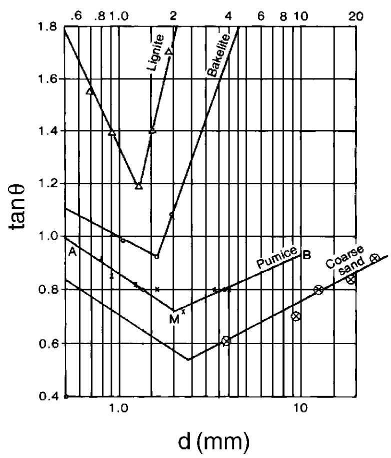

FIGURE 35.2 Angle of repose as a function of size and shape (adapted from Simon and Sentürk, 1992).

with particle size, shape, and density, and empirical curves for some of these variations are given in Fig. 35.2. The angle of repose for riprap, large stones or rock in layer(s) often used for stabilization of erodible banks, was given in simpler form (Fig. 35.3) by Anderson (1973) as part of a procedure for the design of channel linings. A value of 40° for the angle of repose is sometimes suggested as a design value for riprap.

35.3 Flow Characteristics and Dimensionless Parameters; Notation

The important flow characteristics are those associated with open-channel or more generally free-surface flows (see the chapters on Open Channel Hydraulics or the Fundamentals of Hydraulics for more details). These are the total water discharge, Q (or for wide or rectangular channels, the discharge per unit width,

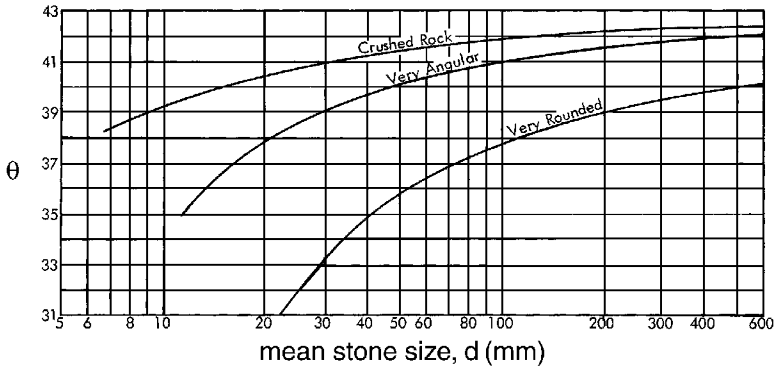

FIGURE 35.3 Angle of repose for riprap (from Anderson, 1973).

$q = Q/B$, where B is the width of the channel), the mean velocity, $V = Q/A$, where A is the channel cross-sectional area, the hydraulic radius, R_h, (or for a very wide channel the flow depth, $R_h \approx H$), and the energy or friction slope, S_f. The total bed shear stress, τ_b, and the related quantities, the *shear velocity*, $u_* = \sqrt{\tau_b/\rho} = \sqrt{gR_hS_f}$, where g is the gravitational acceleration, and friction factor, $f = 8(u_*/V)^2$, are also important.

Much of sediment transport engineering remains highly empirical, and so the organization of information in terms of dimensionless parameters becomes important (see the discussion of dimensional analysis in the chapter on Fundamentals of Hydraulics). Sediment and flow quantities may be combined in several dimensionless parameters that arise repeatedly in sediment transport. A dimensionless bed shear stress, also termed the *Shields parameter* (see Section 35.4), can be defined as

$$\Theta \equiv \frac{\tau_b}{\gamma(s-1)d} = \frac{u_*^2}{g(s-1)d} = \frac{R_hS_f}{(s-1)d} \tag{35.1}$$

Two *grain Reynolds numbers* based on the grain diameter can be usefully defined as

$$\mathrm{Re}_g \equiv \frac{\sqrt{g(s-1)d^3}}{\nu} \quad \text{and} \quad \mathrm{Re}_* \equiv \frac{u_*d}{\nu} \tag{35.2}$$

where ν is the fluid kinematic viscosity. Since $Re_g^2 \propto d^3$, a definition of a dimensionless diameter may be motivated as $d_* = Re_g^{2/3}$.

A *grain 'Froude' number* also based on grain diameter can be defined as

$$Fr_g \equiv \frac{V}{\sqrt{g(s-1)d}} = \left(\frac{V}{u_*}\right)\sqrt{\Theta} = \sqrt{\frac{8\Theta}{f}} \tag{35.3}$$

A dimensionless sediment discharge per unit width, Φ, may be defined as:

$$\Phi \equiv \frac{g_s/\gamma_s}{\sqrt{g(s-1)d^3}} \tag{35.4}$$

where $g_s = \gamma q\bar{C}$ is the weight flux of sediment per unit width and $\bar{C}$ is the flux-weighted mass or weight concentration of sediment (see Section 35.6 for more details).

In the above definitions, various characteristic grain diameters and shear velocities may be used according to the context.

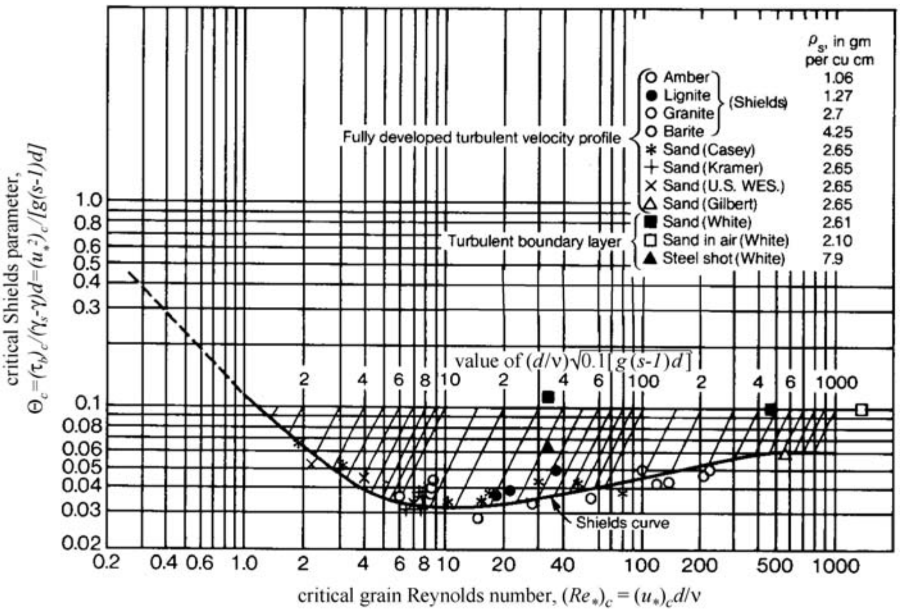

FIGURE 35.4 The Shields diagram relating critical shear stress to hydraulic and particle characteristics (adapted from ASCE Sedimentation Engineering, 1975).

35.4 Initiation of Motion

The Shields Curve and the Critical Shear Stress

A knowledge of the hydraulic conditions under which the transport of sediment in an alluvial channel begins or is initiated is important in numerous applications, such as the design of stable channels, i.e., channels that will not suffer from erosion, or bank stabilization, or remedial measures for scour. A criterion for the initiation of general sediment transport in a turbulent channel flow may be given in terms of a critical bed shear stress, $(\tau_b)_c = \rho(u_*)_c^2$, above which *general* motion of bed sediment of mean diameter, d, is observed. The *Shields curve* (Fig. 35.4) correlates a critical dimensionless bed shear stress, Θ_c, to a critical grain Reynolds number, $(Re_*)_c$, where $(u_*)_c$ is used in defining both Θ_c and $(Re_*)_c$. The Shields curve is an implicit relation, and so a solution for $(u_*)_c$ must be obtained iteratively. For large $(Re_*)_c$ (i.e., for coarse sediment), $\Theta_c \to \approx 0.06$, which provides a convenient initial guess for iteration. Also drawn on Fig. 35.4 are straight oblique lines along which an auxiliary parameter, $(d/\nu)\sqrt{0.1g(s-1)d} = \sqrt{0.1Re_g}$ is constant. This parameter does not involve $(u_*)_c$, and so, provided d and ν are known, $(u_*)_c$ can be directly determined by the intersection of these lines with the Shields curve. Various formulae or curve-fits have been proposed for describing the Shields curve; one example is due to Brownlie (1981) and involves the auxiliary parameter, $Y \equiv Re_g^{-0.6}$,

$$\Theta_c = 0.22Y + 0.06 \times 10^{-7.7Y} \tag{35.5}$$

Example 35.1

Given a sand ($s = 2.65$) grain with $d = 0.4$ mm in water with $\nu = 0.01$ cm^2/s, what is the critical shear stress? The iterative procedure based on the graphical Shields curve starts with an initial guess, $\Theta_c = 0.06$, implying $(u_*^2)_c = 2.0$ cm^2/s^2 and $(Re_*)_c = 7.9$. This is inconsistent with the Shields curve, which indicates $\Theta_c = 0.032$ for $(Re_*)_c = 7.9$. The procedure is iterated by making another guess, $\Theta_c = 0.032$, which yields $(\tau_b)_c = 0.21$ kPa corresponding to $(Re_*)_c = 5.8$. This result is sufficiently consistent with the Shield curve, and so the iteration can be stopped. More directly, the auxiliary parameter, $(d/\nu)\sqrt{0.1g(s-1)d} = 10$, can be computed, and the line corresponding to this value intersects the Shields curve at $\Theta_c = 0.034$. The use of the Brownlie empirical formula (Eq. [35.5]) gives, with $Re_g = 32.2$ and $Y = 0.125$, more directly $\Theta_c = 0.034$.

Instead of using $(\tau_b)_c$, traditional procedures for the design of stable channels have often been formulated in terms of a critical average velocity, V_c, or critical unit-width discharge, q_c, above which sediment transport begins, because these quantities are more easily available than the bed shear stress. If a relationship between V and τ_b, namely a friction or flow resistance law, then V_c can be derived from $(\tau_b)_c$, and this is discussed in Section 35.5.

The Effect of Slope

The above criterion is applicable to grains on a surface with negligible slope, as is usually the case for grains on the channel bed. Where the slope of the surface on which grains are located is appreciable, e.g., on a river bank, its effect cannot be neglected. With the inclusion of the additional gravitational forces, a force balance reveals that $(\tau_b)_c$ is reduced by a fraction involving the angle of repose of the grain, and the ratio of the value of $(\tau_b)_c$ *including slope effects* to its value for a horizontal surface is given by:

$$K_{slope} = \frac{\left[(\tau_b)_c\right]_{slope}}{\left[(\tau_b)_c\right]_{zero\ slope}} = \left(1 - \frac{\sin^2\phi}{\sin^2\theta}\right)^{1/2} \tag{35.6}$$

where ϕ = the angle of the sloping surface
θ = the angle of repose of the grain.

On a horizontal surface, $\phi = 0$, and the ratio is unity, whereas if $\phi = \theta$, then no shear is required to initiate sediment motion (consistent with the definition of the angle of repose).

Summary

Although the Shields curve is widely accepted as a reference, controversy remains concerning its details and interpretation, e.g., its behavior for small $(Re_*)_c$ (Raudkivi, 1990) and the effect of fluid temperature (Taylor and Vanoni, 1972). The random nature of turbulent flow and random magnitudes of the instantaneous bed shear stresses motivate a probabilistic approach to the initiation of sediment motion. The critical shear stress given by the Shields curve can be accordingly interpreted as being associated with a probability that sediment particle of given size will begin to move. It should *not* be interpreted as a criterion for zero sediment transport, and design relations for zero transport, if based on the Shields curve, should include a significant factor of safety (Vanoni, 1975).

35.5 Flow Resistance and Stage-Discharge Predictors

The stage-discharge relationship or rating curve for a channel relating the uniform-flow water level (stage) or hydraulic radius, R_h, to the discharge, Q, is determined by channel flow resistance. For flow conditions above the threshold of motion, the erodible sand bed is continually subject to scour and deposition, so that the bed acts as a deformable or 'movable' free surface. The plane bed, i.e., one in which large-scale features are absent, is often unstable, and bedforms (Fig. 35.5) such as *dunes, ripples,* and *antidunes*, develop. Dunes, which exhibit a mild upstream slope and a sharper downstream slope, are the most commonly occurring of bedforms in sand-bed channels. Ripples share the same shape as dunes, but are smaller in dimensions. They may be found in combination with dunes, but are generally thought to be unimportant except in streams at small depths and low velocities. Antidunes assume a smoother more symmetric sinusoidal shape, which results in less flow resistance, and are associated with steeper streams. Antidunes differ from dunes in moving upstream rather than downstream, and in being associated with water surface variations that are *in phase* rather than out of phase with bed surface variations.

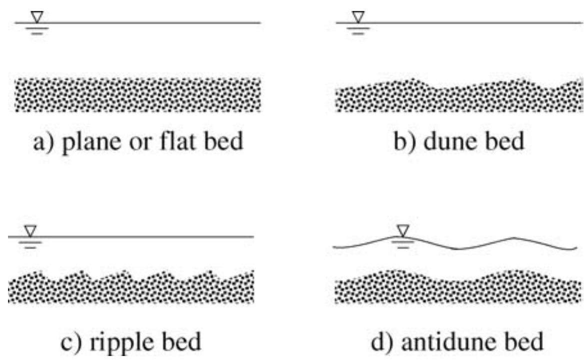

FIGURE 35.5 Various bedforms.

In fixed-bed open-channel flows, resistance is characterized by a Darcy-Weisbach friction factor, f (see chapter on Fundamentals of Hydraulics) or a Manning's n (see chapter on Open Channel Hydraulics), which is assumed to vary only slowly or not at all with discharge. For movable or erodible beds, substantial changes in flow resistance may occur as the bedforms develop or are washed out. Very loosely, as transport intensity (as measured, e.g., by the Shields parameter, Θ) increases, ripples evolve into dunes, which in turn become plane or transition beds, to be followed by antidunes. Multiple depths may be consistent with the same discharge or velocity (Fig. 35.6), and the rating curve (the relationship between stage or depth and discharge or velocity) may exhibit discontinuities. These discontinuities are attributed to a short-term transition from low-velocity high-resistance flow over ripples and dunes, termed *lower-regime flow*, to high-velocity low-resistance flow over plane, transition or antidune bed, termed *upper-regime flow* or vice-versa. Because of these two possible regimes, movable-bed friction formulae (unlike fixed-bed friction formulae) must include a method to determine the flow regime.

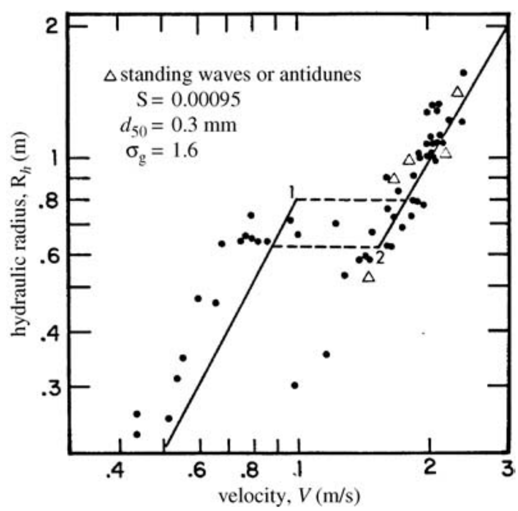

FIGURE 35.6 Stage-discharge data reported by Dawdy (1961) for the Rio Grande River near Bernalillo, New Mexico. (Adapted from Brownlie, 1981.)

Form and Grain Resistance Approach

In flows over dunes and ripples, form resistance due to flow separation from dune tops provides the dominant contribution to overall resistance. Yet the processes involved in determining bedform characteristics are more directly related to the actual bed shear stress (as in the problem of initiation of motion). Much of sediment transport modeling has distinguished between *form and grain* (*skin*) *resistance* (see the section on hydrodynamic forces in the chapter on Fundamentals of Hydraulics for the distinction between the two types of flow resistance). An overall bed shear stress, $(\tau_b)_{overall} \equiv \gamma R_j S_f$, is taken as the sum of a contribution due to grain resistance, τ', and a contribution due to form resistance, τ''. Since $(\tau_b)_{overall}$ is usually correlated empirically with τ', it remains only to determine τ' from given hydraulic parameters. The traditional approach estimates τ' from fixed-bed friction formulae for plane beds. A simple effective example of this approach to stage-discharge prediction is due to Engelund and Hansen (1967) (with extension by Brownlie (1983)) and correlates a total overall dimensionless shear stress, Θ, with a dimensionless grain shear stress, Θ':

Engelund-Hansen formula

$$\Theta = 1.58\sqrt{\Theta' - 0.06}, \qquad 0.06 \le \Theta' \le 0.55, \quad (\text{lower regime}) \tag{35.7a}$$

$$= \Theta', \qquad 0.55 \le \Theta' \le 1, \quad (\text{upper regime}) \tag{35.7b}$$

$$= \left[1.425(\Theta')^{-1.8} - 0.425\right]^{1.8} \qquad 1 \le \Theta' \quad (\text{upper regime}) \tag{35.7c}$$

where $\Theta'(\equiv R_h'S_f/(s-1)\ d_{50})$ is related to V by a friction formula for a plane fixed bed:

$$\frac{V}{\sqrt{gR_h'S_f}} = 5.76\log_{10}\frac{5.51R_h'}{d_{65}} \tag{35.8}$$

The lower regime corresponds to flows over dune-covered beds with dominant contribution due to form resistance, such that $\Theta > \Theta'$ for values of Θ' not too close to 0.06, whereas in the transition or upper regime, corresponding to plane beds or beds with antidunes, $\Theta = \Theta'$, because flow resistance is expected to be due primarily to grain resistance, comparable in this respect to plane beds. The Engelund-Hansen formula was originally developed based on large-flume laboratory data with d_{50} in the range 0.19 mm to 0.93 mm, and σ_g of 1.3 for the finest sediment and 1.6 for the others.

Overall Resistance Approach

The distinction between grain (skin) and form resistance is physically sound, but the use of a plane fixed-bed friction formula such as Eq. (35.8) cannot be justified rigorously for beds with dunes and ripples, and the need for a further correlation between Θ and Θ' is inconvenient. A simpler more direct approach relating Θ (or R_h) directly to Q or other dimensionless parameters may therefore be more attractive from an engineering point of view. Guided by dimensional analysis, Brownlie (1983) performed regression analyses on a large data set of laboratory and field measurements, and proposed the following stage-discharge formulae:

Brownlie formulae

$$\frac{R_h}{d_{50}} = 0.0576(s-1)^{0.95} Fr_g^{1.89} S_f^{-0.74} \sigma_g^{0.3}, \text{ lower regime,} \tag{35.9a}$$

$$= 0.0348(s-1)^{0.83} Fr_g^{1.67} S_f^{-0.77} \sigma_g^{0.21}, \text{ upper regime,} \tag{35.9b}$$

where σ_g = the geometric standard deviation
Fr_g = the grain Froude number (Eq. [35.3])

To determine whether the flow is in lower or upper regime, the following criteria are applied:

- for $S_f > 0.006$, only upper regime flow is observed,
- for $S_f < 0.006$, additional criteria are formulated in terms of a modified grain Froude number, $Fr_g^* \equiv Fr_g/[1.74S_f^{-1/3}]$, and a modified grain Reynolds number, $\Delta \equiv u_*'d_{50}/(11.6\nu)$, where u_*' is the shear velocity corresponding to the upper regime flow, i.e., due primarily to grain resistance.
- the **lower** limit of the **upper regime** is given as

$$\log_{10}\left(Fr_g^*\right)_{up} = -0.0247 + 0.152\log_{10}\Delta + 0.838\left(\log_{10}\Delta\right)^2, \quad \Delta < 2 \tag{35.10a}$$

$$= \log_{10}1.25, \qquad \Delta \ge 2 \tag{35.10b}$$

- the **upper** limit of the **lower regime** is given as

$$\log_{10}\left(Fr_g^*\right)^{low} = -0.203 + 0.0703\log_{10}\Delta + 0.933\left(\log_{10}\Delta\right)^2, \qquad \Delta < 2, \tag{35.11a}$$

$$= \log_{10} 0.8, \qquad \Delta \geq 2. \tag{35.11b}$$

- The range of conditions covered by the data set was: $3 \times 10^{-6} < S_f < 3.7 \text{ x}10^{-2}$, 0.088 mm $< d_{50} <$ 2.8mm, 0.012 m^3/s/m $< q <$ 40 m^3/s/m, $0.025 < R_h <$ 17 m, and temperatures between 0°C and 63°C.

Example 35.2

Given a wide alluvial channel with unit width discharge, q = 1.6 m^3/s/m, and bed slope, S_0 = 0.00025, and sand characteristics, d_{50} = 0.35 mm, σ_g = 1.7, estimate the normal flow depth. The approximation is made that, for a wide channel, $R_h \approx H$ and $V = q/H$, where H is the required flow depth. With the Brownlie formulae, substitution of the given values yields for the lower regime (Eq. [35.9a]), H_{low} = 1.80 m, and for the upper regime (Eq. [35.9b]), H_{up} = 1.30 m. Only *one* of these is the appropriate flow; to determine which, the criteria for the two regimes are examined. Since $u'_* = \sqrt{gH_{up}S_0}$ = 0.056 m/s, $\Delta = u'_* d_{50}/11.6\nu$ = 1.70 assuming $\nu = 10^{-6}$ m^2/s. From Eqs. (35.10) to (35.11), the limits of the flow regimes for Δ = 1.70 are determined as: for the upper regime, $(Fr_g^*)_{up}$ = 1.13, and for the lower regime, $(Fr_g^*)^{low}$ = 0.73. The parameter, $Fr_g^* = (q/H)[1.74S_f^{-1/3}\sqrt{g(s-1)d_{50}}]$ is evaluated using $H = H_{up}$ to be Fr_g^* = 0.59 < (Fr_g^*) = 1.13, i.e., below the lower limit of the upper flow regime, and using $H = H_{low}$ to be Fr_g^* = 0.59 < $(Fr_g^*)^{low}$ = 0.73, i.e., below the upper limit of the lower flow regime. The only consistent solution for H is therefore H_{low}, and hence, according to the Brownlie criteria, the flow must then be in the lower regime, and so the normal flow depth is estimated as $H = H_{low}$ = 1.80 m. In some (hopefully infrequent) cases, these criteria may still not be sufficient to give a unique solution.

The Engelund-Hansen procedure involves two unknowns, H as well as H' (the 'fictitious' depth related to grain resistance), which must be solved with the two available equations, namely, the flow resistance relationships (Eqs. [35.7] and [35.8]), together with the requirement that $q = VH$. The solution for H can be obtained with software tools, such as a spreadsheet simultaneous equation solver, or by the following 'manual' iterative procedure. H' is initially guessed, e.g., 1 m, from which Θ' = 0.43. According to Eq. (35.7), this falls in the lower regime, and from Eq. (35.7a), Θ = 0.96, so that H = 2.22 m. From Eq. (35.8), the mean velocity is computed (assuming a lognormal size distribution, $d_{65} \approx$ 0.43 mm for σ_g = 1.7, see Section 35.2) as V = 1.17 m/s, and hence $q = V H$ = 2.60 m^2/s. Since this is not consistent with the given q = 1.6 m^2/s, the iteration is continued. A final iteration yields Θ = 0.76 and H = 1.75 m, which agrees well with the result using the Brownlie formulae.

Critical Velocity

Given a stage-discharge predictor, a formula for the *critical velocity*, V_c, (see section 35.4 for definition) can be obtained. For example, the Brownlie lower regime equation (Eq. [35.9a]) can be expressed in terms of a critical grain Froude number, $(Fr_g)_c$ as

$$\left(Fr_g\right)_c \equiv \frac{V_c}{\sqrt{g(s-1)d_{50}}} = \frac{4.60\Theta_c^{0.53}}{S_f^{0.14}\sigma_g^{0.16}} \tag{35.12}$$

where Θ_c is obtained from the Shields curve (e.g., Eq. [35.5]).

Based on experiments, Neill (1967) gave a simpler design formula intended for zero transport of coarse particles,

$$\left(Fr_g\right)_c^2 = 2.5\left(\frac{H}{d}\right)^{0.2} \tag{35.13}$$

This gives a more conservative result than the Eq. (35.12). A design equation for sizing riprap, very similar to Eq. (35.13), and recommended by the U. S. Army Corps of Engineers (1995) is

$$\frac{d_{30}}{H} = K_R \left(\frac{V}{\sqrt{K_{slope}(s-1)H}} \right)^{2.5} \tag{35.14a}$$

where K_R is an empirical coefficient correcting for various effects such as the vertical velocity distribution and the thickness of the riprap layer, as well as including a safety factor. Although Eq. (35.6) (with θ = 40°) may be used for evaluating K_{slope}, this is found to be rather conservative, and an empirical curve for this factor has been developed.

Example 35.3

A channel is to be designed to carry a discharge of 5 m^3/s on a slope of 0.001. The bed material has a median diameter, d_{50} = 8 mm, and a geometric standard deviation, σ_g = 3. Determine the width and depth at which the channel will not erode. Assume for simplicity a rectangular channel cross-section and rigid banks. For d_{50} = 8 mm, it is found from Eq. (35.5) that Θ_c = 0.054, so from Eq. (35.12), $(Fr_g)_c$ = 2.18 and V_c = 0.78 m/s. This is substituted into the lower regime friction equation, Eq. (35.9a), to give $R_h = BH/(B+2H)$ = 0.72 m. Since $Q = V_c BH$ = 5 m^3/s, H is determined as 0.9 m, and so B = 7.1 m. If Eq. (35.13) is used with a Manning-Strickler friction law, then H is found to be 0.46 m, and B to be 12.4 m, with V_c = 0.86 m/s. Eq. (35.12), being based on Shields curve, is not intended to be used for design for zero transport (see previous remarks in section 35.4), while Eq. (35.13) was intended as a design equation for zero transport and so gives a more conservative value (smaller H and hence smaller bed shear stress for given bed slope).

Summary

The prediction of flow depth in alluvial channels remains an uncertain art with much room for judgement. Estimates using various predictors should be considered and the use of field data specific to the problem should be exploited where feasible to arrive at a range of predictions. *If* the regime is correctly predicted, the better stage-discharge relations are generally reliable to within 10 to 15% in predicting depth.

35.6 Sediment Transport

Three modes of sediment transport are distinguished: *wash load, suspended load,* and *bed load*. Wash load refers to very fine suspended material, e.g., silt, that because of their very small fall velocities, interacts little with the bed. It will not be further considered since it is determined by upstream supply conditions rather than by local hydraulic parameters. Suspended load refers to material that is transported downstream primarily in suspension far from the bed, but which because of sedimentation and turbulent mixing still interacts significantly with the bed. Finally, bed load refers to material that remains generally close to the bed in the bedload region, being transported mainly through rolling or in short hops (termed saltation). The relative importance of the two modes of sediment transport may be roughly inferred from the ratio of settling velocity to shear velocity, w_s/u_*. For $w_s/u_* < 0.5$, suspended load transport is likely dominant, while for $w_s/u_* > 1.5$, bedload transport is likely dominant. The sum of suspended and bed loads is termed *bed-material load* as distinct from the wash load, which may only be very weakly, if at all, related to material found in bed samples.

The total sediment load or discharge, G_T, is considered here as the sum of only the suspended-load discharge, G_S, and the bedload discharge, G_B, and is defined as the mass or more usually the weight flux of sediment material passing a given cross-section (SI units of kg/s or N/s, English units slugs/s or lb/s). A total sediment discharge (by weight) per unit width, based on the flux over the entire depth. is often used:

$$g_T = \gamma q\overline{C} = \gamma\int_0^H u\,c\,dy \quad (35.14b)$$

where u and c = the mean velocity and mass (or weight) concentration at a point in the water column
$\overline{C}$ = a mean flux-weighted mass (or weight) concentration defined by Eq. (35.14a).

Because of a nonuniform velocity profile, $\overline{C}$ is *not* equal to the *depth-averaged concentration*, $\langle C\rangle \equiv (1/H)\int_0^H c\,dy$.

Suspended Load Models

The prediction of g_T given appropriate sediment characteristics and hydraulic parameters has been attempted by treating bed load and suspended load separately, but such an approach is fraught with difficulties. The traditional approach derives a differential equation for conservation of sediment assuming uniform conditions in the streamwise direction:

$$\varepsilon_s \frac{dc}{dy} + w_s c = 0 \quad (35.15)$$

where ε_s is a turbulent diffusion or mixing coefficient for sediment.

The first term represents a net upward sediment flux due to turbulent mixing, while the second term is interpreted as the net downward flux due to settling. A solution for the vertical distribution of sediment concentration, $c(y)$, depends on a model for ε_s, and a boundary condition at or near the bed. The well-known *Rouse concentration profile*,

$$\frac{c(y)}{c_{ref}} = \left(\frac{H-y}{y}\frac{y_{ref}}{H-y_{ref}}\right)^{Z_R} \quad (35.16)$$

with the *Rouse exponent*, $Z_R \equiv w_s/\beta\kappa u_*$, assumes an eddy viscosity mixing model with $\varepsilon_s = \beta u_*\, y(1 - y/H)$, where β is a coefficient relating momentum to sediment diffusion, and the von Kármán constant, κ, stems from the assumption of a log-law velocity profile. It avoids a precise specification of the bottom boundary condition by introducing a reference concentration, c_{ref}, at a reference level $y = y_{ref}$, taken close to the bed. Here $u_* = \sqrt{gR_hS_f}$ refers to the *overall* shear velocity (i.e., not only the shear velocity associated with grain resistance).

Although Eq. (35.16) can usually be made to fit measured concentration profiles approximately with an appropriate choice of Z_R, its predictive use is limited by the lack of information concerning β, κ, and particularly c_{ref}, which may vary with hydraulic and sediment characteristics. In the simplest models, $\beta = 1$ and $\kappa = 0.4$, which assume that sediment diffusion is identical to momentum diffusion and the velocity profile follows the log-law (see section on turbulent flows in the chapter on fundamentals of hydraulics) profile exactly as in plane fixed-bed flows without sediment. More complicated models (e.g., van Rijn, 1984a) have been proposed in which β is correlated with w_s/u_* and κ varies with suspended sediment concentration. The suspended load discharge per unit width may be computed using Eq. (35.16) as

$$g_s = \gamma\int_{y_{ref}}^H u\,c\,dy \quad (35.17)$$

with u typically assumed to be described by a log-law profile. To determine the total load (per unit width), g_T, a formula for predicting g_B, the transport per unit width in the bed-load region, $0 < y < y_{ref}$, must be coupled with Eq. (35.17), and the reference level, y_{ref}, must be chosen at the limit of the bed load region. In flows with bed forms, neither Eq. (35.15) nor Eq. (35.16) can be rigorously justified, since bed conditions are not uniform in the streamwise direction and the log-law velocity profile is inadequate to describe velocity and stress profiles near the bed (Lyn, 1993).

Bed-Load Models and Formulae

Bed-load models are used either in cases where bed load transport is dominant, or to complement suspended-load models in total-load computations. Most available formulae can be written in terms of the dimensionless bed-load transport, Φ_b, and a dimensionless grain shear stress, Θ' (see Section 35.2 for definitions). Only two such models, one traditional and one more recent, are described. The Meyer-Peter-Muller bed-load formula was based on laboratory experiments with coarse sediments (mean diameter range: 0.4 to 30 mm) with very little suspended load.

Meyer-Peter-Muller bed-load formula

$$\Phi_b = 0.08\left(\frac{\Theta'}{\Theta_c} - 1\right)^{3/2}, \quad \frac{\Theta'}{\Theta_c} \geq 1, \tag{35.18}$$

where the dimensionless critical shear stress, $\Theta_c = 0.047$ (note the difference from the generally accepted Shields' curve value of 0.06 for coarse material) and Θ' is the fraction of the dimensionless total shear stress, $\Theta' = (k/k')^{3/2}\Theta$, that is attributed to grain resistance. Based on a plane fully rough fixed-bed friction law of Strickler type, the fraction, k/k', is computed from

$$\frac{k}{k'} = 0.12\left(\frac{d_{90}}{R_h}\right)^{1/6} \frac{U}{\sqrt{gR_hS_f}} \tag{35.19}$$

In the Meyer-Peter-Muller formula, the characteristic grain size used in defining Φ_b and Θ' is the mean diameter, d_m (which can be related to d_g if necessary, see Section 35.2).

A more recent bed-load model due to van Rijn (1984), intended both for predicting bed-load dominated transport as well as for complementing a suspended-load model, is similar in form:

van Rijn bed-load formula

$$\Phi_b = 0.053\frac{\left[\left(\Theta'/\Theta_c\right) - 1\right]^{2.1}}{\mathrm{Re}_g^{0.2}}, \quad \frac{\Theta'}{\Theta_c} \geq 1. \tag{35.20}$$

Θ_c is determined from a Shields curve relation, and Θ' is computed from a fully rough plane-bed friction formula of log-law form (cf. Eq. [35.8]),

$$\frac{V}{u'_*} = 5.75\ \log_{10}\frac{12R_h}{k_s} \tag{35.21}$$

where the equivalent roughness height, $k_s = 3\ d_{90}$. The median grain diameter, d_{50}, is used in defining Φ_b, Θ', and Re_g. In tests with laboratory and field data, Eq. 35.20 performed on average as well as other well-known bed-models including the Meyer-Peter-Muller formula. Equating q_B to a sediment flux based on a reference mass concentration, c_{ref}, at a reference level ($y = y_{ref}$), van Rijn (1984b) obtained an semi-empirical relation for c_{ref} to be used with a suspended-load model

$$c_{ref} = 0.015\ s\left(\frac{d_{50}}{y_{ref}}\right)\frac{\left[\left(\Theta'/\Theta_c\right) - 1\right]^{1.5}}{Re_g^{0.2}}, \quad \frac{\Theta'}{\Theta_c} \geq 1, \tag{35.22}$$

where y_{ref} is chosen to be one-half of a bed form height for lower regime flows, or the roughness height for upper regime flows with a minimum value chosen arbitrarily to be 0.01 H.

Example 35.4

Given quartz (s = 2.65) sediment with d_{50} = 1.44 mm, σ_g = 2.2, in a uniform flow of hydraulic radius, R_h = 0.62 m, in a wide channel of slope, S = 0.00153, and average velocity, V = 0.8 m/s, what is the sediment discharge per unit width? A bed-load dominated sediment discharge is indicated by $w/u_* \approx$ (16 cm/s)/(9.6 cm/s) = 1.6, based on d_{50}, and $u_* = \sqrt{gHS} = 0.096$ m/s. In the Meyer-Peter-Muller formula, $k/k' = 0.43$, where d_{90} = 3.9 mm and u_* = 0.096 m/s. Hence, since d_{m} = 1.96 mm, it is found that Θ' = 0.082. This gives $\Phi_b = 0.052$ from which $g_B = \gamma_s \Phi_b \sqrt{g(s-1)d_m^3} = 0.47$ N/s/m or $\bar{C} = g_b/\gamma q = 97$ ppm by mass. If the van Rijn formula is used, $u'_* = 0.050$ m/s, so that $\Theta' = 0.106$. From the Shields curve, Θ_c = 0.039 for R_g = 219, so that $\Phi_b = 0.056$ or $g_b = 0.32$ N/s/m or $\bar{C} = 66$ ppm. The given parameter values correspond to field measurements in the Hii River in Japan where the reported $\bar{C}$ was 191 ppm (from the data compiled by Brownlie, 1981), which may have included some suspended load as well as wash load.

Total Load Models

The distinction between suspended load and bed load is conceptually useful, but, as has been noted previously in other contexts, this does not necessarily yield any predictive advantages since neither component can as yet be treated satisfactorily for most practical problems. As such, simpler empirical formulae that directly relate g_T (or equivalently, $\bar{C}$) to sediment and hydraulic parameters remain attractive and have often performed as well or better than more complicated formulae in practical predictions. Only two of the many such formulae will be discussed. The formula of Engelund and Hansen (1967) was developed along with their stage-discharge formula (see Section 35.5 for the range of experimental parameters). The total dimensionless transport per unit width, Φ_T, is related to Fr_g, and Θ, with characteristic grain size, d_g, by

Engelund-Hansen total-load formula

$$\Phi_T = 0.05\, Fr_g^2\, \Theta^{3/2} \tag{35.23}$$

The Brownlie formula was originally stated in terms of the mean sediment transport (mass or weight) concentration, $\bar{C}$, as:

Brownlie total-load formula

$$\bar{C} = 0.00712\, c_f \left[Fr_g - \left(Fr_g\right)_c\right]^{1.98} S_f^{0.66} \left(\frac{d_{50}}{R_h}\right)^{0.33} \tag{35.24}$$

where c_f = 1 for laboratory data and c_f = 1.27 for field data, $(Fr_g)_c$ is the critical grain Froude number corresponding to the initiation of sediment motion given by Eq. (35.12). In term of Φ_T and Θ, Eq. (35.24) can be expressed (assuming $R_h \approx H$) with rounding as

$$\Theta_T = 0.00712\left(\frac{c_f}{s}\right)\left[Fr_g - \left(Fr_g\right)_c\right]^{2.0} Fr_g\left[(s-1)\Theta\right]^{1.5} \tag{35.25}$$

Example 35.5

The total load formulae should also be applicable to bed-load dominated transport as in Example 35.3. In that case, the Engelund-Hansen formula, with Fr_g^2 = 27.5 and Θ = 0.4, predicts Φ_T = 0.35, corresponding to g_T = 2.0 N/s/m and $\bar{C}$ = 411 ppm by weight. This is approximately twice the measured value. The Brownlie formula, with $(Fr_g)_c$ = 1.8 from Eq. 35.12 and c_f = 1.27, yields Φ_T = 0.16, corresponding to g_T = 0.91 N/s/m or in terms of $\bar{C}$ = 189 ppm by weight, which agrees well with the measured value of 191 ppm. This rather close agreement should be considered somewhat fortuitous, and is at least partially attributed to the fact that the observed value was included in the data set on which the Brownlie formulae were based. The performance of the Brownlie formulae in practice is likely to be similar to the better recent proposals.

Measurement of Sediment Transport

In addition to, and contributing to, the difficulties in describing and predicting accurately sediment transport, total load measurements, particularly in the field, are associated with much uncertainty. Natural alluvial channels may exhibit a high degree of spatial and temporal nonuniformities, which are not specifically considered in the 'averaged' models discussed above. Standard methods of suspended load measurements in streams include the use of depth-integrating samplers that collect a continuous sample as they are lowered at a constant rate (depending on stream velocity) into the stream, and the use of point-integrating samplers that incorporate a valve mechanism to restrict sampling, if desired, to selected points or intervals in the water column. Such sampling assumes that the sampler is aligned with a dominant flow direction, and that the velocity at the sampler intake is equal to the stream velocity. In the vicinity of a dune-covered bed, these conditions cannot be fulfilled. The finite size of the suspended load samplers implies that they cannot measure the bedload discharge, which must therefore be measured with a different sampler or estimated with a bedload model. A bedload sampler, such as the U.S.G.S. Helley-Smith sampler, will necessarily interact with and possibly change the erodible bed. Questions also arise concerning the distinction between suspended and bed loads when bedload samplers are used in problems involving suspended loads. Calibration is necessary, e.g., in the laboratory using a sediment trap, but this may vary with several parameters, including the particular type of sampler used, the transport rate, grain size (Hubbell, 1987), and unless full-scale tests are performed, questions of model-prototype similitude also arise.

Expected Accuracy of Transport Formulae

The reliability of sediment transport formulae is relatively poor. Some of this poor performance may be attributed to measurement uncertainties. The best general sediment discharge formulae available have been found to predict values of g_T which are within one-half to twice the observed value for only about 75% of cases (Brownlie, 1981; van Rijn, 1984a, b; Chang, 1988). Circumspection is therefore advised in basing engineering decisions on such formulae, especially when they are imbedded in sophisticated computer models of long-term deposition or erosion. Where feasible, site-specific field data should be exploited, and used to complement model predictions.

35.7 Special Topics

The preceding sections have been limited to the simplest sediment-transport problems involving steady uniform flow. The following deals briefly with more specialized and complex problems.

Local Scour

Hydraulic structures, such as bridge piers or abutments, that obstruct or otherwise change the flow pattern in the vicinity of the structure, may cause localized erosion or scour. Changes in flow characteristics lead to changes in sediment transport capacity, and hence to a local disequilibrium between actual sediment load and the capacity of the flow to transport sediment. A new equilibrium may eventually be restored as hydraulic conditions are adjusted through scour. *Clear-water scour* occurs when there is effectively zero sediment transport upstream of the obstruction, i.e., $Fr_g < (Fr_g)_c$ upstream, while *live-bed scour* occurs when there would be general sediment transport even in the absence of the local obstruction, i.e., $Fr_g > (Fr_g)_c$, upstream. Additional difficulties in treating local scour stem from flow non-uniformity and unsteadiness. The many different types and geometries of hydraulic structures lead to a wide variety of scour problems, which precludes any detailed unified treatment. Design for local scour requires many considerations and the results given below should be considered only as a part of the design process.

Empirical formulae have been developed for special scour problems; only two are presented here, both relevant to problems associated with bridge crossings over waterways, one for contraction scour, and one for scour around a bridge pier. Consider a channel contraction sufficiently long that uniform flow is established in the contracted section, which is uniformly scoured (Fig. 35.7). The entire discharge is assumed to flow through the approach and the contracted channels. Application of conservation of water and sediment (assuming a simple transport formula of power-law form, $g_T \sim V^m$) results in

$$\frac{H_1}{H_2} = \left(\frac{B_2}{B_1}\right)^{\alpha} \tag{35.26}$$

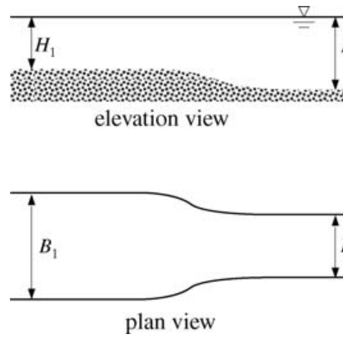

FIGURE 35.7 Channel constriction causing local scour.

where the subscripts, 1 and 2, indicate the contracted (2) or the approach (1) channels, H the flow depth, and B the channel width.

The exponent, α, varies from 0.64 to 0.86, increasing with τ_c/τ_1, where τ_c is the critical shear stress for the bed material, and τ_1 is total bed shear stress in the main channel. A value of $\alpha = 0.64$, corresponding to $\tau_c/\tau_1 \ll 1$, i.e., significant transport in the main channel, is often used.

Scour around bridge piers has been much studied in the laboratory but field studies have been hampered by inadequate instrumentation and measurement procedures. For design purposes, interest is focused on the maximum scour depth at a pier, y_s (see Fig. 35.8 for a definition sketch). A wide variety of formulae have been proposed; only one will be presented here, namely that developed at Colorado State University, and recommended by the U. S. Federal Highway Administration,

$$\frac{y_s}{b} = 2.0\, K_p \left(\frac{H_0}{b}\right)^{0.35} Fr_0^{0.43} \tag{35.27}$$

where b = the pier width
H_0 = the approach flow depth
$Fr_0 = V_0/\sqrt{gH_0}$, the Froude number of the approach flow

The empirical coefficient, K_p, depends on pier geometry, the angle of attack or skew angle (θ in Fig. 35.8) of the flow with respect to the pier, bed condition (plane-bed or dunes), and whether armoring of the bed (see below) may occur; details of the evaluation of K_p may be found in Richardson and Davis (1995).

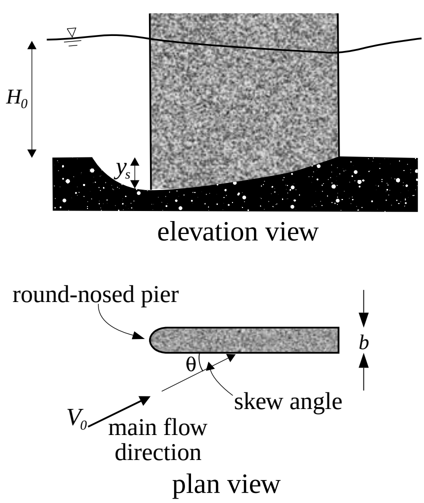

FIGURE 35.8 Bridge pier causing local scour.

Unsteady Aspects

Many problems in channels involve non-uniform flows and slow long-term changes, such as *aggradation* (an increase in bed elevation due to net deposition) or *degradation* (a decrease in bed elevation due to net erosion). The problem is formulated generally in terms of three (differential) balance equations: conservation of mass of water, of momentum (or energy), of sediment. For gradually varied flows, the first two equations are identical in form to those encountered in fixed-bed problems (see the chapter on open channel flows), except that the bed elevation is allowed to change with time.

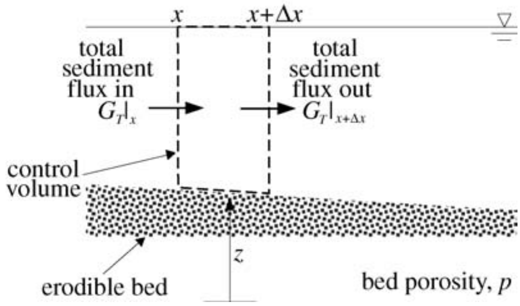

FIGURE 35.9 Control volume used in unsteady analysis.

A control volume analysis of a channel reach of cross-sectional area, A, and bed width, B_b, (Fig. 35.9) shows that conservation (continuity) of sediment over a small reach of length, Δx, in a time interval, Δt, requires

$$\Delta\left[(1-p)z\right](B_b\Delta x)+\ \Delta\langle C\rangle A\Delta x\ +\ \left(G_T\big|_{x+\Delta x}-G_T\big|_x\right)\Delta t\ =\ S_s\Delta t\Delta x \tag{35.28}$$

where p = bed the porosity
z = the bed elevation
t = the time variable
$\langle C\rangle$ = the concentration of sediment averaged over the cross-sectional area
$G_T\big|_x$ = the total sediment discharge evaluated at a cross-section location, x
S_s = included as a possible external sediment source strength per unit length

The first term represents the change over time in the bulk volume of sediment in the bed due to net deposition or erosion (bed storage); the second term represents the change over time in total volume of suspended sediment in the water column (water column storage); the third term stems from differences in sediment discharge between the channel cross-sections bounding the control volume; and the fourth term allows for distributed sediment sources. The second term is often assumed negligible, so that in its differential form (dividing through by $\Delta x\ \Delta t$ and taking the limit as $\Delta x \to 0$, $\Delta t \to 0$), Eq. (35.28) is written as

$$\frac{\partial\left[(1-p)zB_b\right]}{\partial t}+\frac{\partial G_T}{\partial x}=S_s \tag{35.29}$$

which is referred to as the *Exner equation*. A total load computation as in Section 35.6 is performed to determine G_T. This assumes implicitly that a quasi-equilibrium has been established, in which the sediment discharge at any section is equal to the sediment transport capacity as specified by conventional total load computations. Thus, the quality of the predictions of the unsteady model depends not only on the quasi-equilibrium assumption but also on the quality of the estimates of sediment transport by the transport formula applied.

Numerical methods are used to solve Eq. (35.29) simultaneously with the flow equations (water continuity and the momentum/energy equations). In practice, numerical models often solve the flow equations first, and then the sediment continuity equation, under the implicit assumption that changes in bed elevations occur much more slowly than changes in water-surface elevation. Of the many unsteady alluvial-river models described in the literature, HEC-6 for scour and deposition in rivers and reservoirs, may be mentioned as a member of the well-known HEC series of channel models (Hydrologic Engineering Center, 1991) and hence perhaps the most widely adopted. There are plans to incorporate sediment transport capabilities to the new generation of HEC software, HEC-RAS, but as of this writing, this has not yet been performed. An early evaluation (Comm. on Hydrodynamic Models for Flood Insurance Studies, 1983) of several models, including HEC-6, noted the following general deficiences: unreliable formulation and/or inadequate understanding of sediment-transport capacity, of flow resistance, of armoring (see below), and of bank erosion. In spite of the intervening years, this evaluation may still be taken as a cautionary note in using such models.

Effects of a Nonuniform Size Distribution

Natural sediments exhibit a size distribution (also termed gradation), and, since the grain diameter profoundly influences transport, the effects of size distribution are likely substantial. The crudest models of such effects incorporate distribution parameters, such as the geometric standard deviation, σ_g, in empirical formulae, e.g., the Brownlie formulae. An alternative approach more appropriate for computer modeling divides the distribution into a finite number of discrete size classes. Each size class is characterized by a single grain diameter, and results such as the Shields curve or the Rouse equation are applied to each separate size class, where they are presumably more valid. Total transport is then determined by a summation of the transport in each size class.

The heterogeneous bed material, which constitutes a source or sink of grains of different size classes, must be taken into account. Conventional bed load or total load transport equations or even initiation of motion criteria may not necessarily apply to individual size classes in a mixture. The transport or entrainment into suspension of one size class may influence transport or entrainment of other size classes, so individual size classes may not be treated independently of each other. This is often handled by the use of empirical 'hiding' coefficients. Finer bed material may under erosive conditions be preferentially entrained into the flow, with the result that the remaining bed material becomes coarser. This will reduce the rate of erosion relative to the case where the bed consists of uniformly sized fine material. If the available fine material is eventually depleted, suspended load transport will be reduced or in the limit entirely suppressed. Eventually, a layer of coarse material termed the armor layer consisting of material that is not erodible under the given flow condition may develop, which protects or 'armors' the finer material below it from erosion, thereby substantially reducing sediment transport and local scour. Armoring may also have consequences for flow resistance, since size distribution characteristics of the bed will vary with varying bed shear stress, and hence affect bed roughness and bed forms. In this way, episodic high-transport events such as floods may have an enduring impact on sediment transport as well as flow depths. Various detailed numerical models of the armoring process have been developed, and the reader is directed to the literature for further information (Borah et al., 1982; Sutherland, 1987; Andrews and Parker, 1987; Holly and Rahuel, 1990a, b; Hydrological Engineering Center, 1991).

Gravel-Bed Streams

Channels in which the bed material consists primarily of coarse material in the gravel and larger range are typically situated in upland mountain regions with high bed slopes ($S > 0.005$), in contrast to sand-bed channels, which are found on flatter slopes of lower lying regions. The same basic concepts summarized in previous sections apply also to gravel-bed streams, but the possibly very wide range of grain sizes introduces particular difficulties. Bedforms such as dunes play less of a role, and so grain resistance can often be assumed dominant; hence an upper regime stage-discharge relationship can be applied. The effects of large-scale roughness elements such as cobbles and boulders that may even protrude through the water surface may however not be well described by formulae based primarily on data from sand-bed channels. Instead of a gradually varying bed elevation, riffle-pool (or step-pool) sequences of alternating shallow and deep flow regions may occur. The wide size range results in transport events that may be highly non-uniform across the stream, and highly unsteady in the sense of being dominated by episodic events. Armoring may also need to be considered. The coarse grain sizes increase the relative importance of bedload transport. The highly non-uniform and unsteady nature of the transport hinders reliable field measurements. Much debate has surrounded the topic of appropriate sampling of the bed surface material to characterize the grain size distribution. The traditional grid method of Wolman (1954) draws a regular grid over the bed of the chosen reach, with the gravel (cobble or boulder) found at each of gridpoint being included in the sample.

A friction law proposed specifically for mountain streams is that of Bathurst (1985) based on data from English streams (60 mm $< d_{50} <$ 343 mm, $0.0045 < S < 0.037$, 0.3 m^3/s $< Q <$ 195 m^3/s) for which the friction factor, f, is given by

$$\sqrt{\frac{8}{f}} = 5.62 \log_{10} \frac{H}{d_{84}} + 4 \tag{35.30}$$

with a reported uncertainty of ±30%. An earlier formula due to Limerinos (1970) is identical in form except that R_h is used instead of the depth, H, and Manning's n is sought rather than f:

$$\frac{K_M}{\sqrt{g}} \frac{R_h^{1/6}}{n} = \sqrt{\frac{8}{f}} = 5.7 \log_{10} \frac{R_h}{d_{84}} + 3.4 \tag{35.31}$$

K_M is the dimensional constant associated with Manning's equation (see chapter on Open Channel Hydraulics). Using laboratory and field data, Bathurst et al. (1987) assessed various criteria for the initiation of motion and bedload discharge formulae (including the Meyer-Peter-Muller formula, Eq. [35.18]). They recommended a modified Schoklisch formula for larger rivers ($Q > 50$ m^3/s) where sediment supply is not a constraint:

$$\left(q_s\right)_b = \frac{2.5}{s} S_f^{3/2} \left(q - q_c\right) \tag{35.32}$$

where the critical unit-width discharge, q_c, is given by

$$q_c = 0.21 \frac{\sqrt{g d_{16}^3}}{S_f^{1.1}} \tag{35.33}$$

Here, $(q_s)_b$ is the *volumetric* unit width bedload discharge, and the units are metric in both equations. These gravel-bed formulae, while representative, are not necessarily the best for all problems; and they should be applied with caution and a dose of skepticism.

Defining Terms

Aggradation — Long-term increase in bed-level over an extended reach due to sediment deposition

Armoring — A phenomenon in which a layer of coarser particles that are non-erodible under the given flow condition protects the underlying layer of finer erodible particles

Bed forms — Features on an erodible channel bed which depart from a plane bed, e.g., dunes or ripples

Bed load — That part of the total sediment discharge which is transported primarily very close to the bed

Critical shear stress — The bed shear stress above which general sediment transport is said to begin

Critical velocity — The mean velocity above which general sediment transport is said to begin

Local scour — Erosion occurring over a region of limited extent due to local flow conditions, such as may be caused by the presence of hydraulic structures

Sediment discharge — The downstream mass or weight flux of sediment

Suspended load — That part of the total sediment discharge which is transported primarily in suspension

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Further Information

Several general books or book chapters on various aspects of sediment transport are available:

1. ASCE *Sedimentation Engineering*, V. A. Vanoni, Ed., (1975), ASCE Manual No. 54 is a standard comprehensive account, with very broad coverage of topics related to sediment transport. A new ASCE manual, covering topics of more recent interest, is due out shortly.
2. *Fluvial Processes in River Engineering,* H. H. Chang (1988), Prentice-Hall, Englewood Cliffs, NJ.
3. *Loose Boundary Hydraulics,* 3rd ed., A. J. Raudkivi (1990), Pergamon Press, New York.
4. *Sediment Transport Technology,* D. B. Simons and F. Sentürk (1992), rev. ed., Water Resources Publications.
5. *Sediment Transport Theory and Practice*, C. T. Yang (1996), McGraw-Hill, New York.
6. "Sedimentation and Erosion Hydraulics," M. H. Garcia, Chap. 6, in *Hydraulic Design Handbook*, Larry W. Mays, Ed., (1999), McGraw-Hill, New York.

Special topics are dealt with in

1. *Scouring*, H. N. C. Breusers and A. J. Raudkivi (1991), Balkema, discusses a variety of problems involving scour.
2. *Highways in the River Environment*, E. V. Richardson, D. B. Simons, and P. Y. Julien (1990), FHWA-HI-90–016, and *Evaluating Scour at Bridges*, HEC-18, E. V. Richardson and S. R. Davis (1995), are documents produced for the U.S. Federal Highway Administration, and discuss in great detail hydraulic considerations in the design and siting of bridges, including scour, and the recommended design practice in the U.S.
3. *Sediment Transport in Gravel-Bed Rivers*, C. R. Thorne, J. C. Bathurst, and R. D. Hey, Eds., (1987) John Wiley & Sons, New York, provides information on problems in gravel-bed streams.
4. *Sedimentation: Exclusion and Removal of Sediment from Diverted Water*, A. J. Raudkivi (1993), Balkema, discusses settling basins and sediment traps.
5. *Reservoir Sedimentation Handbook*, G. L. Morris and J. Fan, McGraw-Hill, New York, presents an exhaustive discussion of sedimentation in reservoirs.
6. *Field Methods for Measurement of Fluvial Sediment,* H. P. Guy and V. W. Norman (1970) in the series Techniques of Water-Resources Investigations of the United States Geological Survey, Book 3, Chap. C2, gives practical advice regarding field measurements of sediment transport, including site selection and sampling methods.