31

Surface Water Hydrology

A. Ramachandra Rao
Purdue University

31.1 Introduction

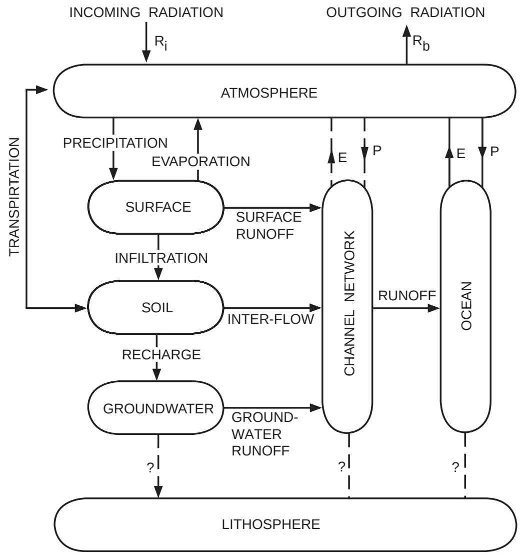
Hydrologic Cycle

Hydrologic cycle is the name given to the continuum of the movement of water in the atmosphere, hydrosphere and lithosphere. It is an intricate cycle of great complexity. A schematic diagram of the hydrologic cycle is shown in Fig. 31.1.

Water evaporates from the oceans and land and becomes a part of the atmosphere. The water vapor is either carried in the atmosphere or it returns to the earth in the form of precipitation. A portion of precipitation falling on land may be intercepted by vegetation and returned back directly to the atmosphere as evaporation. Precipitation that reaches the earth may evaporate or be transpired by plants; or it may flow over the ground surface and reach streams as surface water; or it may infiltrate the soil. The infiltrated water may flow over the upper soil regions and reach surface water or it may percolate into deeper zones and become groundwater. Groundwater may reach the streams naturally or it may be pumped, used, and discarded to become a part of the surface water system.

The general volume balance equation for the hydrologic cycle may be written as in Eq. (31.1),

$$\frac{dS}{dt} = I(t) - O(t) \tag{31.1}$$



FIGUREW 31.1 A systems representation of the hydrologic cycle. (From Dooge, 1973.)

where S is the storage including the surface, soil moisture, groundwater and interception storage, t the time. $I(t)$ is the input, which includes precipitation in all its forms and $O(t)$ is the output, which includes surface and subsurface runoff, evaporation, transpiration, and infiltration.

According to the world water balance studies (UNESCO, 1978), about 127 cm of precipitation falls on oceans of area 361.3 million km.[2] The corresponding values for land areas are 78.74 cm and 148.8 million km.[2] The evaporation rates are 55 and 19 in. respectively from the oceans and land areas. Of the 12 in. of water that remains on land, 0.2 in. becomes groundwater and 11.8 inches reaches the oceans as surface water. Due to the fact that a large part of the precipitation falling on the earth remains as ice, the amount of surface water that is available for use is quite limited. In this chapter several aspects of surface water hydrology are discussed.

Historical Development

Chow (1964) has classified the development of hydrology into eight periods. The first of these is called the period of speculation (Ancient - 1400 AD) during which there were many speculations about the concept of the hydrologic cycle. However, during this period, practical aspects of hydrologic knowledge were studied and used to build civil works. During the period 1400 to 1600 AD (period of observation), hydrological variables were simply observed. Leonardo da Vinci and Bernard Palissy understood the hydrological cycle during this period. Hydrologic measurements started during the period of measurement (1600–1700) and the science of hydrology may be said to have begun during this period. It continued up to nineteenth century with experiments, a period called the period of experimentation (1700–1800). The foundations of modern hydrology were laid during the period of modernization (1800–1900). Hydrology was mostly empirical during the period of empiricism (1900–1930), which gradually gave into the period of rationalization (1930–1960), during which theoretical developments in hydrograph, infiltration and groundwater processes took place. The theoretical developments accelerated from 1950 to the present, a period called the period of theorization. The recent development of computers has made it possible to construct and verify theories of increasing complexity, an endeavor that has become the mainstay of modern hydrologic research.

31.2 Precipitation

Atmospheric Processes

Air masses must be lifted and cooled for precipitation to occur. Air mass cooling can occur during the passage of fronts when warm air is lifted over cooler air (frontal cooling); the passage of warm air over mountain ranges (orographic cooling); or the lifting of air masses due to localized heating such as that in the center of a thunderstorm cell (convective cooling).

As the air is cooled, water condenses on microscopic sized particles, called nuclei and this process is called nucleation. Dust and salt particles are common condensation nuclei. Water particles resulting from nucleation grow by condensation and by coming into contact with neighboring particles. They start to descend as they become heavier and may coalesce with other water drops or they may decrease in size during descent because of evaporation. If conditions are favorable, these water drops reach the ground as rain, snow, or sleet. The particular form taken by precipitation is dictated by the atmospheric conditions extant during the descent of water drops.

Measurement of Precipitation

Rainfall and snowfall are commonly measured. Both nonrecording and recording gages are used for rainfall measurement. Recording rain gages are used to measure rainfall depth at predetermined time intervals. These intervals can be as small as a minute. Nonrecording rain gages are read at larger time intervals. Common recording rain gages are of the weighing, the tipping bucket and the float types. In each of these, a record of rainfall depth against time is obtained. Depth and density of snow packs, in addition to the water equivalent of snow, are also commonly measured, as these are useful in estimating the water yield from snow packs. Measurement of snow depth is complicated because of the strong effect of wind on snow (Garstka, 1964).

Temporal Variation of Precipitation

The unit of rainfall measurement is depth, in inches or millimeters. The rates are usually expressed as in/hr or mm/hr, although longer durations such as days, months, and years are also used. Rainfall rate, especially when the time duration is an hour, is called the intensity of rainfall. In general, rainfall intensity is highly variable with time. A plot of intensity against duration is called a hyetograph of rainfall. A plot of the sum of the rainfall depth against time is called as the mass curve. A mass curve whose abscissa and ordinate are dimensionless is called the dimensionless mass curve. Typical hyetograph, mass curve, and dimensionless mass curve are shown in Fig. 31.2.

Spatial Variation of Precipitation

Rainfall measurements are taken at different points in an area. The spatial structure of storms and their internal variation cannot be adequately represented by a point measurement or even by many point measurements made over a region. Consequently, there have been attempts to relate point rainfall measurements to spatial average rainfall. As the area represented by a point measurement increases, the reliability of the data from a point as a representative of the average over a region decreases. As drainage areas become larger than a few square miles, point data must be adjusted to estimate areal data (Hershfield, 1961). If the drainage area is larger than 8 mi^2, an area reduction factor is applied to the point rainfall depth values obtained from the rainfall atlas (Hershfield, 1961). Recently there have been attempts to measure rainfall by using radar.

Average Rainfall over an Area

The *arithmetic average* method, the *Thiessen polygon* method, and the *isohyetal* methods are commonly used to compute average rainfall over an area. These and other methods have been investigated by Singh and Chowdhury (1986) who concluded that they give comparable results, especially for longer storm durations.

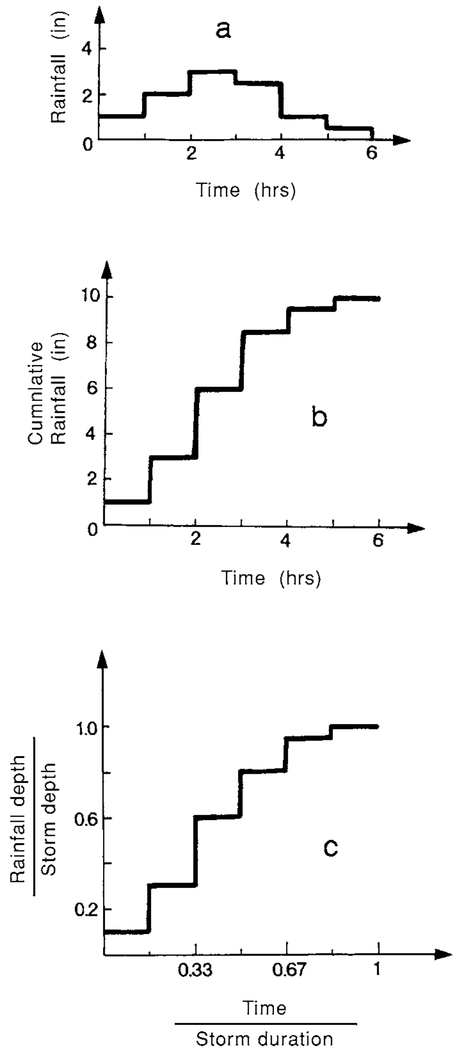


FIGURE 31.2 (a) Hyetograph: (b) mass curve; (c) dimensionless mass curve.

Let the *average rainfall* over an area be $\bar{P}$. Let the rainfall measured by N rain gages over an area A be P_1, P_2,…P_N. The general expression for $\bar{P}$ is given in Eq. (31.2), where W_j are the weights.

$$\bar{P} = \sum_{j=1}^{N} W_j P_j, \quad W_j \, \varepsilon(0,1) \tag{31.2}$$

Different methods of estimation of rainfall give different W_j values. For the arithmetic average, the weights W_j are the same and are equal to $(1/N)$. To compute the *Thiessen* average rainfall, the locations of rain gages are joined by straight lines on a map of the area. These are bisected to develop a Theissen polygon such that each rain gage with rainfall P_j is located in a part of a watershed of area A_j. The sum of areas A_j equal the watershed area A. The Thiessen weights W_i are given by A_i/A and W_j add up to unity. The Thiessen average does not consider the spatial distribution of rainfall but takes into account the spatial distribution of rain gages.

In the *Isohyetal* method, lines of equal rainfall depth or *isohyetal lines* are first estimated. The variation in rainfall between rain gages may be assumed to be linear for interpolation purposes. The areas A_j between isohyetal lines are measured and these add up to A. The number of areas enclosed within isohyets is usually not equal to number of rain gages. The weights W_j are equal to A_j/A. The rainfall values P_i in Eq. (31.2) are the average rainfall values between the isohyets. In the isohyetal method the spatial distribution of rainfall variation is explicitly considered.

Intensity - Duration - Frequency (i-d-f) Curves

The largest rainfall depth measured over a specified duration, during a year, is an extreme value. Because the durations are fixed, we will have a series of rainfall extreme values. For example, we will have extreme rainfall depths corresponding to 30 min, 1 hr, 6 hr, etc. These extreme values — which are also the annual maximum values in this case — are random variables denoted by *x*. The probability that x is larger than a value x_T is called the exceedance probability and is indicated by $P(x \geq x_T)$ or *p*. The recurrence interval *T* is the average time elapsed between occurrences of $x \geq x_T$. The *return period* or *recurrence interval* between events exceeding or equalling x_T is the *average time T* between exceedances of the event. The exceedance probability and the return period are inversely related. The return period is also called the *frequency*.

By analyzing annual maximum rainfall intensities corresponding to a duration — such as 1 hr or 6 hr — the rainfall-intensity-frequency relationships are obtained. A set of rainfall intensity (i) frequency (f) relationships corresponding to different rainfall durations (d) are called the intensity (i) - duration (d) - frequency (f) curves. The i-d-f curves for Indianapolis, presented in Fig. 31.3, are typical of these.

It is difficult to use the i-d-f curves in computer analysis. Consequently, they are represented as empirical equations such as Eq. 31.3 where i is the intensity in in./hr, *t* is the duration in hours, *T* is the frequency in years, and *C* and *d* are constants corresponding to a location and *m* and *n* are exponents. For Indianapolis, $C = 1.5899$, $d = 0.725$, $m = 0.2271$ and $n = 0.8797$, and the resulting equation is valid for durations between 1 and 36 hours.

$$i = \frac{CT^m}{(t+d)^n} \tag{31.3}$$

These empirical equations give results that are more accurate than those obtained by interpolation of graphs.

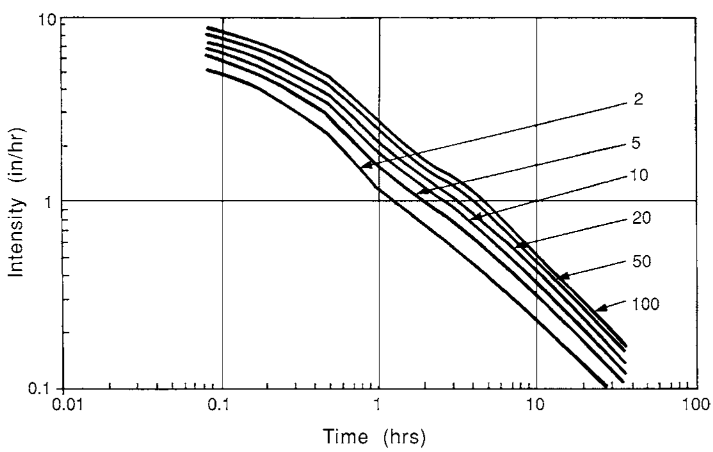


FIGURE 31.3 Intensity-duration-frequency curves for Indianapolis. (From Purdue, A.M., G.D. Jeong, A.R. Rao [1992] "Statistical Characteristics of Short Time Increment Rainfall," *Tech. Rept. CE-EHE*-92–09, School of Civil Engineering, Purdue University, W. Lafayette, IN., pp. 64. With permission.)

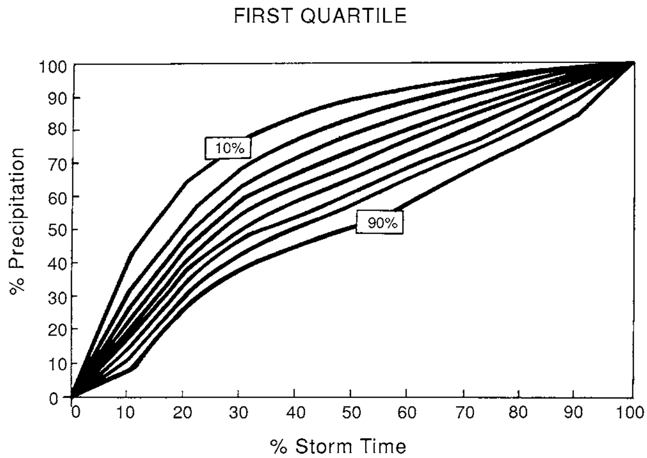


FIGURE 31.4 Dimensionless first quartile cumulative rainfall curves for Indianapolis.

TABLE 31.1 Dimensionless Mass Curves (10% Level) for Four Quartiles

% Storm Time	I	II	III	IV
0	0.00	0.00	0.00	0.00
10	42.00	16.36	14.36	19.35
20	64.35	32.73	25.26	30.00
30	76.36	58.10	33.33	36.36
40	83.84	76.36	41.82	43.53
50	89.63	87.50	54.72	50.00
60	92.50	92.54	78.18	54.55
70	95.00	95.69	92.35	63.64
80	97.00	97.00	96.43	82.50
90	98.57	98.67	98.73	96.73
100	100.00	100.00	100.00	100.00

Dimensionless Mass Curves

To estimate hyetographs from rainfall depth-duration data, dimensionless mass curves are used. These dimensionless mass curves are developed by classifying observed rainfall data into different quartiles. For example, if the largest rainfall depth occurs during the first quartile of a storm duration then it is called a first quartile storm. The cumulative rainfall in these storms is made dimensionless by dividing the rainfall depths by the total rainfall depth and corresponding times by the storm duration. These dimensionless mass curves of rainfall are analyzed to establish their frequencies of occurrence. These are published in graphical form as shown in Fig. 31.4 and Table 31.1.

Given a rainfall depth and duration, the dimensionless mass curve information is used to generate hyetographs. These hyetographs may be used as inputs to rainfall-runoff models or with unit hydrographs to generate runoff hydrographs. The dimensionless hyetographs thus provide an easy method to generate rainfall hyetographs (Purdue et al. 1992).

Chen (1983) developed a method of generating intensity-duration curves for different frequencies or recurrence intervals. The method is based on 10 year - 1 hr, (R_1^{10}), 10 year - 24 hr, (R_{24}^{10}) and 100 year - 1 hr (R_1^{100}) rainfall *depths* for the location of interest. These are available from the NWS publications such

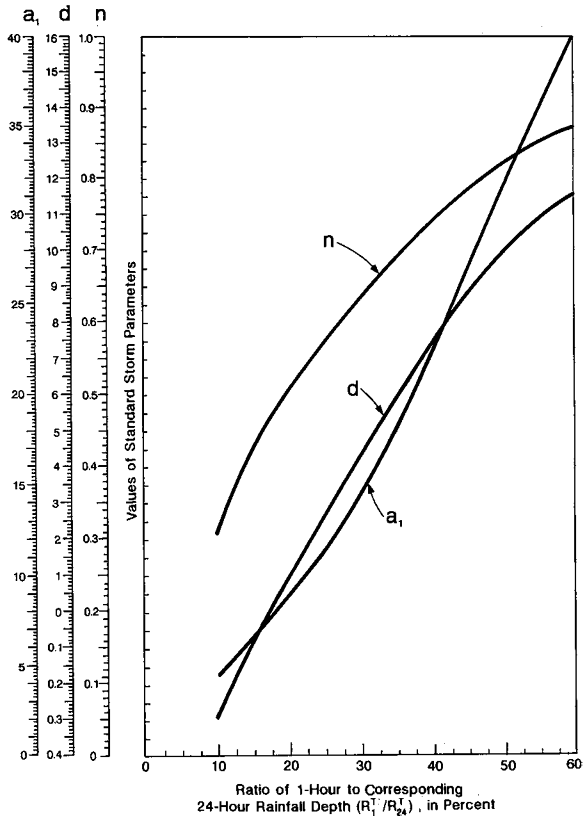


FIGURE 31.5 Coefficients and exponents for use with Chen's (1983) method.

as TP-40 or NWS HYDRO-35 or NOAA Atlas 2 in the U.S. (Hershfield, 1961; Frederick et al. 1977; Miller et al. 1973). The intensity-duration-frequency relationship used in this study is of the form of Eq. (31.3).

In this method, the ratios, R_1^{10}/R_{24}^{10} and $R_1^{100}/R_1^{10} = x$ are formed. The R_1^{10}/R_{24}^{10} ratio is the x axis of Fig. 31.5, and by using this value, a_1, d and n are read off from Fig. 31.5. The rainfall intensity corresponding to duration t and recurrence interval T is estimated by using Eq. (31.4), where r_1^{10} is the 10 year - 1 hour rainfall intensity which is the same as R_1^{10} because the duration is 1 hour.

$$r_t^T = \frac{a_1\left(r_1^{10}\right)\log\left(10^{2-x}\,T^{x-1}\right)}{\left(t+d\right)^n} \tag{31.4}$$

Chen's (1983) method has been demonstrated to give very good estimates of rainfall intensity-duration-frequency relationships.

Other Probabilistic Aspects

Numerous probabilistic models (Table 31.2) have been developed and used to characterize various rainfall properties. For analyzing annual maximum rainfall data, log-normal and extreme value (III) distributions have been successfully used. The Weibull distribution has been used to characterize the time between precipitation events. Two parameter gamma distribution and bivariate exponential distribution have been used to characterize the storm depths and durations. A discussion of these models is found in Eagleson (1970) and Bras (1990).

TABLE 31.2 Probability Distributions for Fitting Hydrologic Data

Distribution	Probability Density Function	Range	Parameters
Normal	$F(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$	$-\infty \le x \le \infty$	μ, σ $\mu = \bar{x}, \sigma = s_x$
Lognormal	$f(x) = \frac{1}{x\sigma\sqrt{2\pi}} \exp\left(-\frac{(y-\mu_y)^2}{2\sigma_y^2}\right)$ where $y = \log x$	$x > 0$	μ_y, σ_y $\mu_y = \bar{y}, \sigma_y = s_y$
Gamma	$f(x) = \frac{\gamma^\beta x^{\beta-1} e^{-\gamma x}}{\Gamma(\beta)}$ where $\Gamma = \gamma$ function	$x \ge 0$	λ, β $\beta = \left(\frac{\bar{x}}{s_x}\right)^2; \lambda = \frac{\bar{x}}{s_x^2}$
Pearson Type III (three parameter gamma).	$f(x) = \frac{\gamma^\beta (x-\varepsilon)^{\beta-1} e^{-\gamma(x-\varepsilon)}}{\Gamma(\beta)}$	x, ε	$\lambda, \beta, \varepsilon$ $\lambda = \frac{Sy}{\sqrt{p}}; \beta = \left(\frac{2}{C_s 2}\right)^2$ $\varepsilon = \bar{x} - s_x \sqrt{\beta}$
Log Pearson Type III	$f(x) = \frac{\gamma^\beta (y-\varepsilon)^{\beta-1} e^{-\gamma(y-\varepsilon)}}{x\,\Gamma(\beta)}$ where $y = \log x$	log x, ε	$\lambda, \beta, \varepsilon$ $\lambda = s_y / \sqrt{\beta}; \beta = \left[\frac{2}{C_{sy}}\right]^2$ $\varepsilon = \bar{y} - s_y\sqrt{\beta}$
Extreme Value Type I	$f(x) = \frac{1}{\alpha} \exp\left[-\frac{x-u}{\alpha} - \exp\left(-\frac{x-u}{\alpha}\right)\right]$	$-\infty < x < \infty$	α, u $\alpha = \frac{\sqrt{6}\, s_x}{\pi}$ $u = \bar{x} - 0.5772\alpha$

31.3 Evaporation and Transpiration

Evaporation

Evaporation is the process by which water is removed from an open water surface. The rate of evaporation depends on two factors: the energy available to provide the latent heat of vaporization, and the rate of transport of water vapor from the water surface. Evaporation from ponds, rivers, and lakes depends on solar radiation and wind velocity. The gradient of *specific humidity*, which is the ratio of the water vapor pressure, *e*, to the atmospheric pressure, *p*, also affects the evaporation rate.

Evaporation from a body of water can be estimated by using the law of conservation of energy. By using a control volume and estimating the energy inputs to and outputs from it, the evaporation rate E_1 (mm/day) can be shown to be as in Eq. 31.5, where R_w is the net short wave radiation in W/m^2,

$$E_1 = \frac{R_W}{l_v p_w} \tag{31.5}$$

where l_v = the latent heat of vaporization (kJ/kg)
ρ_w = the density of water in kg/m^3.

In deriving Eq. (31.5), the sensible heat flux and the heat transfer from the water to the ground are neglected. These assumptions are not realistic. Furthermore, there is no provision in Eq. (31.5) for removal of water vapor from the water surface. To eliminate this strong drawback, Thornthwaite and Holzman (1939) derived an equation which includes the wind velocity to estimate evaporation from open water surfaces. This equation has been simplified for operational application and is given in Eq. (31.6) (Chow et al. 1988). In Eq. (31.6), k is the von Karman constant, usually assumed to be 0.4, ρ_a the density of air, (kg/m^3) u_2 is the wind speed (m/sec) at height z_2 (ρ_w is the density of water in kg/m^3), z_0 is a roughness height, p is the atmospheric pressure (Pa), e_{sat} is the saturated vapor pressure (Pa) and e the actual vapor pressure (Pa), E_2 is in mm/day.

$$E_2 = \frac{0.622\, k^2 \rho_a u_2}{p\rho_w \left[\ln(z_2/z_0)\right]^2}\left(e_{sat} - e_a\right) \tag{31.6}$$

Penman (1948) developed a comprehensive theory of evaporation and his equation involves both the estimates E_1 and E_2. The Penman evaporation estimate E is given in Eq. (31.7), where γ is the psychrometric constant and Δ is the slope of the saturated vapor pressure curve at air temperature T_a.

$$E = \frac{\Delta}{\Delta+\gamma}E_1 + \frac{\gamma}{\Delta+\gamma}E_2 \tag{31.7}$$

$$\gamma = \frac{C_p K_h p}{0.622\ l_v K_w} \tag{31.8}$$

$$D = \frac{4098 e_{sat}}{(237.3+T)^2} \tag{31.9}$$

In Eq. (31.8) and Eq. (31.9), C_p is the specific heat at constant pressure, K_h is the heat diffusivity, K_w is the vapor eddy diffusivity and the ratio (K_h/K_w) is usually assumed to be unity, and T is the temperature (°C). Priestley and Taylor (1972) analyzed Eq. (31.7) and found that for evaporation over large areas, the second term in Eq. (31.7) is about 30% of the first one. Based on this observation they developed Eq. (31.10), the Priestley-Taylor Equation, which reduces the computations involved.

$$E = 1.3\frac{\Delta}{\Delta+\gamma}E_1 \tag{31.10}$$

The Penman equation is the most accurate of all equations used to estimate evaporation from open water surfaces. However, the data required to use it, such as solar radiation, humidity and wind speed may not be easily available. In such cases, simpler evaporation equations (ASCE, 1973; Doorenbos and Pruitt, 1977) are used.

Most commonly, evaporation is measured by using evaporation pans. The class A pan used in the U.S. is 4 ft. (120.67 cm) in diameter and 10 in (25.4 (cm)) deep. The pan is made of Monel metal or of unpainted galvanized iron. It is placed on a wooden support to facilitate air circulation beneath it. In addition to the pan, an anemometer to measure the wind speed, a precipitation gage, thermometers to measure water and air temperatures are also used. Water level in the pan is measured and reset to a fixed level each day. The change in water level, adjusted to account for precipitation, is the evaporation which has occurred during that day. Further details about evaporation pans used both in the U.S. and other countries are found in WMO (1981).

The rates measured in evaporation pans are usually greater than those measured in larger water bodies. The ratio of lake evaporation to pan evaporation is called the *pan coefficient*. To estimate evaporation from a larger water body, the pan evaporation is multiplied by the pan coefficient. In the U.S., an average pan coefficient is approximately 0.7, which varies with locations and at a location with seasons.

Evapotranspiration

The process by which water in soil, vegetation, and land surface is converted into water vapor is called evapotranspiration. Both the transpiration of water by vegetation and evaporation of water from soil, vegetation and water surfaces are included in evapotranspiration. Because it includes water vapor generated by all the mechanisms, evapotranspiration plays a major role in water balance computations. Consumptive use includes evapotranspiration and the water used by plant tissue. In practice evapotranspiration and consumptive use are used interchangeably.

The process by which plants transfer water from roots to leaf surfaces, from which it evaporates is called transpiration. The rate of transpiration greatly depends on sunshine and on seasons and moisture availability. Transpiration rates of different plant types vary. As transpiration ends up in evaporation, transpiration rates are affected by the same meteorologic variables as evaporation. Therefore, it is common practice to combine transpiration and evaporation and express the total as evapotranspiration.

Potential evapotranspiration (PET) (Thornthwaite et al., (1944)) is a concept in common usage in evapotranspiration computations. The evapotranspiration rate that occurs when the moisture supply is unlimited is called the PET. The PET is a good indicator of optimum crop water requirements. The concept of reference crop evapotranspiration (ET_0) was introduced by Doorenbos and Pruitt (1977). The reference crop evapotranspiration is similar to PET. The rate of evapotranspiration from an extended surface of 8- to 15-cm tall green grass cover of uniform height, actively growing, completely shading and not short of water is called the reference crop evapotranspiration. Thus, ET_0 is the PET of short green grass which is the reference crop.

The PET is equivalent to evaporation from free water surface of extended proportions. However, the heat storage capacity of this water body is assumed to be negligibly small. Consequently methods used to estimate PET and evaporation are similar. Evapotranspiration and PET are based on (1) temperature, (2) radiation, (3) combination or Penman, and (4) pan-evaporation methods. These are considered below.

Blaney-Criddle formula (Eq. [31.11]), which is widely used to estimate crop water requirements is typical of evapotranspiration formulas based only on temperature.

$$F = PT \tag{31.11}$$

In Eq. (31.11), F is the evapotranspiration for a given month in inches, P is the ratio of total daytime hours in a given month to the total daytime hours in a year and T is the monthly mean temperature in degrees Fahrenheit. Doorenbos and Pruitt (1977) modified the Blaney-Criddle formula to include actual insolation time, minimum relative humidity and daytime wind speed. Another well known temperature-based evapotranspiration estimation method is that proposed by Thornthwaite et al. (1944). In this method, the heat Index I_j is defined in terms of the mean monthly temperature T_j(°C). The annual temperature efficiency index J is the sum of 12 monthly heat indices I_j.

$$I_j = \left(\frac{T_j}{5}\right)^{1.514}; \quad J = \sum_{j=1}^{12} I_j \tag{31.12}$$

The potential evapotranspiration is computed by Eq. (31.13) and Eq. (31.14)

$$PET = K^* PET_0 \tag{31.13}$$

$$PET_0 = 1.6\left(\frac{10\,T}{J}\right)^c \tag{31.14}$$

$$c = 0.000000\,675\,J^3 - 0.0000\,771\,J^2 + 0.49239.$$

PET_0 is the PET at 0° latitude in centimeters per month. K^* in Eq. (31.13) varies from month to month and is given in Thornthwaite et al. (1944) and in Ponce (1989).

Priestley and Taylor (1972) developed a formula to compute PET, which is based only on the radiation part of the Penman's equation Eq. (31.7). Priestley and Taylor's formula is given in Eq. (31.15).

$$PET = \frac{1.260\,\Delta\left(R_n/l_v\,\rho_w\right)}{\Delta + \gamma} \tag{31.15}$$

The Penman equation is also used to calculate PET. Penman (1952) suggested the use of crop coefficients (0.6 in winter and 0.8 in summer) to compute the evapotranspiration rates. In pan evaporation models, the PET is given by the formula Eq. (31.16), where K_p is the pan coefficient and E_p is the pan evaporation.

$$PET = K_p E_p \tag{31.16}$$

The pan evaporation is widely used to estimate PET. Guidelines to choose appropriate pan coefficients are found in Doorenbos and Pruitt (1977).

31.4 Infiltration

Process and Variability

Water on the soil surface enters the soil by infiltration. Percolation is the process by which water moves through the soil because of gravity. As the soil exposed to atmosphere is not usually saturated, flow near the ground surface is through unsaturated medium. The percolated water may reach the ground water storage or it may transpire back to the surface.

As water travels from the surface to the groundwater, two forces act on it. The gravity forces attract the flow towards groundwater and the capillary forces attract it to capillary spaces. Consequently, rate of percolation decreases with the passage of time and leads to decreasing rates of infiltration. Infiltration capacity is the maximum rate at which infiltration can occur. The infiltration capacity f_p is affected by conditions such as the soil moisture. The actual rate of infiltration rate is f_i. The infiltration rate and capacity are the same when the rate of supply of water i_s is equal to or greater than f_p. Infiltration theories assume that i_s is equal to or greater than f_p. Under these conditions, the maximum infiltration rate f_o occurs at the beginning of a storm and approaches a constant rate f_c as the soil becomes saturated. The rate at which f_o approaches f_c and the final value of f_c depend on the characteristics of the soil and initial soil moisture (Fig. 31.6).

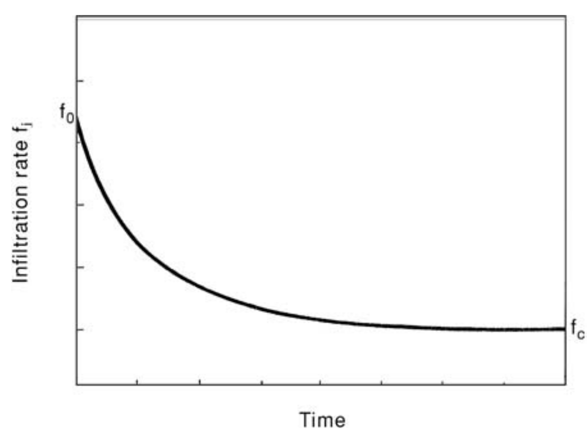


FIGURE 31.6 Infiltration curve.

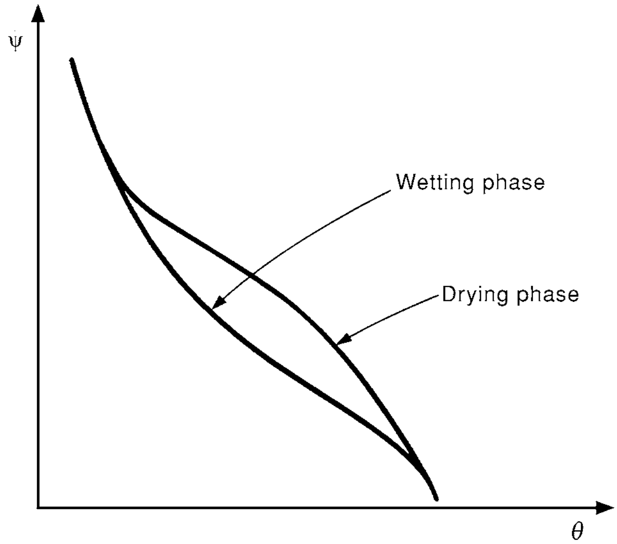


FIGURE 31.7 Capillary potential vs. soil moisture.

The energy possessed by the fluid due to soil suction forces $\psi(\theta)$ of a soil is a function of volumetric soil moisture θ and is called the capillary potential. The ratio of the volume of water in a soil volume is the moisture content θ. The capillary potential $\psi(\theta)$ is related to the piezometric head h as in Eq. (31.17), where z is the elevation head, h is the piezometric head and p_c/γ is the pressure head.

$$h = \frac{p_c}{\gamma} + z = \psi(\theta) + z \tag{31.17}$$

p_c is negative and hence p_c/γ is called the suction head. The capillary potential, for the same soil moisture θ, depends on whether the soil is in the wetting or drying phase. For the same θ, $\psi(\theta)$ is higher when the soil is drying than when it is wetting (Fig. 31.7)

Infiltration Models

Considering only the flow in the vertical direction z, the infiltration of water is governed by Eq. (31.18), where t is the time, $D_z(\theta)$ is the diffusivity and $K_z(\theta)$ is the hydraulic conductivity in the z direction (Bras, 1990).

$$\frac{\partial\theta}{\partial t} = \frac{\partial}{\partial z}\left[D_z(\theta)\frac{\partial\theta}{\partial z} + K_z(\theta)\right] \tag{31.18}$$

The diffusivity $D_z(\theta)$ is given by Eq. (31.19), where the hysterisis effects of $\psi(\theta)$ (Fig. 31.7) are ignored and $\psi(\theta)$ is considered as a single valued function of θ.

$$D_z(\theta) = K_z(\theta)\frac{\partial\psi(\theta)}{\partial\theta} \tag{31.19}$$

To simplify Eq. (31.18), $K_z(\theta)$ is considered to be a constant or that it is small in comparison with $D_z(\theta)$ $(\partial\theta/\partial z)$. If it is further assumed that $D_z(\theta)$ is constant and equal to D, Eq. (31.18) reduces to the diffusion Eq. (31.20). Assuming a semi-infinite soil system and the boundary conditions in Eq. (31.21), Eagleson (1970) shows that the solution of Eq. (31.20) is given by Eq. (31.22)

$$\frac{\partial\theta}{\partial t} = D\frac{\partial^2\theta}{\partial z^2} \tag{31.20}$$

$$\theta = \begin{cases} \theta_i & z \le 0; \quad t = 0 \\ \theta_o & z = 0; \quad t > 0 \end{cases} \tag{31.21}$$

$$f_p = f_c + \left(f_o - f_c\right) e^{-Dl^2 t} \tag{31.22}$$

where

$$f_p = f_c \qquad z = 0, \qquad t = \infty$$

$$f_p = f_o \qquad z = 0, \qquad t_o = 0.$$

The solution in Eq. (31.22) is applicable at the soil surface ($z = 0$). The variable l in Eq. (31.22) is a characteristic length dependent on z. Equation (31.22) is a form of the infiltration equation derived by Horton (1939, 1940). Philip (1960) presented an analytical solution to the infiltration equation when the initial and boundary conditions correspond to those in Eq. (31.21) with θ_o equal to porosity n. An approximate form of Philip's solution is given in Eq. (31.23) where S, called the sorptivity, and K are parameters related to diffusivity, hydraulic conductivity and initial soil moisture.

$$f_p(t) = \frac{1}{2}St^{-1/2} + K \tag{31.23}$$

Empirical Infiltration Equations

Although there is a strong theoretical basis from which infiltration equations such as Horton's and Philip equations are derived, there are numerous empirical equations in use. Most of these are based on the observation that there is an initial infiltration rate f_o, which depends on the soil type and antecedent moisture conditions, which decreases to infiltration capacity f_c under a supply rate i_s which is higher than f_o. The third parameter besides f_o and f_c is the rate of decay. More popular of the empirical infiltration equations are considered below. These models have also been derived by using a systems approach. Quite often, the supply rate is smaller than the infiltration capacity. In such a case all the rainfall supply is assumed to infiltrate into the soil until such time when the total rainfall and the infiltration depths are the same. The time at which the infiltration and rainfall depths are the same is called the ponding time.

Horton Equation

Horton's equation is given in Eq. (31.24), which is the same as Eq. (31.22) with $Dl^2 = k$.

$$f_p(t) = f_c + \left(f_o - f_c\right)e^{-kt} \tag{31.24}$$

The cumulative infiltration $F_p(t)$ corresponding to Eq. (31.24) is given by Eq. 31.25). If a constant rainfall rate i_s is assumed, the ponding time t_p is given by Eq. (31.26).

$$F_p = f_c t + \frac{\left(f_o - f_c\right)}{k} \tag{31.25}$$

$$t_p = \frac{1}{i_s k}\left[f_o - i_s + f_c \ln\left(\frac{f_o - f_c}{i_s - f_c}\right)\right] \tag{31.26}$$

Phillip's Infiltration Equation

Phillip's equation is not an empirical equation. The cumulative infiltration and the time of ponding for rainfall of constant intensity i_s for this equation are given by Eq. (31.27) and Eq. (31.28)

$$F_p = S t^{1/2} + At \tag{31.27}$$

$$t_p = S^2(i_s - A/2) \big/ 2i_s(i_s - A)^2, \; i_s > A \tag{31.28}$$

Green and Ampt Model

Green and Ampt (1911) proposed an infiltration model that is given by Eq. (31.29), where the cumulative infiltration is given by Eq. (31.30)

$$f_p = K\left(\frac{\psi\Delta\theta}{F(t)} + 1\right) \tag{31.29}$$

$$F_p = Kt + \psi\Delta\theta \ln\left(1 + \frac{F_p}{\psi\Delta\theta}\right) \tag{31.30}$$

In Eqs. (31.29) and (31.30), K is the hydraulic conductivity, $\Delta\theta$ is the difference between the soil porosity n and moisture content θ, ψ is the soil suction head. In practice, the hydraulic conductivity K, and the soil suction head ψ and the porosity are obtained from published sources such as Rawls et al. (1983). Instead of using $\Delta\theta$, it is expressed in terms of effective porosity θ_e and effective saturation S_e as in Eq. (31.31).

$$\Delta\theta = (1 - s_e)\theta_e. \tag{31.31}$$

Effective porosity θ_e is the difference between soil porosity and the residual moisture content θ_r after it has been thoroughly drained (Eq. [31.32]).

$$\theta_e = n - \theta_r \tag{31.32}$$

The effective saturation s_e is given by Eq. (31.33).

$$s_e = \frac{\theta - \theta_r}{n - \theta_r} \tag{31.33}$$

θ_e values are listed for different soils by Rawls et al. (1983). Consequently by knowing the effective saturation s_e of the soil and the other parameters listed by Rawls et al., (1983), the Green and Ampt model may be used to estimate infiltration.

31.5 Surface Runoff

Process and Measurement

An area that drains into a stream at a given location via a network of streams is called a watershed. Rainfall that falls on a watershed fills the depression storage, which consists of storage provided by natural depressions in the landscape, it is temporarily stored on vegetation as interception and it infiltrates into the soil. After these demands are satisfied, water starts flowing over the land and this process is called overland flow. Water that is stored in the upper soil layer may emerge from the soil and join the overland flow. The overland flow lasts only for short distances after which it is collected in small channels called rills. Flows from these rills reach channels. Flow in channels reaches the mainstream.

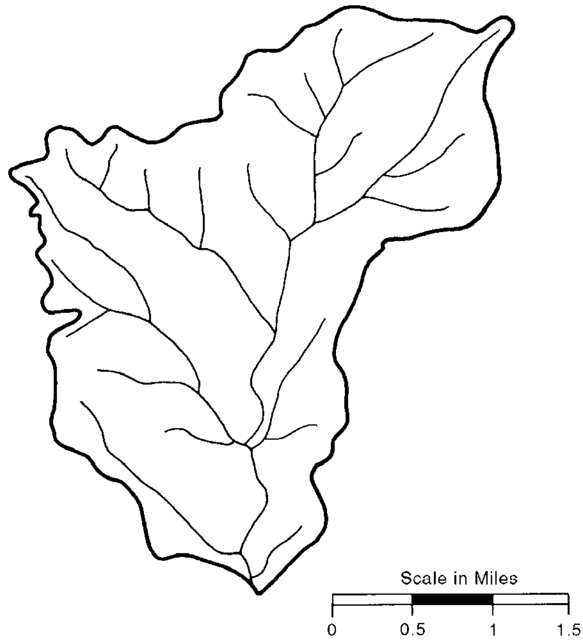


FIGURE 31.8 Drainage map of Bear Creek Basin, Indiana.

When rainfall is of low intensity, the overland-rill-channel flow sequence may not occur. In such cases, only the land near the streams contributes to the flow. These areas are called variable source or partial areas. Only a small area of watersheds contribute to stream flows in a humid region.

The transformation of rainfall to runoff is affected by the stream network, by precipitation, by soil, and land use. A watershed consists of a network of streams as shown in Fig. 31.8. Channels that start from upland areas are called the first order channels. Horton (1945) developed a stream order system, in which when two streams of order *i* join together the resulting stream is of order $i + 1$. There are several laws of stream orders developed by Horton (1945).

If a watershed has N_i streams of order *i* and N_{i+1} of order $i + 1$, the ratio N_i/N_{i+1} is called the bifurcation ratio R_B, the ratio of stream lengths L_{i+1} and L_i belonging to orders i+1 and i the ratio of stream lengths R_L, and the ratio of areas R_A and R_{A+1} the area ratio. These ratios vary over a small range for each watershed. The drainage density D of a watershed is the ratio of total stream length to the area of the watershed. Higher values of D represent a highly developed stream network and vice versa. Plots of L_i, A_i and N_i against the order i for an Indiana watershed are shown in Fig. 31.9.

The second factor that significantly affects runoff is rainfall. The spatial and temporal rainfall distribution and the history of rainfall preceding a storm affect runoff from watersheds. Rainfall is usually treated as a lumped variable because spatial rainfall data are not commonly available.

The third factor that affects runoff characteristics is the land use. As watersheds are changed from rural to urban or from forested to clear cut condition, runoff from these watersheds changes drastically. For example, when a rural watershed is urbanized, the peak discharges from the urban watershed may be more than 100% higher than runoff from the rural watershed for the same rainfall. The time to peak discharge would also be considerably shorter and the runoff volume much larger in urban watersheds compared to rural watersheds.

A plot of variation of discharge with time is called a hydrograph. A hydrograph may have different time scales such as hourly, daily, etc. Hydrographs that result from storms are called storm hydrographs (Fig. 31.10). A typical storm hydrograph may have a small flow before the discharge increases on the rising limb, reaches a peak and decreases along the recession limb. The flow that exists before the

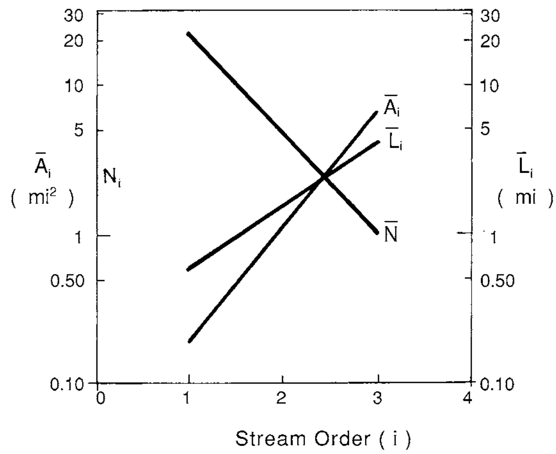


FIGURE 31.9 Horton's ratios for Bear Creek Basin, Indiana.

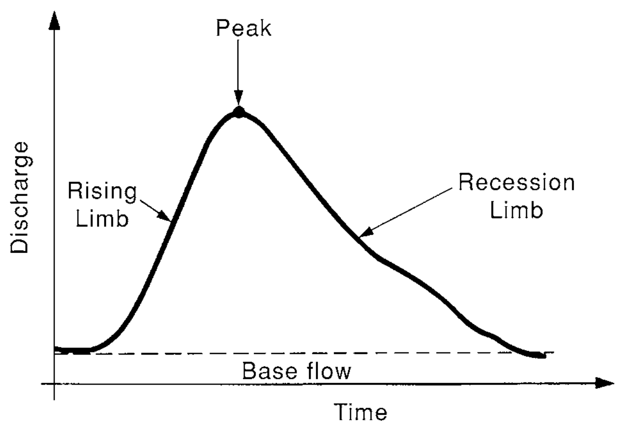


FIGURE 31.10 A single peaked hydrograph.

hydrograph starts rising is contributed by the groundwater and is called the baseflow and is not considered to be generated by the storm.

Streamflows are measured by using current meters. A stable cross section of the stream is selected and divided into a number of sections. Velocities in each section are measured and averaged. The product of the average velocity and the area of the section give the discharge in that section. Sum of discharges measured in different sections gives the discharge in the stream at that cross section.

Discharges are uniquely related to the water levels in stable stream channels. A plot of discharges against water level elevations, called river stages, is called a rating curve of a stream at a gaging station (Fig. 31.11). Once a rating curve is established for a river cross section, only the stages are measured and discharges are computed by using the rating curve. Discharges are recorded continuously or at finite time intervals. They can also be transmitted electronically to a central location where they are recorded for dissemination. Details about river gaging are found in Rantz et al. (1982).

Unit Hydrographs

One of the common and important problems in hydrology is the estimation of runoff hydrographs that result from rainfall. These hydrographs are needed for various purposes such as design of drainage and hydraulic structures and flood flow forecasting. Unit hydrograph theory is commonly used to estimate runoff hydrographs.

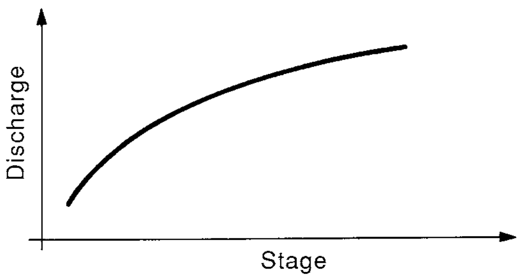


FIGURE 31.11 A rating curve.

Although surface runoff has several components such as baseflow, interflow, overland flow, etc., two components of surface runoff are commonly recognized. These are the baseflow and direct runoff. There are numerous methods of separation of base flow from the observed hydrograph to obtain the direct runoff hydrograph. The simplest among these assumes a constant baseflow value. The volume under the direct runoff hydrograph may be expressed in units of depth (centimeters or inches) by dividing it by the watershed area.

The rainfall hyetograph associated with the surface runoff hydrograph may also be separated into two components as the effective rainfall hyetograph and losses. The ϕ-index method is commonly used to derive the effective rainfall hyetograph. The rainfall depth under an effective rainfall hyetograph is the same as the depth of direct runoff.

The unit hydrograph is the direct runoff hydrograph resulting from one unit (1 in. or 1 cm) of effective rainfall occurring uniformly in space and time over a unit period of time. The duration of effective rainfall is the "unit" for which the unit hydrograph is estimated. A unit hydrograph is derived by dividing the direct runoff hydrograph ordinates by the direct runoff depth.

If a unit hydrograph of duration D_1 is available for a watershed, a unit hydrograph of duration D_2 for the same watershed may be developed by using the S-curve method. To develop an S-curve, unit hydrographs are displaced in time (or lagged) by a time interval D_1. Unit hydrograph ordinates at a given time are summed to obtain the S-curve., The S-curve is displaced by duration D_2 and the difference between the two S-curve ordinates is multiplied by D_1/D_2 to get the unit hydrograph of duration D_2.

The dependence of unit hydrographs on effective rainfall duration can be eliminated by assuming the interval between the ordinates of the runoff hydrograph to be the duration of the unit hydrograph and also of the effective rainfall pulses. Under these assumptions, the direct runoff $\boldsymbol{Q}$ is related to the effective rainfall $\boldsymbol{P}$ and unit hydrograph $\boldsymbol{U}$, as in Eq. (31.34), where $\boldsymbol{Q}$, $\boldsymbol{P}$ and $\boldsymbol{U}$ are vectors and matrices.

$$Q = P\,U \tag{31.34}$$

In Eq. (31.34), the direct runoff ordinates are Q_1, $Q_2 \ldots Q_i$, the effective rainfall ordinates are P_1, $P_2, \ldots P_j$ and the unit hydrograph ordinates are U_1, $U_2, \ldots U_k$. Equation (31.34) may be expressly written as in Eq. (31.35).

$$\begin{pmatrix} Q_1 \\ Q_2 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ Q_i \end{pmatrix} \begin{pmatrix} P_1 & 0 & 0 & & \ldots & 0 \\ P_2 & & & & \ldots & 0 \\ \cdot & & & & & \\ \cdot & & & & & \\ \cdot & & & & & \\ \cdot & & & & & \\ 0 & 0 & \ldots & P_{k-1} & \ldots & P_{j-1} \\ 0 & 0 & \ldots & & \ldots & P_j \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ u_{k-1} \\ u_k \end{pmatrix} \tag{31.35}$$

The relationship between *i, j* and *k* is given in Eq. (31.36)

$$i = j + k - 1 \tag{31.36}$$

Expanding Eq. (31.35) we get Eq. (31.37). These equations are used to compute direct runoff given effective precipitation ***P*** and unit hydrograph ordinates ***U***. They may also be used to estimate unit hydrograph ordinates by forward substitution as shown below.

$$\begin{aligned} Q_1 &= P_1\ U_1 \\ Q_2 &= P_2\ U_1 + P_1\ U_2 \\ Q_3 &= P_3\ U_1 + P_2\ U_2 + P_1\ U_3 \\ &. \\ &. \\ &. \\ U_1 &= Q_1 / P_1 \\ U_2 &= \left(Q_2 - P_2\ U_1\right) / P_1 \\ U_3 &= \left(Q - P_3\ U_1 - P_2\ U_2\right) / P_1 \end{aligned} \tag{31.37}$$

The major problem with the forward substitution method of computing unit hydrograph ordinates is that the errors in the estimated unit hydrograph ordinates propagate and magnify.

To avoid the amplification of errors and to get stable unit hydrograph estimates, several other methods have been developed (Singh, 1988). One of the commonly used methods is the least squares method. The least squares estimate of the unit hydrograph is given by Eq. (31.38), where *T* is the vector transpose.

$$U = \left[P^T\ P\right]^{-1} P^T Q \tag{31.38}$$

Synthetic Unit Hydrographs

For many, especially small watersheds, rainfall-runoff data may not be available to develop unit hydrographs and use them to estimate runoff. In such cases, relationships developed between unit hydrograph characteristics derived by using observed rainfall-data and watershed and effective rainfall characteristics are used to generate unit hydrographs. These hydrographs are called synthetic unit hydrographs.

Snyder (1938) developed synthetic unit hydrographs by using data from Appalachian highlands. Watersheds, the data from which were used by Snyder, varied in size from about 10 to 10,000 mi^2. A number of studies following Snyder's study followed. Many of these are designed to develop unit hydrographs from urban watersheds. A few, representative synthetic unit hydrograph methods are discussed below.

Sarma et al. (1969) provided a set of equations to estimate the watershed time lag — defined as the time interval between the centroids of effective rainfall and direct runoff — the time to peak T_p and peak discharge Q_p of observed runoff (Eqs. [31.39] to [31.41]). In Eqs. (31.39) to (31.41), *A* is the area (mi^2), *U* is the fraction of imperviousness of the watershed, P_E is the effective rainfall depth (inches) and T_R is the duration of effective rainfall (hours).

$$t_L = 0.831\ A^{0.458}\ \left(1+U\right)^{-1.662}\ P_E^{-0.267}\ \mathrm{T}_\mathrm{R}^{0.371} \tag{31.39}$$

$$Q_p = 484.1\ A^{0.723}\ \left(1+U\right)^{1.516}\ \ P_E^{1.113}\ \ T_R^{-0.403} \tag{31.40}$$

$$T_p = 0.775\ A^{0.323}\left(1+U\right)^{-1.285}\ P_E^{-0.195}\ T_R^{0.634} \tag{31.41}$$

The time lag t_L is the parameter k in the instantaneous unit hydrograph of the single linear reservoir model in Eq. (31.42). The unit hydrograph of an urban watershed can thus be estimated by Eq. (31.42) and the direct runoff can be computed by using Eq. (31.34).

$$u\left(t\right) = \frac{1}{k}\, e^{-t/k} \tag{31.42}$$

Espey et al. (1977) developed another set of equations to estimate synthetic unit hydrographs along the lines of Snyder (1938). In these (Eqs. [31.43 to– [31.37]) L is the length along the main channel (ft.), S is the main channel slope determined by $H/0.8L$ where H is the difference along two points on the main channel. The first point on the channel bottom is at a distance of 0.22 L from the downstream from the watershed boundary and the second point is on the channel bottom at the downstream point. I (percent) is the watershed impervious area (equal to 5 percent for undeveloped watershed), φ is a dimensionless conveyance factor, A is the (mi^2) watershed area, t_p (min) is the time to peak, U_p (cfs) is the peak flow of U.H., t_b (min.) is the U.H. base time, and W_{75} and W_{50} are the 75-percent (w_{75}) and 50 percent (w_{50}) of unit hydrograph peak discharge.

$$t_p = 3.1\ L^{0.23}\ S^{-0.25}\ I^{0.18}\ \varphi\ 1.57 \tag{31.43}$$

$$U_p = 31.62\times 10^3\ A^{0.96}\ T_p^{-1.07} \tag{31.44}$$

$$t_b = 125.89\times 10^3\ A\ Q_p^{-0.95} \tag{31.45}$$

$$W_{50} = 16.22\times 10^3\ A^{0.93}\ Q_p^{-0.92} \tag{31.46}$$

$$W_{75} = 3.24\times 10^3\ A^{0.79}\ Q_p^{-0.78} \tag{31.47}$$

The watershed conveyance factor φ is estimated from Fig. 31.12.

SCS Method

The SCS method is based on the time of concentration t_c (hours) and the watershed area A (square miles). The duration of the unit hydrograph ΔD (hours) is given by Eq. (31.48), the time to peak t_p (hours) and the peak discharge q_p (cfs) of the unit hydrograph are given by Eq. (31.49) and Eq. (31.50), and the base time units by Eq. (31.51).

$$\Delta D = 0.133\ t_c \tag{31.48}$$

$$t_p = 0.67\ t_c \tag{31.49}$$

$$q_p = \frac{484\,A}{t_p} \tag{31.50}$$

$$t_b = 2.67\ t_p. \tag{31.51}$$

A triangular unit hydrograph of duration ΔD can be constructed by using Eqs. (31.49) to (31.51). A unit hydrograph can also be generated by the dimensionless unit hydrograph given in SCS (1972).

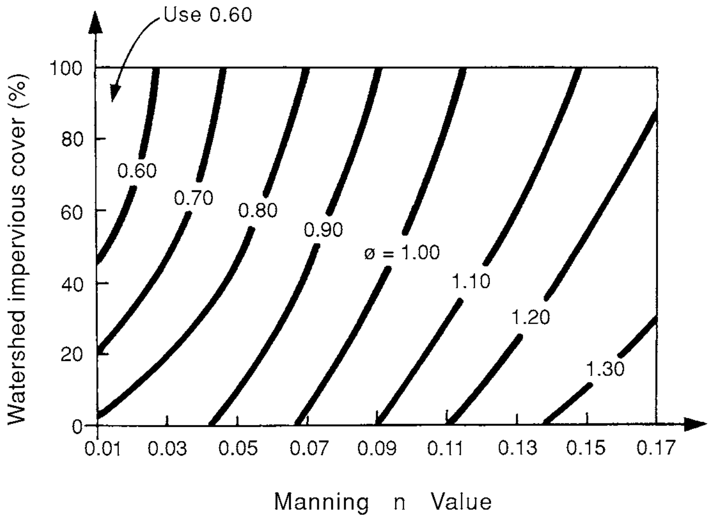


FIGURE 31.12 Watershed conveyance factor φ. (From Espey, W.H., Jr., Altman, D.G. and Graves, C.B., [1977] "Nomographs for Ten-Minute Unit Hydrographs for Small Urban Watersheds," Tech. Memo 32, *Urban Water Resources,* Res. Prog., ASCE, New York, NY. With permission.)

31.6 Flood Routing Through Channels and Reservoirs

As runoff from land enters into channels, the volume of water temporarily stored in the channel increases. After the end of precipitation water moves down the channel and the discharge decreases. At a cross section of a channel, this increase in stage and its decrease at the end of a storm is analogous to the passage of a wave and hence these are called flood waves.

Whether a flood wave moves down a channel or through a reservoir, water is temporarily stored in the channel or in the reservoir and is naturally drained out or released. Flood routing is the name given to a set of techniques that are developed to analyze the passage of a flood wave through the system. Hydraulic routing is flood routing in which equations which govern the motion of a flood wave — the St. Venant's equations — are used. In hydrologic routing, one dimensional, lumped, continuity equation is solved to estimate the passage of a flood wave. Reservoir routing is similar to hydrologic routing of flood waves.

Hydraulic Routing

The St. Venant equations that are the basic equations of motion describing the passage of a flood wave down a channel are in Eq. (31.52) and Eq. (31.53), where Eq. (31.52) is the continuity equation and Eq. (31.51) the momentum equation.

$$\frac{\partial Q}{\partial x}+\frac{\partial A}{\partial t}=0 \tag{31.52}$$

$$\frac{1}{A}\frac{\partial Q}{\partial t}+\frac{1}{A}\frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right)+g\frac{\partial y}{\partial x}-g\left(s_o-s_f\right)=0 \tag{31.53}$$

In Eq. (31.52) and Eq. (31.53), Q is the discharge, A the cross-sectional area of flow, t is the time, x is the distance along the channel, y is the depth, g is the acceleration due to gravity, s_o and s_f are the slopes of channel bottom and of the energy grade line, respectively. These equations are first order, nonlinear, hyperbolic partial differential equations. Given a set of boundary and initial conditions, they can be solved numerically for discharge and depth (Lai, 1986). Another input is the roughness coefficient, Manning's *n*. St. Venant equations are often simplified and solved as approximate solutions are sufficient in many cases. Two of these simplified solutions are the kinematic wave and diffusion approximations.

Kinematic wave approximation

In this case the momentum equation is reduced to Eq. (31.54) and the continuity equation is retained. The resulting equations are:

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = 0 \tag{31.52}$$

$$s_o = s_f \tag{31.54}$$

In this approximation, the dynamic terms in the momentum equation are ignored. Consequently, the discharge and stage variation both in time and space must be small for the kinematic wave approximation to be valid.

Diffusion Approximation

In this case, in addition to the slope terms, the term involving $\partial y/\partial x$ in Eq. (31.53) is retained to give the system of Eqs. (31.52) and (31.55). When the stage variation with distance is significant the diffusion wave approximation gives better results than the kinematic wave approximation.

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = 0 \tag{31.52}$$

$$\frac{\partial y}{\partial x} - s_o + s_f = 0 \tag{31.55}$$

Kinematic wave approximation has been used in urban hydrology and for modeling flows from small areas. The diffusion wave approximation is used to route floods through streams. Although a few closed form solutions are available for simple cases of kinematic wave routing, numerical methods are used to solve even these simpler equations (Singh, 1988). Hydrologic routing is often used because of its simplicity.

Hydrologic Channel Routing

The storage *S* in a channel reach is a function of both the inflow *I* and outflow *O* from a reach. The continuity equation for this system is given in Eq. (31.56)

$$\frac{dS}{dt} = I - 0 \tag{31.56}$$

The routing problem is, given the inflow and an initial outflow, to estimate the outflow. In the Muskingum method, the storage function is assumed to be a linear function of inflow and outflow as in Eq. (31.57).

$$S = K\left[xI + (1-x)0\right] \tag{31.57}$$

Writing Eq. (31.57) in a discrete form in terms of storage at times *j* and *j*+1 we get Eq. (31.58), where Δt is the time interval between *j* and *j*+1.

$$S_{j+1} - S_j = \frac{\left[I_{j+1} + I_j\right]}{2}\Delta t - \frac{\left[O_{j+1} + O_j\right]}{2}\Delta t \tag{31.58}$$

Substituting Eq. (31.59) and Eq. (31.60) into Eq. (31.58), we get Eq. (31.61), where C_1, C_2 and C_3 are defined in Eq. (31.62),

$$S_{j+1} = K\left[x\, I_{j+1} + (1-x)\, O_{j+1}\right] \tag{31.59}$$

$$S_j = K\left[x\, I_j + (1-x)\, O_j\right] \tag{31.60}$$

$$Q_{j+1} = C_1\, I_{j+1} + C_2\, I_j + C_3\, O_j \tag{31.61}$$

$$C_1 = \frac{\Delta t - 2Kx}{N},\quad C_2 = \frac{\Delta t + 2Kx}{N},\quad C_3 = \frac{2K(1-x) - \Delta t}{N} \tag{31.62}$$

where

$$N = 2K - 2Kx + \Delta t \tag{31.63}$$

$$\text{and } C_1 + C_2 + C_3 = 1. \tag{31.64}$$

To use Eq. (31.61), the coefficients C_1, C_2 and C_3, which are dependent on K, Δt and x must be known. To estimate K, by using a set of inflow and outflow hydrographs, different values of x are assumed and $x\, I_j + (1 - x)\, O_j$ values are plotted against computed storage values. The value of x, which gives a linear relationship between observed and computed storages, is selected and the slope of the fitted line is the best estimate of the K value. K and x values are also estimated by using the method of moments, least squares and optimization methods. If several sets of inflow and outflow hydrographs are used, average K and x values are used as the best estimates for the reach. These K and x values, the inflow hydrograph and the initial outflow value are used to compute the outflow hydrograph from the reach, thus completing the streamflow routing through the channel reach. A detailed discussion of these methods is found in Singh (1988).

Reservoir Routing

Reservoir routing is the procedure by which the outflow hydrograph from a reservoir is computed given the inflow hydrograph, the initial outflow or reservoir level, and the storage characteristics of the reservoir. A linear relationship between storage and inflow and outflow such as that assumed in channel routing cannot be assumed in this case.

The continuity equation, Eq. (31.56) forms the basis of the routing method in this case also. The discrete form of the continuity equation Eq. (31.58) is rewritten as Eq. (31.65).

$$\left(\frac{2S_{j+1}}{\Delta t} + o_{j+1}\right) = \left(I_{j+1} + I_j\right) + \left(\frac{2S_j}{\Delta t} - o_j\right) \tag{31.65}$$

By using the relationship between the storage and outflow relationships that are unique for each reservoir system, Eq. (31.65) is solved iteratively for discharge. Various methods have been developed to solve Eq. (31.65) and one of these methods — storage indication method — is as follows.

By using the reservoir elevation-discharge and elevation-storage data (see chapter on Hydraulic Structures), a curve relating $(2S/\Delta t + O)$ and discharge O is developed. By using the initial storage, outflow information, $2S_0/\Delta t + O_0$ value is estimated. The $2S_1/\Delta t + O_1$ is then estimated by Eq. (31.65), and by using the $2S/\Delta t + O_1$ vs. O relationship, the discharge O_1, and hence $2S/\Delta t - O_1$ is computed. By using

O_1, I_2 and I_1 in Eq. (31.65) is computed and O_2 is evaluated by the curve of vs. O. The procedure is repeated until the end of the inflow hydrograph.

31.7 Statistical Analysis of Hydrologic Data

Important hydrologic processes such as floods or droughts, which are extreme events, are treated as random events. The theory of probability is used to estimate the probabilities of occurrence of these events. The emphasis in statistical analysis is on events rather than on the physical processes that generate them. In the frequency analysis of floods the emphasis is on the frequency of occurrence of these events.

Probability Distributions and Parameter Estimates

The common probability distributions used in hydrologic analysis and their parameters are listed in Table 31.3. There are various methods of parameter estimation and the simplest among these is the method of moments. In this method, moments estimated from the data are equated to the expressions of moments of distributions and the resulting equations are solved for the distribution parameters. The mean, standard deviation and the skewness coefficient are the three moments commonly computed from the data.

The mean $\bar{x}$, standard deviation s_x and the skewness coefficient CS_x computed by using the data x_1, $x_2, \ldots x_N$ are given in Eqs. (31.66) to (31.68).

$$\bar{x} = \frac{\sum_{i=1}^{N} x_i}{N} \tag{31.66}$$

$$s_x = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}\left(x_i - \bar{x}\right)^2} \tag{31.67}$$

$$CS_x = \frac{N\sum_{i=1}^{N}\left(x_i - \bar{x}\right)^3}{(N-1)(N-2)\,S_x^3} \tag{31.68}$$

TABLE 31.3 Frequency Factors for Commonly Used Distributions

Distribution	k_T
Normal	For a given T, compute p = 1/T and $a = \sqrt{\ln\left(1/p^2\right)}$
	$k_{TN} = a - \left(\frac{2.525517 + 0.802853a + 0.010328\,a^2}{1 + 1.432788a + 0.189269\,a^2 + 0.001308\,a^3}\right)$
	If $p > 0.5$, $a = \sqrt{\ln\left(1/(1-p)^2\right)}$ 1 and $x_T = -x_T$.
EV(I)	$k_{TE} = -\frac{\sqrt{6}}{\pi}\ 0.5772 + \ln\left\{\ln\left(\frac{T}{T-1}\right)\right\}\Big]$
LP (III)	For a given T, compute k_{TN}. $b = c_{sy}/6$,
	$k_{TL} = k_{TN} + \left(k_{TN}^2 - 1\right)b + \frac{1}{3}\left(k_{TN}^3 - 6k_{TN}\right)b^2 - \left(k_{TN}^2 - 1\right)\,b^3 + b\,k_{TN}^4 + \frac{1}{3}k_{TN}^5.$

Coefficient of variation CV is the ratio of the standard deviation to mean of the data. If CS_x is zero, then the distribution of x_i is symmetric; otherwise it is positively or negatively distributed depending on the sign of CS_x. Expressions relating the moments in Eqs. (31.66) to (31.68) and the parameters of the distributions are also shown in Table 31.3.

Frequency Analysis of Hydrologic Data

Frequency analysis of hydrologic data is conducted to estimate the magnitude of the variate corresponding to a recurrence interval T. The recurrence interval and the exceedance probability p or $P(x \geq x_T)$, are inversely related.

$$p = P(x \geq x_T) = \frac{1}{T} \tag{31.69}$$

The probability of occurrence of an event x_T, $F_x(x_T)$, is the probability that the random variable x is smaller than x_T, $p\ (x < x_T)$. Therefore, the recurrence interval T and the probability of occurrence of an event are related to each other as in Eq. (31.70).

$$P(x < x_T) = F_x(x_T) = 1 - \frac{1}{T} = \frac{T-1}{T} \tag{31.70}$$

The relationships in Eq. (31.69) and Eq. (31.70) are used to derive the relationships between x_T and the corresponding T. For example, the probability distribution of type I extreme value distribution, or the EV(I) distribution is in Eq. (31.71) (Gumbel, 1958). The function $(x_T - u)/\alpha$ is called the reduced variate y_T.

$$F_x(x_T) = \exp\left[-\exp\left(-\frac{x_T - u}{\alpha}\right)\right] \tag{31.71}$$

Solving Eq. (31.71) for y_T and substituting Eq. (31.70) for $F_x\ (x_T)$ in the resulting equation, Eq. (31.72) is obtained.

$$y_T = -\ln\left[\ln\left(\frac{T}{T-1}\right)\right] \tag{31.72}$$

$$x_T = u + \alpha\ y_T \tag{31.73}$$

Therefore, for specific values of T, u and α, y_T is computed by Eq. (31.72) and the corresponding value of the variate is computed by Eq. (31.73). If annual maximum flood data are analyzed, the corresponding resulting flood magnitude x_T is called the T-year flood.

Chow (1951) generalized relationships such as those in Eq. (31.73) for use in hydrologic frequency analysis. If observed data x_i are used, then the relationship between the magnitude of the variable x_T and the corresponding recurrence interval T is given by Eq. (31.74), where k_T is the hydrologic frequency factor which is related to T and $F_x(x_T)$ as in Eq. (31.70).

$$x_T = \bar{x} + k_T\ s_x \tag{31.74}$$

If log transformed data are used in the frequency analysis, $y_i = \log x_i$, the relationship corresponding to Eq. (31.74) is given in Eq. (31.75), where $\bar{y}$ and s_y are the mean and standard deviation of log transformed data.

$$y_T = \bar{y} + k_T\ S_y \tag{31.75}$$

The skewness coefficient of log transformed data is denoted by C_{sy}. The relationship between k_T and T for normal, EV(1) and log Pearson type (III) [LP(III)] distribution are shown in Table 31.3.

Hydrologic data are plotted on probability papers in which the ordinates represent x_T and the abscissa represent either the exceedance probability or the cumulative probability of a distribution. These exceedance probabilities are estimated by using plotting position formulas. A general representation of the plotting position formulas is given in Eq. (31.76), where i is the rank of the variate, with i = 1 for the largest value. Several commonly used plotting position formulas may be derived from Eq. (31.76). For example, C is equal to 0.5 for Hazen's, 0 for Weibull's, 0.3 for Chegodayev's, 0.375 for Blom's, 0.33 for Tukey's and 0.44 for Gringorten's formulas.

$$P\left(x \geq x_T\right) = \frac{1}{T} = \frac{i - C}{N + 1 - 2C} \tag{31.76}$$

If the data obey the distribution whose paper they are plotted on, they fall approximately on a straight line. Straight lines may be fitted to these data and extrapolated to estimate the x_T values corresponding to recurrence interval T.

Water Resources Council Method

The U.S. Water Resources Council (Benson, 1968) has recommended that the LP (III) distribution be used for flood frequency analysis. In this method, the annual maximum flood data are log transformed and the statistics $\bar{y}$, s_y and C_{sy} are calculated. The variance of the "station skew" C_{sy}, denoted by $V(C_{sy})$ is given by Eq. (31.77), where A and B are defined in Eqs. (31.78a-d).

$$V\left(C_{sy}\right) = 10^{A - B\,\log_{10}\,(N/10)} \tag{31.77}$$

$$A = -0.33 + 0.08\left|C_{sy}\right| \quad \text{for} \quad \left|C_{sy}\right| \leq 0.9 \tag{31.78a}$$

$$A = -0.52 + 0.3\left|C_{sy}\right| \quad \text{for} \quad \left|C_{sy}\right| > 0.9 \tag{31.78b}$$

$$B = 0.94 - 0.26\left|C_{sy}\right| \quad \text{for} \quad \left|C_{sy}\right| \leq 1.50 \tag{31.78c}$$

$$B = 0.55 \qquad\qquad \text{for} \quad \left|C_{sy}\right| > 1.5 \tag{31.78d}$$

The map skewness (C_m) is interpolated from Fig. 31.13. The variance of the map skewness $V(C_m)$ for the U.S. is 0.3025. The weighted skewness coefficient C_{sw} is given by Eq. (31.79).

$$C_{sw} = \frac{V\left(C_m\right) C_{sy} + V\left(C_{sy}\right) C_m}{V\left(C_m\right) + V\left(C_{sy}\right)} \tag{31.79}$$

This skewness coefficient C_{sw} is used to compute b in Table 31.3 and k_{TL} values corresponding to recurrence interval T. The logarithm of the flow is computed by Eq. (31.75), which is used to compute the value of the flow x_T. A computer program HECWRC (U.S. Army Corps of Engineers, 1982) and its derivatives exist to perform these computations.

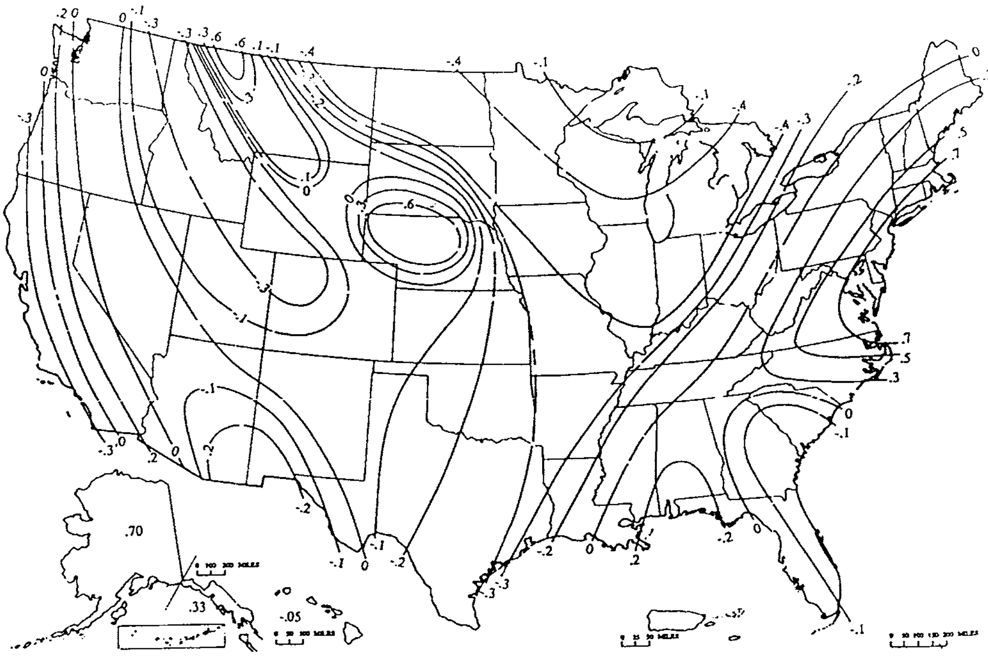


FIGURE 31.13 Map of skewness coefficients of annual maximum flows.

Defining Terms

Evapotranspiration — The process by which water in soil, vegetation and land surface is converted to water vapor.

Flood routing — A technique used to analyze the passage of a flood wave through a channel or a reservoir.

Frequency Analysis — A probabilistic analysis of hydrologic data conducted to estimate the magnitude of a variate corresponding to a recurrence interval.

Hydrograph — A plot of variation of discharge with time.

Hydrologic cycle — Continuum of the movement of water in the atmosphere, hydrosphere and lithosphere.

Infiltration — Process by which water on the soil surface enters the soil.

Isohyetal lines — Lines of equal values of precipitation measured or observed.

Pan coefficient — Ratio of lake evaporation to pan evaporation.

Percolation — Process by which water moves through the soil because of gravity.

Potential evapotranspiration — Evapotranspiration rate that occurs when the moisture supply is unlimited.

Precipitation — Water in its different phases descending from the sky.

Psychometric constant — A constant involving specific heat, latent heat of vaporization and other variables which is used in evaporation estimation.

Rainfall frequency — Time interval between occurrence of a rainfall depth over a specific duration.

Rainfall intensity — Rate of occurrence of rainfall.

SCS — Soil Conservation Service, a division of the U.S. Department of Agriculture.

Specific humidity — Ratio of water vapor pressure to the atmospheric pressure.

Surface runoff — Movement of water over land surface.

Unit hydrograph — A direct runoff hydrograph resulting from one unit of effective rainfall occurring uniformly in space and time over a unit period of time.

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Further Information

The following is a list of handbooks and textbooks in hydrology that contain further details of the topics discussed in this section.

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