19

Consolidation and Settlement Analysis

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In order to evaluate the suitability of a foundation or earth structure, it is necessary to design against both bearing capacity failure and excessive settlement. For foundations on cohesive soils, the principal design criterion is typically the latter—the control of expected settlements within the limits considered tolerable for the structure. As a result, once allowable foundation displacements have been established, the estimate of total settlement over the service life of the structure is a major factor in the choice of foundation design.

The purpose of this chapter is to present the fundamental concepts regarding settlement analysis for saturated, inorganic, cohesive soils. In addition, the recommended procedure for estimation of foundation settlements is described. Much of this chapter is based on Leonards [1968], Perloff [1975], and Holtz [1991]. Readers may refer to these works for additional information on consolidation and settlement analysis.

19.1 Components of Total Settlement

During construction, surface loads from foundations or earth structures are transmitted to the underlying soil profile. As a result, stresses increase within the soil mass and the structure undergoes a time-dependent vertical settlement. In general, this time–settlement curve can be represented conceptually as shown in Fig. 19.1. The **total settlement**, S, is calculated as the sum of the following three components:

$$S = S_i + S_c + S_s \tag{19.1}$$

where S_i is the **immediate settlement**, S_c is the **consolidation settlement**, and S_s is the **secondary compression settlement**.

Immediate settlement is time-independent and results from shear strains that occur at constant volume as the load is applied to the soil. Although this settlement component is not elastic, it is generally calculated using elastic theory for cohesive soils such as clays. Both consolidation and secondary compression settlement components are time-dependent and result from a reduction of void ratio and concurrent expulsion of water from the voids of the soil skeleton. For consolidation settlement, the rate of void ratio

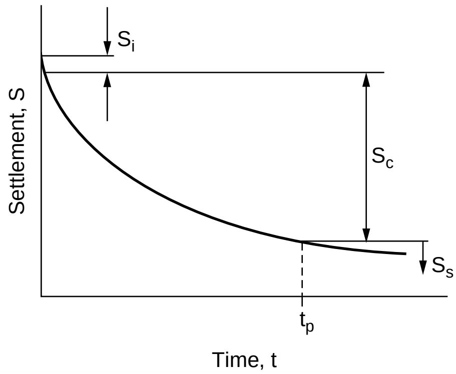

FIGURE 19.1 Time–settlement curve showing total settlement components.

reduction is controlled by the rate at which water can escape from the soil. Therefore, during consolidation, pore water pressure exceeds the steady state condition throughout the depth of the layer. Over time, the rate of consolidation settlement continuously decreases as effective stresses increase to approach their equilibrium values. Once the consolidation process is completed at time t_p, settlement continues in the form of secondary compression. During secondary compression, the rate of void ratio reduction is controlled by the rate of compression of the soil skeleton itself. As such, it is essentially a creep phenomenon that occurs at constant vertical effective stress and without sensible excess pressure in the pore water.

The time–settlement relationship shown in Fig. 19.1 is conceptually valid for all soil types. However, large differences exist in the magnitude of the components and the rate at which they occur for different soils. For granular soils, such as sands, the hydraulic conductivity is sufficiently large that consolidation occurs nearly instantaneously with the applied load. In addition, although granular soils do exhibit creep effects, secondary compression is generally insignificant. For cohesive soils, such as clays, hydraulic conductivity is very small and the consolidation of a thick deposit may require years or even decades to complete. Secondary compression can be substantial for cohesive soils. Different from both sands and clays, peats and organic soils generally undergo rapid consolidation and extensive, long-term secondary compression.

The first step in a settlement analysis is a careful study of the changes in applied loads and the selection of appropriate fractions of live load pertinent to each of the three total settlement components. Often, insufficient attention is given to this aspect of the problem. In general, immediate settlements should be computed using 100% of both live and dead loads of the structure. Consolidation and secondary compression settlements should be calculated using 100% of the dead load and permanent live load, but only a reasonable fraction of the transient live load. The proper estimate of this fraction should be made in consultation with the structural engineer on the project [Leonards, 1992].

19.2 Immediate Settlement

For saturated or nearly saturated cohesive soils, a linear elastic model is generally used for the calculation of immediate settlement. Although clays do not behave as linear elastic materials, the rationale for the use of elastic theory has been the availability of solutions for a wide variety of boundary conditions representative of foundation engineering problems. In general, the elastic approximation performs reasonably well in the case of saturated clays under monotonic loading conditions not approaching failure. In addition, for these same conditions, the elastic parameters can generally be assumed as approximately constant throughout an otherwise homogeneous soil mass [Perloff, 1975].

For cohesionless soils, in which the equivalent elastic modulus depends markedly on confinement, the use of linear elastic theory coupled with the assumption of material homogeneity is inappropriate.

Immediate settlement on granular soils is most often estimated using the procedure of Schmertmann [1970]. Holtz [1991] reviews this and other available methods in detail.

For those cases in which a linear elastic model is acceptable, solutions for stress distribution and surface deflection under a variety of flexible and rigid surface loading configurations can be found in Harr [1966], Perloff [1975], Poulos and Davis [1974], and Holtz [1991]. One particularly useful relationship is provided herein for the immediate settlement of a circular or rectangular footing at the surface of a deep isotropic stratum. In this case, the immediate vertical displacement is given by

$$S_i = C_s q B \left(\frac{1 - \nu^2}{E_u} \right) \tag{19.2}$$

where

S_i = immediate settlement of a point on the surface
C_s = shape and rigidity factor
q = equivalent uniform stress on the footing (total load/footing area)
B = characteristic dimension of the footing
ν = Poisson's ratio
E_u = undrained elastic modulus (Young's modulus)

The coefficient C_s is a function of the shape and rigidity of the loaded area and the point on the footing for which the immediate settlement estimate is desired. Thus, Eq. (19.2) can be used for both rigid and flexible footings, with the appropriate values of C_s given in Table 19.1. The characteristic footing dimension, B, is taken by convention as the diameter of a circular footing or the short side length of a rectangular footing. For saturated cohesive soils, constant volume strain is usually assumed and Poisson's ratio, ν, is set equal to 0.5. For soils that are nearly saturated, ν will be less than 0.5. However, using $\nu = 0.5$ is generally acceptable since the magnitude of computed immediate settlement is not especially sensitive to small changes in ν.

Reliable evaluation of the remaining soil parameter, the undrained elastic modulus, E_u, is critical for a good estimate of immediate settlement. In general, E_u is the slope of the undrained stress–strain curve for a stress path representative of the field condition. Figure 19.2 illustrates the measurement of E_u from a plot of principal stress difference, $\Delta\sigma$, versus axial strain, ε_a, as typically obtained from an undrained triaxial test. Principal stress difference is defined as $\sigma_1 - \sigma_3$, where σ_1 and σ_3 are the major and minor

TABLE 19.1 Values of C_s

Shape and Rigidity	Center	Corner	Edge/Middle of Long Side	Average
Circle (flexible)	1.00		0.64	0.85
Circle (rigid)	0.79		0.79	0.79
Square (flexible)	1.12	0.56	0.76	0.95
Square (rigid)	0.82	0.82	0.82	0.82
Rectangle (flexible):				
length/width				
2	1.53	0.76	1.12	1.30
5	2.10	1.05	1.68	1.82
10	2.56	1.28	2.10	2.24
Rectangle (rigid):				
length/width				
2	1.12	1.12	1.12	1.12
5	1.6	1.6	1.6	1.6
10	2.0	2.0	2.0	2.0

Source: Holtz, R. D. 1991. Stress distribution and settlement of shallow foundations. In *Foundation Engineering Handbook,* 2nd ed., ed. H.-Y. Fang. Van Nostrand Reinhold, New York.

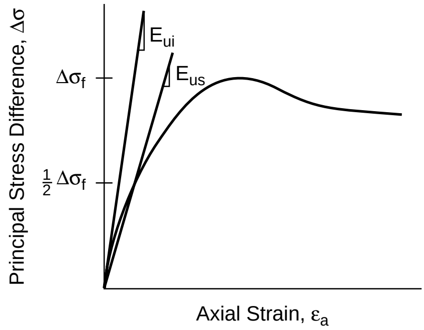

FIGURE 19.2 Definitions of the initial tangent modulus and secant modulus.

principal stresses, respectively. The initial tangent modulus, E_{ui}, is determined from the initial slope of the curve. The secant modulus, E_{us}, is sometimes used instead of E_{ui} when there is severe nonlinearity in the stress–strain relationship over the stress range of interest. Generally, the secant modulus would be taken at some predetermined stress level, such as 50% of the principal stress difference at failure, $\Delta\sigma_f$, in Fig. 19.2.

As a first approximation, the undrained elastic modulus can be estimated from the undrained shear strength using [Bjerrum, 1972]

$$E_u = 500c_u \quad \text{to} \quad E_u = 1500c_u \tag{19.3}$$

where c_u is the undrained shear strength determined from a field vane shear test. In general, E_u depends strongly on the level of shear stress. The lower value in Eq. (19.3) corresponds to highly plastic clays where the applied stress is relatively large as compared to the soil strength. The higher value is for low plasticity clays under small shear stress. In addition, the E_u/c_u ratio decreases with increasing **overconsolidation ratio** for a given stress level [Holtz and Kovacs, 1981]. Thus, Eq. (19.3) can provide a rough estimate of E_u suitable only for preliminary design computations.

In situations where a field loading test is not warranted, the undrained modulus should be estimated from a consolidated undrained (CU) triaxial test in the laboratory. The following procedure is recommended [Leonards, 1968]:

1. Obtain the highest quality soil samples. If possible, use a large-diameter piston sampler, or excavate blocks by hand from a test pit. Optimally, the laboratory test should be performed the same day as the field sampling operations.
2. Reconsolidate the specimen in the triaxial cell to the estimated initial *in situ* state of stress. If possible, anisotropic K_o consolidation is preferred to isotropic consolidation. Undrained modulus values determined from unconfined compression tests will significantly underestimate the actual value of E_u, and thereby overestimate the immediate settlement.
3. Load and unload the specimen in undrained axial compression to the expected *in situ* stress level for a minimum of 5 cycles. For field loading conditions other than a structural foundation, a different laboratory stress path may be needed to better match the actual *in situ* stress path.
4. Obtain E_u from the fifth (or greater) cycle in similar fashion to that shown in Fig. 19.2.

For sensitive clays of low plasticity, CU triaxial tests will likely yield somewhat low values of E_u, even if the specimens are allowed to undergo appreciable aging and E_u is determined at a low stress level. For highly plastic clays and organic clays, CU tests may yield stress–strain curves that are indicative of *in situ* behavior. However, it may be difficult to represent the nonlinear behavior with a single modulus value [D'Appolonia et al., 1971].

The undrained elastic modulus is best measured directly from field tests. For near surface clay deposits having a consistency that does not vary greatly with depth, E_u may be obtained from a plate load test placed at footing elevation and passed through several loading–unloading cycles (ASTM D1194). In this case, all the parameters in Eq. (19.2) are known except the factor $(1 - \nu^2)/E$, which can then be calculated. Because of the relatively shallow influence of the test, it may be advisable to use a selection of different size plates and then scale $(1 - \nu^2)/E$ to the size of the prototype foundation. In situations where the loaded stratum is deep or displays substantial heterogeneity, plate load tests may not provide a representative value for E_u. Large-scale loading tests utilizing, for example, an embankment or a large tank of water may be warranted. In this case, the immediate settlement of the proposed foundation is measured directly without requiring Eq. (19.2). Measurement of stress–strain behavior using field tests is preferred to laboratory tests because of the many difficulties in determining the appropriate modulus in the laboratory. The most important of these is the invariable disturbance of soil structure that occurs during sampling and testing. Of the many soil properties defined in geotechnical engineering, E_u is one of the most sensitive to sample disturbance effects [Ladd, 1964].

For many foundations on cohesive soils, the immediate settlement is a relatively small part of the total vertical movement. Thus, a detailed study is seldom justified unless the structure is very sensitive to distortion, footing sizes and loads vary considerably, or the shear stresses imposed by the foundation are approaching a failure condition.

19.3 Consolidation Settlement

Different from immediate settlement, consolidation settlement occurs as the result of volumetric compression within the soil. For granular soils, the consolidation process is sufficiently rapid that consolidation settlement is generally included with immediate settlement. Cohesive soils have a much lower hydraulic conductivity, and, as a result, consolidation requires a far longer time to complete. In this case, consolidation settlement is calculated separately from immediate settlement, as suggested by Eq. (19.1).

When a load is applied to the ground surface, there is a tendency for volumetric compression of the underlying soils. For saturated materials, an increase in pore water pressure occurs immediately upon load application. Consolidation is then the process by which there is a reduction in volume due to the expulsion of water from the pores of the soil. The dissipation of excess pore water pressure is accompanied by an increase in effective stress and volumetric strain. Analysis of the resulting settlement is greatly simplified if it is assumed that such strain is one-dimensional, occurring only in the vertical direction. This assumption of one-dimensional compression is considered to be reasonable when (1) the width of the loaded area exceeds four times the thickness of the clay stratum, (2) the depth to the top of the clay stratum exceeds twice the width of the loaded area, or (3) the compressible material lies between two stiffer soil strata whose presence tends to reduce the magnitude of horizontal strains [Leonards, 1976].

Employing the assumption of one-dimensional compression, the consolidation settlement of a cohesive soil stratum is generally calculated in two steps:

1. Calculate the total (or "ultimate") consolidation settlement, S_c, corresponding to the completion of the consolidation process.
2. Using the theory of one-dimensional consolidation, calculate the fraction of S_c that will have occurred by the end of the service life of the structure. This fraction is the component of consolidation settlement to be used in Eq. (19.1).

In actuality, the total amount of consolidation settlement and the rate at which this settlement occurs is a coupled problem in which neither quantity can be calculated independently from the other. However, in geotechnical engineering practice, total consolidation settlement and rate of consolidation are almost always computed independently for lack of widely accepted procedures to solve the coupled problem. This will also be the approach taken here. The calculation of total consolidation settlement will be presented first, followed by procedures to calculate the rate at which this settlement occurs.

Total Consolidation Settlement

Total one-dimensional consolidation settlement, S_c, results from a change in void ratio, Δe, over the depth of the consolidating layer. The basic equation for calculating the total consolidation settlement of a single compressible layer is

$$S_c = \frac{\Delta e H_o}{1 + e_o} \tag{19.4}$$

where e_o is the initial void ratio and H_o is the initial height of the compressible layer.

Consolidation settlement is sometimes calculated using H_o for the entire consolidating stratum and stress conditions acting at the midheight. This procedure will underestimate the actual settlement, and the error will increase with the thickness of the clay. As Δe generally varies with depth, settlement calculations can be improved by dividing the consolidating stratum into n sublayers for purposes of analysis. Equation (19.4) is then applied to each sublayer and the cumulative settlement is computed using the following equation:

$$S_c = \sum_{i=1}^{n} \frac{\Delta e_i H_{oi}}{1 + e_{oi}} \tag{19.5}$$

where Δe_i, is the change in void ratio, H_{oi} is the initial thickness, and e_{oi} is the initial void ratio of the ith sublayer.

The appropriate Δe_i for each sublayer within the compressible soil must now be determined. To begin, both the initial vertical effective stress, σ'_{vo}, and the final vertical effective stress (after excess pore pressures have fully dissipated), σ'_{vf}, are needed. The distribution of σ'_{vo} with depth is usually obtained by subtracting the *in situ* pore pressure from the vertical total stress, σ_v. Vertical total stress at a given depth is calculated using the following equation:

$$\sigma_v = \sum_{j=1}^{m} \gamma_j z_j \tag{19.6}$$

where

γ_j = unit weight of the jth stratigraphic layer
z_j = thickness of the jth stratigraphic layer
m = number of layers above the depth of interest

It should not be assumed a priori that *in situ* pore pressures are hydrostatic. Rather, significant upward or downward groundwater flow may be present. For important structures, the installation of piezometers to measure the *in situ* distribution of pore pressure is warranted. In addition, these piezometers will also provide a valuable check on the estimated initial excess pore pressures and indicate when the consolidation process is complete.

The final vertical effective stress is equal to the initial vertical effective stress plus the change of vertical effective stress, $\Delta\sigma'_v$, due to loading:

$$\sigma'_{vf} = \sigma'_{vo} + \Delta\sigma'_v \tag{19.7}$$

For truly one-dimensional loading conditions, such as a wide fill, $\Delta\sigma'_v$ is constant with depth and equal to the change in total stress applied at the surface of the soil stratum. For situations in which the load is applied over a limited surface area, such as a spread footing, $\Delta\sigma'_v$ will decrease with depth as the surface load is transmitted to increasingly larger portions of the soil mass. In this case, the theory of elasticity can be used to estimate $\Delta\sigma'_v$ as a function of depth under the center of the loaded area.

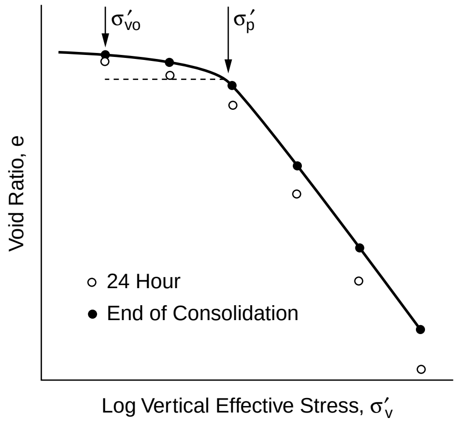

FIGURE 19.3 Typical laboratory compressibility curve.

Once initial and final stress conditions have been established, it is necessary to determine the relationship between void ratio and vertical effective stress for the *in situ* soils. This information is generally obtained from a laboratory consolidation test (ASTM D2435). The general consolidation testing procedure is to place successive loads on an undisturbed soil specimen (typically 25.4 mm high with a diameter-to-height ratio of at least 2.5 to 1) and measure the void ratio corresponding to the end of consolidation for each load increment. The load increment ratio (LIR) is defined as the added load divided by the previous total load on the specimen. The load increment duration (LID) is the elapsed time permitted for each load increment. For the standard consolidation test, the load is doubled every day, giving LIR = 1 and LID = 24 hours. Detailed procedures for specimen preparation and performance of the laboratory consolidation test are found in Bowles [1992].

A typical laboratory compressibility curve is shown in Fig. 19.3. Void ratio, e, is plotted as a function of vertical effective stress, σ'_v, on a semilogarithmic scale. The open points are void ratios measured at the end of 24 hours for each load increment. They include the contribution from all previous immediate and secondary compression settlement. The solid points represent the sum of changes in void ratio during consolidation alone, and are calculated by subtracting out the immediate and secondary compression from all previous load increments. As indicated in Fig. 19.3, the laboratory compressibility curve is best drawn through the solid points. The reason for this procedure is that S_i, S_c, and S_s are computed separately and then summed to calculate total settlement S using Eq. (19.1). Therefore, immediate and secondary compression settlements should likewise be removed from the laboratory compressibility curve to compute S_c.

For the compressibility curve in Fig. 19.3, both the **preconsolidation pressure**, σ'_p and the *in situ* initial vertical effective stress are indicated. The stress history of a soil layer is generally expressed by its **overconsolidation ratio** (OCR), which is the ratio of these two values:

$$\text{OCR} = \frac{\sigma'_p}{\sigma'_{vo}} \tag{19.8}$$

Normally consolidated soils have OCR = 1, while soils with an OCR > 1 are *preconsolidated* or *overconsolidated*. For the example shown in Fig. 19.3, the soil is overconsolidated. In addition, a soil can be underconsolidated if excess pore pressures exist within the deposit (i.e., the soil is still undergoing consolidation). With the exception of recently deposited materials, soils in the field are very often overconsolidated as a result of unloading, desiccation, secondary compression, or aging effects [Brumund et al., 1976].

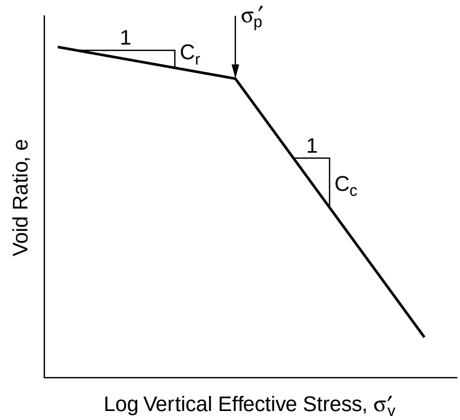

FIGURE 19.4 Simplified approximation of a laboratory compressibility curve.

The preconsolidation pressure is the stress at which the soil begins to yield in volumetric compression, and it therefore separates the region of small strains ($\sigma'_v < \sigma'_p$) from the region of large strains ($\sigma'_v > \sigma'_p$) on the $e - \log \sigma'_v$ diagram. As a result, for a given initial and final stress condition, the total consolidation settlement of a compressible layer is highly dependent on the value of the preconsolidation pressure. If a foundation applies a stress increment such that the final stress is less than σ'_p, the consolidation settlement will be relatively small. However, if the final stress is larger than σ'_p, much larger settlements will occur. Therefore, accurate determination of the preconsolidation pressure, and its variation with depth, is the most important step in a settlement analysis. The determination of σ'_p is generally performed using the Casagrande graphical construction method [Holtz and Kovacs, 1981].

For analysis purposes, the laboratory compressibility curve is usually approximated as linear (in log scale) for both the overconsolidated and normally consolidated ranges. A typical example is shown in Fig. 19.4. The slope of the overconsolidated range is the *recompression index, C_r*. Although this portion of a compressibility curve is generally not linear on the semilog plot, a line of constant C_r is usually fitted to the data for simplicity. The slope of the normally consolidated portion of the compressibility curve is the *compression index, C_c*. C_c is often constant over typical stress ranges of engineering interest. Both C_c and C_r are calculated using the same formula:

$$C_c = -\frac{\Delta e}{\Delta \log \sigma'_v} \quad \text{normally consolidated stress range} \tag{19.9}$$

$$C_r = -\frac{\Delta e}{\Delta \log \sigma'_v} \quad \text{overconsolidated stress range} \tag{19.10}$$

In many clay deposits, σ'_p, C_c, and C_r vary considerably with depth and the practice of performing one or two consolidation tests to evaluate the entire profile is seldom satisfactory. The following procedure is recommended for performing and interpreting the consolidation test to best obtain these parameters:

1. Obtain the highest quality undisturbed specimens, each representative of one sublayer in the profile, and begin testing the same day if possible. The value of σ'_p determined in the laboratory is very sensitive to sample disturbance and will generally be underestimated from data obtained using poor-quality soil samples.
2. Perform a consolidation test in which each load increment is placed on the specimen at the end of consolidation for the previous increment. The final void ratio obtained from each load will then directly provide the solid points in Fig. 19.3. The end of consolidation can be determined from pore pressure measurements or, less accurately, from graphical procedures such as the Casagrande log time or Taylor square root of time methods. LIR = 1 is satisfactory; however, to

more accurately measure the value of σ'_p, it may be advantageous to run a second test with smaller load increments in the vicinity of the preconsolidation pressure. If a standard consolidation test is performed using LID = 24 hours, plot the cumulative void ratio reduction during consolidation for each increment as shown by the solid dots in Fig. 19.3.

3. Obtain C_r by unloading from σ'_p to σ'_{vo} and then reloading. This path is indicated by the dashed line in Fig. 19.3. Using the initial reloading curve in Fig. 19.3 will yield too large a value of C_r. When the value of C_r is critical to a particular design, a backpressure oedometer should be used for testing. A C_r value obtained by unloading and reloading in a conventional oedometer (without backpressure) is about twice the value obtained in a backpressure oedometer due to expansion of gas within the pore water.
4. Reconstruct the *in situ* compressibility curve using the methods of Schmertmann [1955] as described in Holtz and Kovacs [1981], among others.

Once the *in situ* compressibility curve has been established for a given sublayer, the change of void ratio can be calculated knowing $\Delta\sigma'_v$. For normally consolidated conditions, the change of void ratio for the ith sublayer is

$$\Delta e_i = C_{ci} \log\left(\frac{\sigma'_{vfi}}{\sigma'_{voi}}\right) \tag{19.11}$$

Substituting Eq. (19.11) into Eq. (19.5), the ultimate consolidation settlement for a normally consolidated soil is

$$S_c = \sum_{i=1}^{n} \frac{C_{ci} H_{oi}}{1+e_{oi}} \log\left(\frac{\sigma'_{vfi}}{\sigma'_{voi}}\right) \tag{19.12}$$

where the summation is performed over n sublayers.

In the case of overconsolidated clays, the change of void ratio for a given $\Delta\sigma'_v$ is

$$\Delta e_i = C_{ri} \log\left(\frac{\sigma'_{vfi}}{\sigma'_{voi}}\right) \tag{19.13}$$

if $\sigma'_{vf} < \sigma'_p$ and

$$\Delta e_i = C_{ri} \log\left(\frac{\sigma'_{pi}}{\sigma'_{voi}}\right) + C_{ci} \log\left(\frac{\sigma'_{vfi}}{\sigma'_{voi}}\right) \tag{19.14}$$

if $\sigma'_{vf} > \sigma'_p$. Substituting Eqs. (19.13) and (19.14) into Eq. (19.5), the total consolidation settlement of an overconsolidated soil is

$$S_c = \sum_{i=1}^{n} \frac{C_{ri} H_{oi}}{1+e_{oi}} \log\left(\frac{\sigma'_{vfi}}{\sigma'_{voi}}\right) \tag{19.15}$$

if $\sigma'_{vf} < \sigma'_p$ and,

$$S_c = \sum_{i=1}^{n} \frac{C_{ri} H_{oi}}{1+e_{oi}} \log\left(\frac{\sigma'_{pi}}{\sigma'_{voi}}\right) + \frac{C_{ci} H_{oi}}{1+e_{oi}} \log\left(\frac{\sigma'_{vfi}}{\sigma'_{voi}}\right) \tag{19.16}$$

if $\sigma'_{vf} > \sigma'_p$.

As noted earlier, this discussion of consolidation settlement has been limited to conditions of one-dimensional compression. In those cases where the thickness of the compressible strata is large relative to the dimensions of the loaded area, the three-dimensional nature of the problem may influence the magnitude and rate of consolidation settlement. The best approach for problems of this nature is a three-dimensional numerical analysis, but these have not yet become generally accepted in practice. As an alternative, the semiempirical approach of Skempton and Bjerrum [1957] and the stress path method [Lambe, 1967] are more commonly used to take these effects into account.

Rate of Consolidation Settlement

The preceding discussion has described the calculation of ultimate consolidation settlement corresponding to the complete dissipation of excess pore pressure and the return of the soil to an equilibrium stress condition. At any time during the process of consolidation, the amount of settlement is directly related to the proportion of excess pore pressure that has been dissipated. The theory of consolidation is used to predict the progress of excess pore pressure dissipation as a function of time. Therefore, the same theory is also used to predict the rate of consolidation settlement. The one-dimensional theory of Terzaghi is most commonly used for prediction of consolidation settlement rate. The assumptions of the classical Terzaghi theory are as follows:

1. Drainage and compression are one-dimensional.
2. The compressible soil layer is homogenous and completely saturated.
3. The mineral grains and pore water are incompressible.
4. Darcy's law governs the outflow of water from the soil.
5. The applied load increment produces only small strains. Therefore, the thickness of the layer remains unchanged during the consolidation process.
6. The hydraulic conductivity and compressibility of the soil are constant.
7. The relationship between void ratio and vertical effective stress is linear and unique. This assumption also implies that there is no secondary compression settlement.
8. Total stress remains constant throughout the consolidation process.

Accepting these assumptions, the fundamental governing equation for one-dimensional consolidation is

$$\frac{\partial u}{\partial t} = c_v \frac{\partial^2 u}{\partial z^2} \tag{19.17}$$

where

$$c_v = \frac{k(1 + e_o)}{\gamma_w a_v} \tag{19.18}$$

and

k = hydraulic conductivity
γ_w = unit weight of water
e_o = initial void ratio
$a_v = -de/d\sigma'_v$ = coefficient of compressibility

The parameter c_v is called the coefficient of consolidation and is mathematically analogous to the diffusion coefficient in Fick's second law. It contains material properties that govern the process of consolidation and has dimensions of area per time. In general, c_v is not constant because its component parameters vary during the consolidation process. However, in order to reduce Eq. (19.17) to a linear form that is more easily solved, c_v is assumed constant for an individual load increment.

The consolidation equation [Eq. (19.17)] can be solved analytically using the Fourier series method. In the course of the solution, the following dimensionless quantities are defined:

$$Z = \frac{z}{H_{dr}} \tag{19.19}$$

$$T = \frac{c_v t}{H_{dr}^2} \tag{19.20}$$

$$U_z = 1 - \frac{u}{u_i} \tag{19.21}$$

where

z = depth below top of the compressible stratum
H_{dr} = length of the longest pore water drainage path
t = elapsed time of consolidation
u = excess pore pressure at time t and position z
u_i = initial excess pore pressure at position z

Z is a measure of the dimensionless depth within the consolidating stratum, T is the time factor and serves as a measure of dimensionless time, and U_z is the consolidation ratio. U_z is a function of both Z and T, and thus it varies throughout the consolidation process with both time and vertical position within the layer. U_z expresses the progress of consolidation at a specific point within the consolidating layer. The value of H_{dr} depends on the boundary drainage conditions for the layer. Figure 19.5 shows the two typical drainage conditions for the consolidation problem. A single-drained layer has an impervious and pervious boundary. Pore water can escape only through the previous boundary, giving $H_{dr} = H_o$. A double-drained layer is bounded by two pervious strata. Pore water can escape to either boundary, and therefore $H_{dr} = H_o/2$.

Figure 19.6 shows the solution to Eq. (19.17) in terms of the above dimensionless parameters. For a double-drained layer, pore pressure dissipation is modeled using the entire figure. However, for a single-drained layer, only the upper or lower half is used. As expected, U_z is zero for all Z at the beginning of the consolidation process ($T = 0$). As time elapses and pore pressures dissipate, U_z gradually increases to 1.0 for all points in the layer and σ_v' increases accordingly. From Fig. 19.6, it is possible to find the consolidation ratio (and therefore u and σ_v') at any time t and any position z within the consolidating layer after the start of loading. The time factor T can be calculated from Eq. (19.20) given the c_v for a particular deposit, the total thickness of the layer, and the boundary drainage conditions.

Figure 19.6 also provides some insight as to the progress of consolidation with time. The isochrones (curves of constant T) represent the percent consolidation for a given time throughout the compressible layer. For example, the percent consolidation at the midheight of a doubly drained layer for $T = 0.2$ is approximately 23% (see point A in Fig. 19.6). However, at $Z = 0.5$, $U_z = 44\%$ for the same time factor. Similarly, near the drainage surfaces at $Z = 0.1$, the clay is already 86% consolidated. This also means,

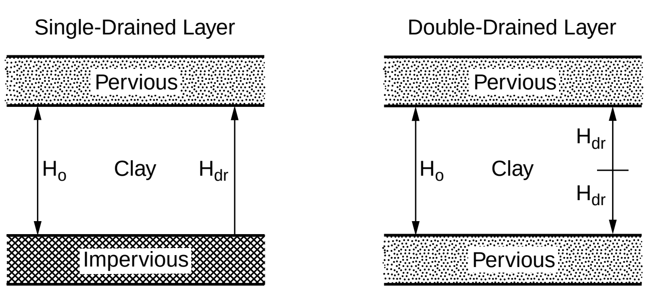

FIGURE 19.5 Boundary drainage conditions for the consolidation problem.

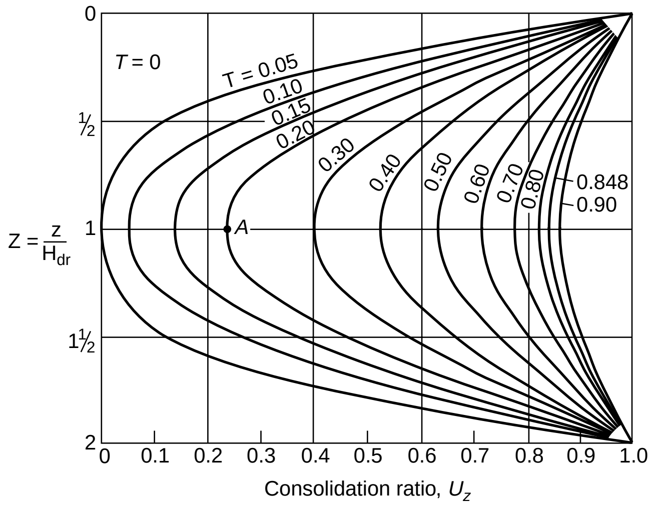

FIGURE 19.6 Consolidation ratio as a function of *Z* and *T*. (*Source:* Taylor, D. W. 1948. *Fundamentals of Soil Mechanics*. John Wiley & Sons, New York.)

at that same depth and time, 86% of the original excess pore pressure has dissipated and the effective stress has increased by a corresponding amount.

For settlement analysis, the consolidation ratio U_z is not the quantity of immediate interest. Rather, the geotechnical engineer needs to know the average degree of consolidation for the entire layer, U, defined as

$$U = \frac{s(t)}{S_c} \tag{19.22}$$

where $s(t)$ is the consolidation settlement at time t and S_c is the total (ultimate) consolidation settlement.

The following approximations can be used to calculate U. For $U < 0.60$,

$$T = \frac{\pi}{4}U^2 \tag{19.23}$$

and for $U > 0.60$,

$$T = 1.781 - 0.933 \log(100(1 - U)) \tag{19.24}$$

Provided S_c is known, Eqs. (19.22), (19.23), and (19.24) can be used to predict consolidation settlement as a function of time.

19.4 Secondary Compression Settlement

Secondary compression settlement results from the time-dependent rearrangement of soil particles under constant effective stress conditions. For highly compressible soils, such as soft clays and peats, secondary compression is important whenever there is a net increase in σ'_v due to surface loading. Although structures of any consequence would seldom be founded on these soils, highways, for example, must commonly cross areas of compressible soils that are either too deep or too extensive to excavate.

Secondary compression settlement can be predicted using the secondary compression index, C_α, defined as the change of void ratio per log cycle of time:

$$C_\alpha = -\frac{\Delta e}{\Delta \log t} \tag{19.25}$$

Laboratory values for the secondary compression index should be measured at a stress level and temperature corresponding to that expected in the subsurface.

As a first approximation, C_α can be calculated from the compression index using the ratio C_α/C_c. This method has the advantage of not requiring prolonged periods of secondary compression in the laboratory consolidation test. Recommended values for C_α/C_c are [Mesri and Castro, 1987]:

$$\frac{C_\alpha}{C_c} = 0.04 \pm 0.01 \quad \text{for soft inorganic clays} \tag{19.26}$$

$$\frac{C_\alpha}{C_c} = 0.05 \pm 0.01 \quad \text{for highly organic plastic clays} \tag{19.27}$$

Once a C_α value has been selected, secondary compression settlement S_s is calculated using the following equation:

$$S_s = \frac{C_\alpha H_o}{1 + e_o} \log \frac{t_f}{t_p} \tag{19.28}$$

where t_p is the time at the end of consolidation, and t_f is the final time for which secondary compression settlement is desired (typically the design life of the structure).

For all loading conditions, including one-dimensional compression, LIR decreases with depth. This is especially true for foundations where the load is spread over a limited surface area. For important structures, it is recommended to account for the effect of LIR on S_s. To begin, the secondary compression settlement per log cycle of time, R_s, is defined as follows,

$$R_s = \frac{\Delta S_s}{\Delta \log t} \tag{19.29}$$

Leonards and Girault [1961] demonstrated that a consistent relationship exists between R_s/S_c and LIR for Mexico City clay, as shown in Fig. 19.7. If a corresponding relationship can be obtained from laboratory consolidation tests, it can be used to estimate secondary compression settlement in the field in the following manner:

1. Divide the compressible strata into sublayers and compute the ultimate consolidation settlement S_{ci} for each ith sublayer using the procedure previously described.
2. Calculate LIR for each sublayer.
3. Obtain R_{si}/S_{ci} using the LIR for each sublayer.
4. Multiply the values of S_{ci} from step 1 by the values of R_{si}/S_{ci} obtained in step 3 to calculate R_{si} for each sublayer.
5. Sum the value of R_{si} for all sublayers to obtain R_s, the total secondary compression settlement per log cycle of time.
6. Calculate the secondary compression settlement using the following equation:

$$S_s = R_s \log \frac{t_f}{t_p} \tag{19.30}$$

Discrepancies of up to 75% may be expected using this procedure. This is indicative of the present state-of-the-art in predicting secondary compression settlement.

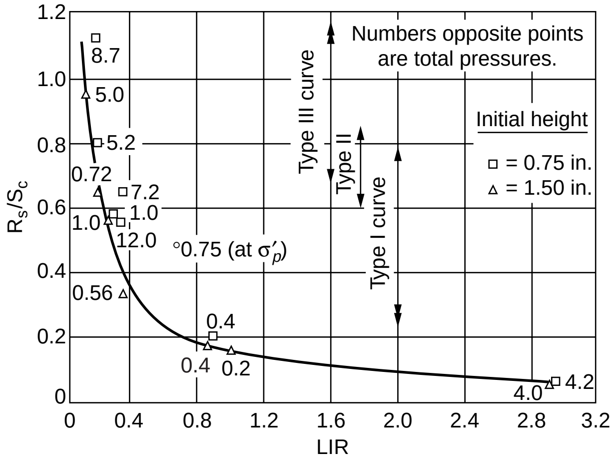

FIGURE 19.7 Effect of LIR on rate of secondary compression for undisturbed Mexico City clay. (*Source:* Leonards, G. A. and Girault, P. 1961. A study of the one-dimensional consolidation test. In *Proc. 5th Int. Conf. Soil. Mech. Found. Eng.,* Paris, 1:213–218.)

In many cases of practical interest, secondary compression is a minor effect relative to the magnitude of consolidation settlement. However, in some instances where very soft soils are involved or where deep compressible strata are subjected to small LIR, secondary compression may account for the majority of total settlement.

Defining Terms

Consolidation settlement — The time-dependent component of total settlement that results from the dissipation of excess pore pressure from within the soil mass.

Immediate settlement — The time-independent component of total settlement that occurs at constant volume as the load is applied to the soil.

Normally consolidated — A condition in which the initial vertical effective stress is equal to the preconsolidation pressure.

Overconsolidated — A condition in which the initial vertical effective stress is less than the preconsolidation pressure.

Overconsolidation ratio — The value of the preconsolidation pressure divided by the initial vertical effective stress.

Preconsolidation pressure — The vertical effective stress at which the soil begins to yield in volumetric compression.

Secondary compression — The time-dependent component of total settlement which occurs after consolidation and results from creep under constant effective stress.

Total settlement — The total vertical displacement of a foundation or earth structure that takes place after construction.

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