15

Soil Relationships and Classification

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15.1 Soil Classification

There are two soil classification systems in common use for engineering purposes. The **Unified Soil Classification System** [ASTM D 2487-93] is used for virtually all geotechnical engineering work except highway and road construction, where the **AASHTO classification system** [AASHTO M 145-87] is used. Both systems use the results of **grain-size analysis** and determinations of **Atterberg limits** to determine a soil's classification. Soil components may be described as **gravel, sand, silt,** or **clay.** A soil comprising one or more of these components is given a descriptive name and a designation consisting of letters or letters and numbers which depend on the relative proportions of the components and the **plasticity** characteristics of the soil.

Grain-Size Characteristics of Soils

Large-grained materials such as **cobbles** and **boulders** are sometimes considered to be soil. The differentiation of cobbles and boulders depends somewhat on local practice, but boulders are generally taken to be particles larger than 200 to 300 mm or 9 to 12 in. The Unified Soil Classification System suggests that boulders be defined as particles that will not pass a 12-in. (300 mm) opening. Cobbles are smaller than boulders and range down to particles that are retained on a 3-inch (75 mm) sieve. Gravels and sands are classified as **coarse-grained** soils; silts and clays are **fine-grained** soils. For engineering purposes, gravel is defined as soil that passes a 3-inch (75 mm) sieve and is *retained* by a No. 4 sieve (4.75 mm or 0.187 in.) or No. 10 sieve (2.00 mm or 0.078 in.), depending on the classification system. Sand is defined as soil particles smaller than gravel but retained on a No. 200 sieve (0.075 mm or about 0.003 in.). Soils passing the No. 200 sieve may be silt or clay. Although grain-size criteria were used in some older classification systems to differentiate silt from clay, the two systems described herein make this differentiation based on plasticity rather than grain size.

TABLE 15.1 Opening Sizes of Commonly Used Sieves

Inches	Millimeters
1.5	37.5
1	25
0.75	19
0.5	12.5
Sieve No.	**Millimeters**
4	4.75
10	2.00
20	0.850
40	0.425
70	0.212
100	0.150
200	0.075

The grain-size characteristics of soils that are predominantly coarse grained are evaluated by a **sieve analysis.** A **nest of sieves** is prepared by stacking sieves one above the other with the largest opening at the top followed by sieves of successively smaller openings and a catch pan at the bottom. Opening sizes of commonly used sieves are shown in Table 15.1. A sample of dry soil is poured onto the top sieve, the nest is covered, and it is then shaken by hand or mechanical shaker until each particle has dropped to a sieve with openings too small to pass, and the particle is *retained.* The cumulative weight of all material larger than each sieve size is determined and divided by the total sample weight to obtain the *percent retained* for that sieve size, and this value is subtracted from 100% to obtain the *percent passing* that sieve size. Results are displayed by plotting the percent passing (on a linear scale) against the sieve opening size (on a log scale) and connecting the plotted points with a smooth curve referred to as a **grain-size distribution curve.** A sample of some grain-size distribution curves is presented in Fig. 15.1.

The notation D_{xx} refers to the size D, in mm, for which *xx* percent of the sample by weight passes a sieve with an opening equal to D. The **D_{10} size,** sometimes called the **effective grain size,** is the grain diameter for which 10% of the sample (by weight) is finer. It is determined from the grain-size distribution curve at the point where the curve crosses a horizontal line through the 10% passing value on the y axis. Other D sizes are found in a similar manner. The D_{50} size, called the **median grain size,** is the grain diameter for which half the sample (by weight) is smaller and half is larger.

Two parameters are used to describe the general shape of the grain-size distribution curve. The **coefficient of uniformity,** C_u, is:

$$C_u = \frac{D_{60}}{D_{10}} \tag{15.1}$$

The **coefficient of curvature,** C_c, is:

$$C_c = \frac{D_{30}^2}{D_{60} \times D_{10}} \tag{15.2}$$

Atterberg Limits and Plasticity

Atterberg limits, named after the Swedish soil scientist A. Atterberg, are water content values at which notable changes in soil behavior occur. The **liquid limit**, denoted LL or w_L, marks the transition between liquid and plastic behavior. At water contents above the liquid limit the soil behaves as a viscous liquid; below the liquid limit the soil behaves as a plastic solid. The liquid limit is determined in the laboratory by partly filling a standard brass cup with wet soil and cutting a groove of a standard dimension in the

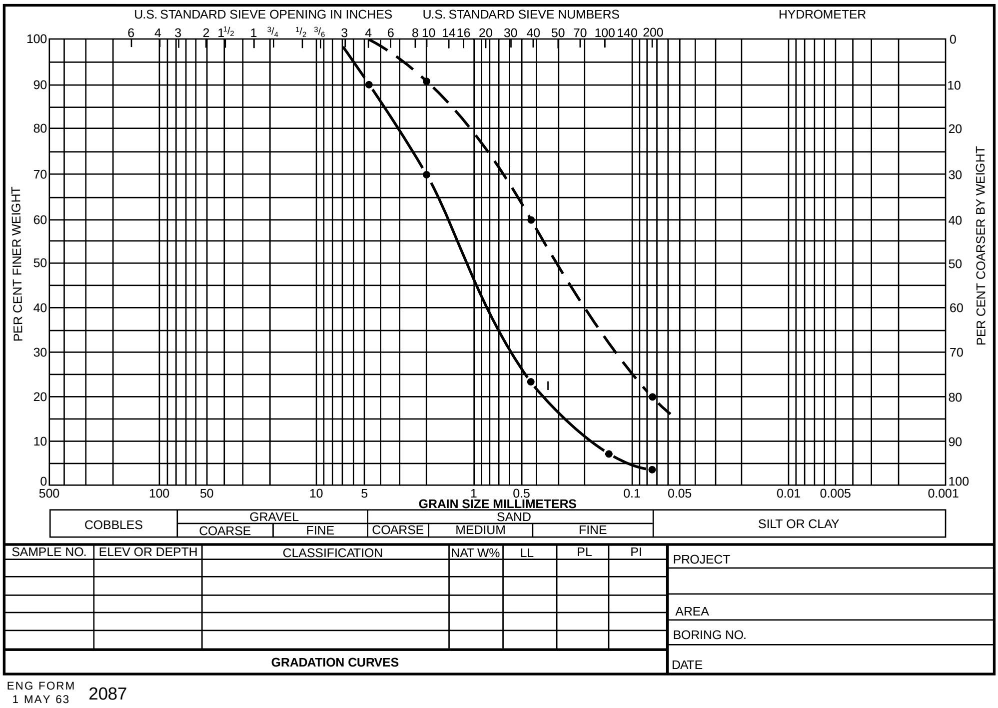

FIGURE 15.1 Typical grain size distribution curves. (From U.S. Army, 1970.)

soil. The liquid limit is taken as the water content at which the groove closes a specified amount when the cup is lifted and dropped 1 cm exactly 25 times. The details of the test are given in AASHTO T 89 and ASTM D 4318-93. The **plastic limit**, denoted PL or w_p, is the transition between plastic and brittle behavior. It is determined in the laboratory as the water content at which a 1/8-inch diameter thread of soil begins to crumble when rolled under the palm of the hand. Details of the liquid limit and plastic limit tests are provided by AASHTO T 90 and ASTM D 4318-93. The **shrinkage limit**, denoted SL or w_S, is the water content below which the soil no longer reduces in volume when the water content is reduced. Although Atterberg limits are water contents and are properly decimals or percentages, they are usually expressed as an integer percentage without a percent sign. Thus, a liquid limit of 40% is usually reported as LL = 40.

The **plasticity index**, denoted PI or I_P, is the difference of the liquid limit and the plastic limit:

$$\mathrm{PI} = \mathrm{LL} - \mathrm{PL} \tag{15.3}$$

The **liquidity index**, denoted LI or I_L, is a measure of the natural water content (w) relative to the plastic limit and the liquid limit:

$$\mathrm{LI} = I_L = \frac{w - \mathrm{PL}}{\mathrm{LL} - \mathrm{PL}} \tag{15.4}$$

The Unified Soil Classification System (USCS)

The Unified Soil Classification System is based on the airfield classification system developed by A. Casagrande during World War II. With some modification it was jointly adopted by several U.S. government agencies in 1952. Additional refinements were made and it is currently standardized as ASTM D 2487-93. It is used in the U.S. and much of the world for geotechnical work other than roads and highways.

In the unified system soils are designated by a two-letter symbol: the first identifies the primary component of the soil, and the second describes its grain size or plasticity characteristics. For example, a poorly graded sand is designated SP and a low plasticity clay is CL. Five first-letter symbols are used:

G for gravel
S for sand
M for silt
C for clay
O for organic soil

Clean sands and gravels (having less than 5% passing the No. 200 sieve) are given a second letter P if poorly graded or W if well graded. Sands and gravels with more than 12% by weight passing the No. 200 sieve are given a second letter M if the fines are silty or C if fines are clayey. Sands and gravels having between 5 and 12% are given dual classifications such as SP-SM. Silts, clays, and organic soils are given the second letter H or L to designate high or low plasticity. The specific rules for classification are summarized as follows and described in detail in ASTM D 2487.

Organic soils are distinguished by a dark-brown to black color, an organic odor, and visible fibrous matter.

For soils that are not notably organic the first step in classification is to consider the percentage passing the No. 200 sieve. If less than 50% of the soil passes the No. 200 sieve, the soil is *coarse grained*, and the first letter will be G or S; if more than 50% passes the No. 200 sieve, the soil is *fine grained* and the first letter will be M or C.

For coarse-grained soils, the proportions of sand and gravel in the **coarse fraction** (not the total sample) determine the first letter of the classification symbol. The coarse fraction is that portion of the total sample retained on a No. 200 sieve. If more than half of the coarse fraction is gravel (retained on the No. 4 sieve), the soil is *gravel* and the first letter symbol is G. If more than half of the coarse fraction is sand, the soil is *sand* and the first letter symbol is S.

For sands and gravels the second letter of the classification is based on gradation for clean sands and gravels and plasticity of the fines for sands and gravels with fines. For clean sands (less than 5% passing the No. 200 sieve), the classification is well-graded sand (SW) if $C_u \geq 6$ *and* $1 \leq C_c \leq 3$. *Both* of these criteria must be met for the soil to be SW, otherwise the classification is poorly graded sand (SP). Clean gravels (less than 5% passing the No. 200 sieve) are classified as well-graded gravel (GW) if $C_u \geq 4$ *and* $1 \leq C_c \leq 3$. If both criteria are not met, the soil is poorly graded gravel (GP).

For sands and gravels where more than 12% of the total sample passes the No. 200 sieve, the soil is a clayey sand (SC), clayey gravel (GC), silty sand (SM), or silty gravel (GM). The second letter is assigned based on whether the fines classify as clay (C) or silt (M) as described for fine-grained soils below.

For sands and gravels having between 5 and 12% of the total sample passing the No. 200 sieve, both the gradation and plasticity characteristics must be evaluated and the soil is given a dual classification such as SP-SM, SP-SC, GW-GC, etc. The first symbol is always based on gradation, whereas the second is always based on plasticity.

For fine-grained soils and organic soils, classification in the unified system is based on Atterberg limits determined by the fraction passing the No. 40 sieve. The liquid limit and plasticity index are determined and plotted on the plasticity chart (Fig. 15.2). The vertical line at LL = 50 separates high-plasticity soils from low-plasticity soils. The *A-line* separates clay from silt. The equation of the A-line is $PI = 0.73(LL - 20)$. The U-line is not used in classification but is an upper boundary of expected results for natural soils. Values plotting above the U-line should be checked for errors.

Inorganic soils with liquid limits below 50 that plot above the A-line and have PI values greater than 7 are **lean clays** and are designated CL; those with liquid limits above 50 that plot above the A-line are **fat clays** and are designated CH. Inorganic soils with liquid limits below 50 that plot below the A-line are silt and are designated ML; those with liquid limits above 50 that plot below the A-line are elastic silts and are designated MH. The plasticity chart has a shaded area; soils that plot in this area (above the A-line with PI values between 4 and 7) are silty clay and are given the dual symbol CL-ML. If the soil

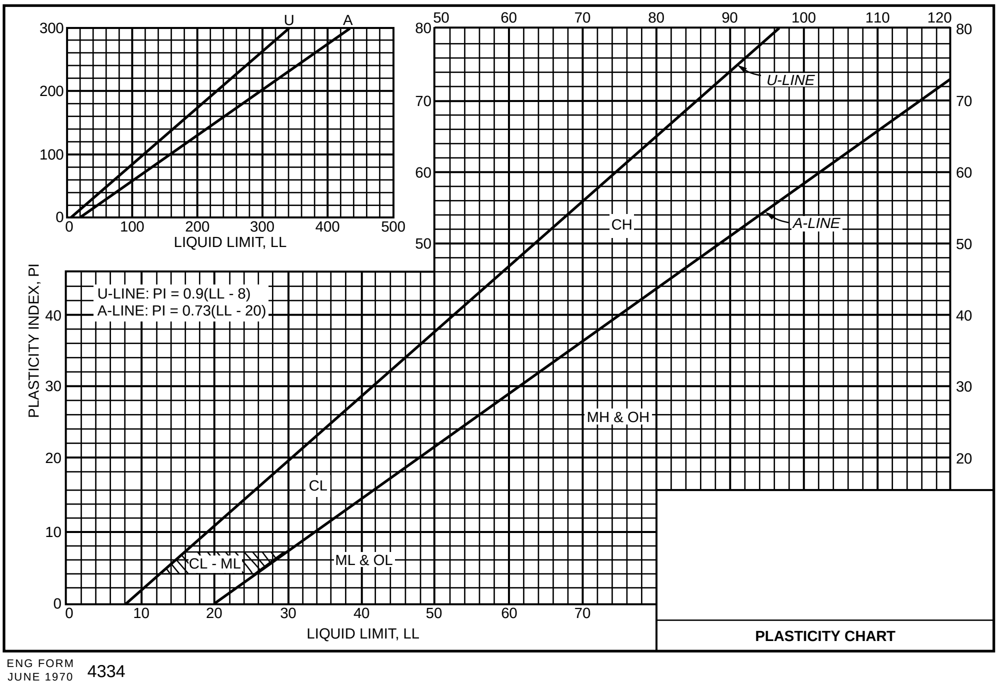

FIGURE 15.2 Plasticity chart for Unified Soil Classification System. (From U.S. Army, 1970.)

under consideration is the fines component of a dually classified sand or gravel, the soil is classified as SM-SC or GM-GC.

Soils with sufficient organic contents to influence properties that have liquid limits below 50 are classified as OL; those with liquid limits above 50 are classified as OH. Soils that are *predominantly* organic, with visible vegetable tissue, are termed **peat** and given the designation Pt.

The AASHTO Classification System

The AASHTO system classifies soils into seven primary groups, named A-1 through A-7, based on their relative expected quality for road embankments, subgrades, subbases, and bases. Some of the groups are in turn divided into subgroups, such as A-1-a and A-1-b. Furthermore, a *group index* may be calculated to quantify a soil's expected performance within a group.

To determine a soil's classification in the AASHTO system, one first determines the relative proportions of gravel, coarse sand, fine sand, and silt-clay. In the AASHTO system gravel is material smaller than 75 mm (3 in.) but retained on a No. 10 sieve; coarse sand is material passing a No. 10 sieve but retained on a No. 40 sieve; and fine sand is material passing a No. 40 sieve but retained on a No. 200 sieve. Material passing the No. 200 sieve is *silt-clay* and is classified based on Atterberg limits. It should be noted that the division between gravel and sand is made at a smaller size (No. 10 sieve) in the AASHTO system than in the unified system (No. 4 sieve). Secondly, if any fines are present, Atterberg limits are determined and the plasticity index is calculated.

A soil is a **granular material** if less than 35% of the soil by weight passes the No. 200 sieve. Granular materials are classified into groups A-1 through A-3. Soils having more than 35% passing the No. 200 sieve are silt-clay and fall in groups A-4 through A-7.

Having the proportions of the components and the plasticity data, one enters one of the two alternative AASHTO classification tables (Tables 15.2 and 15.3) and checks from left to right until a classification is found for which the soil meets the criteria. It should be noted that, in this scheme, group A-3 is checked before A-2. The AASHTO plasticity criteria are also illustrated in Fig. 15.3.

TABLE 15.2 Classification of Soils and Soil-Aggregate Mixtures by the AASHTO System

General Classification	Granular Materials (35% or Less Passing 0.075 mm)			Silt-Clay Materials (More than 35% Passing 0.075 mm)			
Group Classification	A-1	A-3[a]	A-2	A-4	A-5	A-6	A-7
Sieve analysis, percent passing:							
2.00 mm (No. 10)	—	—	—	—	—	—	—
0.425 mm (No. 40)	50 max.	51 min.	—	—	—	—	—
0.075 mm (No. 200)	25 max.	10 max.	35 max.	36 min.	36 min.	36 min.	36 min.
Characteristics of fraction passing 0.425 mm (No. 40)							
Liquid limit	—	—		40 max.	41 min.	40 max.	41 min.
Plasticity index	6 max.	N.P.	[b]	10 max.	10 max.	11 min.	11 min.
General rating as subgrade	Excellent to good			Fair to poor			

[a] The placing of A-3 before A-2 is necessary in the "left to right elimination process" and does not indicate superiority of A-3 over A-2.

[b] See Table 15.3 for values.

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Soils classified as A-1 are typically well-graded mixtures of gravel, coarse sand, and fine sand. Soils in subgroup A-1-a contain more gravel whereas those in A-1-b contain more sand. Soils in group A-3 are typically fine sands that may contain small amounts of nonplastic silt. Group A-2 contains a wide variety of "borderline" granular materials that do not meet the criteria for groups A-1 or A-3.

Soils in group A-4 are silty soils, whereas those in group A-5 are high-plasticity elastic silt. Soils in group A-6 are typically lean clays, and those in group A-7 are typically highly plastic clays.

Within groups containing fines, one may calculate a *group index* to further evaluate relative quality and supporting value of a material as subgrade. The group index is calculated according to the following empirical formula:

$$\text{Group index} = (F - 35)[0.2 + 0.005(\text{LL} - 40)] + 0.01(F - 15)(\text{PI} - 10) \tag{15.5}$$

In this equation F is the percentage of fines (passing the No. 200 sieve) expressed as a whole number. When calculating the group index for A-2-6 and A-2-7, only the PI term is used. The group index is rounded to the nearest whole number and, if negative, it is taken as zero. The expected performance is inversely related to group index. A value of zero indicates a good subgrade material and a value above 20 indicates a very poor material.

Example 15.1

Classify the soil shown by the solid curve in Fig. 15.1. Assume the soil is nonplastic. The following data are obtained:

Percent passing No. 4 sieve: 90%
Percent passing No. 10 sieve: 70%
Percent passing No. 40 sieve: 23%
Percent passing No. 200 sieve: 4%
D_{60} size: 1.50 mm
D_{30} size: 0.61 mm
D_{10} size: 0.18 mm

To classify the soil in the Unified Soil Classification System, the percent passing the No. 200 sieve is first checked. As only 4% passes the No. 200 sieve, the soil is coarse grained. The coarse fraction makes up 96% of the soil, with 10% being gravel (larger than the No. 4 sieve) and 86% being sand (the difference between the Nos. 4 and 200 sieves). As there is more sand than gravel, the soil is a sand and the first letter is S.

TABLE 15.3 Classification of Soils and Soil-Aggregate Mixtures

General Classification	Granular Materials (35% or Less Passing 0.075 mm)							Silt-Clay Materials (More than 35% Passing 0.075 mm)			
	A-1			A-2							A-7
Group Classification	A-1-a	A-1-b	A-3	A-2-4	A-2-5	A-2-6	A-2-7	A-4	A-5	A-6	A-7-5, A-7-6
Sieve analysis, percent passing:											
2.00 mm (No. 10)	50 max.	—	—	—	—	—	—	—	—	—	—
0.425 mm (No. 40)	30 max.	50 max.	51 min.	—	—	—	—	—	—	—	—
0.075 mm (No. 200)	15 max.	25 max.	10 max.	35 max.	35 max.	35 max.	35 max.	36 min.	36 min.	36 min.	36 min.
Characteristics of fraction passing 0.425 mm (No. 40)											
Liquid limit	—		—	40 max.	41 min.	40 max.	41 min.	40 max.	41 min.	40 max.	41 min.
Plasticity index	6 max.		N.P.	10 max.	10 max.	11 min.	11 min.	10 max.	10 max.	11 min.	11 min.[a]
Usual types of significant constituent materials	Stone fragments, gravel, and sand		Fine sand	Silty or clayey gravel and sand				Silty soils		Clayey soils	
General rating as subgrade	Excellent to good							Fair to poor			

[a] Plasticity index of A-7-5 subgroup is equal to or less than LL minus 30. Plasticity index of A-7-6 subgroup is greater than LL minus 30.

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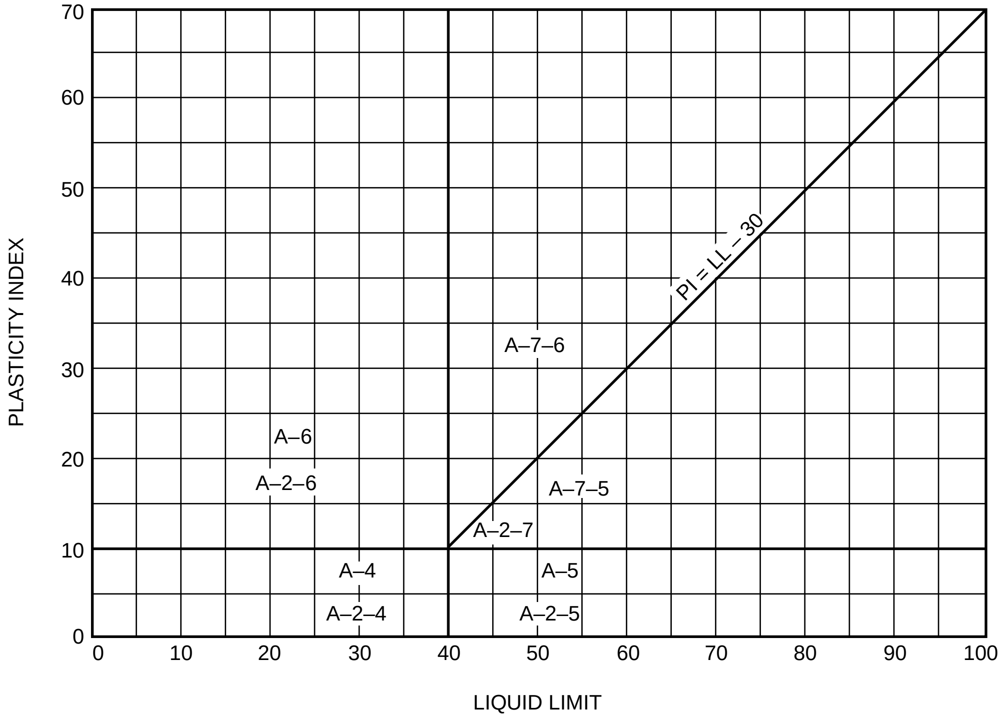

FIGURE 15.3 Plasticity chart for AASHTO Classification System. (From *Standard Specification for Transportation Materials and Methods of Sampling and Testing*. Copyright 1990 by the American Association of State Highway and Transportation Officials, Washington, D.C. Used by permission.)

As less than 5% passes the No. 200 sieve, the sand is clean (SP or SW). The coefficient of uniformity is $(1.5)/(0.18) = 8.33$. The coefficient of curvature is $0.61^2/(1.5)(0.18) = 1.38$. As the coefficient of uniformity is greater than 6.0 and the coefficient of curvature is between 1 and 3, the soil meets both of the criteria for SW and is so classified.

In the AASHTO system the soil is 30% gravel, 47% coarse sand, 19% fine sand, and 4% silt-clay. Proceeding from left to right on the classification chart, the soil cannot be classified as A-1-a because the soil has 70% passing the No. 10 and that classification permits a maximum of 50%. The soil meets the classification criteria for A-1-b and is so classified.

Example 15.2

Classify the soil represented by the dashed curve in Fig. 15.1. The liquid limit and plastic limit are found to be 30 and 20, respectively. The following data are obtained:

Percent passing No. 4 sieve: 100%
Percent passing No. 10 sieve: 91%
Percent passing No. 40 sieve: 60%
Percent passing No. 200 sieve: 20%
D_{60} size: 0.41 mm
D_{30} size: 0.12 mm
D_{10} size: <0.074 mm
Liquid limit: 30
Plastic limit: 20

First the soil is classified by the Unified Soil Classification System. As only 20% of the soil is smaller than a No. 200 sieve, the soil is coarse grained. All of the coarse fraction is smaller than the No. 4 sieve, so the soil is sand (first letter S).

As the percentage passing the No. 200 sieve is greater than 12%, the gradation characteristics are not considered, and the Atterberg limits are examined to determine whether the sand is a clayey sand or silty sand. The plasticity index is calculated as 30 – 20 = 10, and the coordinates LL = 30, PI = 10 are entered on the plasticity chart. As this plots in the CL region, the fines are clay and the soil is a clayey sand SC.

Next the soil is classified according to the AASHTO system. Following the classification table from left to right, group A-1 is eliminated due to too much material passing the No. 40 sieve, and group A-3 is eliminated due to too much material passing the No. 200 sieve. The soil passes the criteria for A-2-4 and is so classified. The group index is calculated as:

$$\text{Group index} = (F - 35)[0.2 + 0.005(\text{LL} - 40)] + 0.01(F - 15)(\text{PI} - 10)$$

where $F = 20$, the percent passing the No. 200 sieve. Thus:

$$\begin{aligned}\text{Group index} &= (20 - 35)[0.2 + 0.005(30 - 40)] + 0.01(20 - 15)(10 - 10)\\ &= (15)[0.2 + (-0.05)] + 0\\ &= 15(0.15)\\ &= 2.25\end{aligned}$$

This is rounded to the nearest whole number, 2, and the soil classification is reported as A-2-4(2).

Example 15.3

A fine-grained soil has the following properties:

Percent passing No. 200 sieve:	65%
Liquid limit:	60
Plastic limit:	28

First the soil is classified according to the unified system. As more than 50% of the sample passes the No. 200 sieve, the soil is fine grained and the plasticity chart is used. The plasticity index is 60 – 28 = 32. The coordinates (60, 32) plot in the CH region, so the soil is a high-plasticity clay (CH).

To classify the soil in the AASHTO system, one notes that more than 35% passes the No. 200 sieve, so the soil is silt-clay. Entering the coordinates (60, 32) on the AASHTO plasticity chart, the classification is A-7-6. The group index is calculated as follows:

$$\begin{aligned}\text{Group index} &= (F - 35)[0.2 + 0.005(\text{LL} - 40)] + 0.01(F - 15)(\text{PI} - 10)\\ &= (65 - 35)[0.2 + 0.005(60 - 40)] + 0.01(65 - 15)(32 - 10)\\ &= 30[0.2 + 0.1] + 0.01(50)(22)\\ &= 9.0 + 11.0\\ &= 20\end{aligned}$$

The complete classification is A-7-6(20), which indicates a poor quality soil for highway construction.

15.2 Weight, Mass, and Volume Relationships

In an engineering context, soil comprises three components: solid particles, water, and air. Many problems in soil mechanics and construction quality control involve making calculations and communicating information regarding the relative proportions of these components and the volumes they occupy, individually or in combination. Given some known values of mass, weight, volume, or density (or relationships among them), it is often necessary to calculate other values. This section defines the terms commonly used in geotechnical engineering to describe such relationships and provides worked examples of typical calculations.

The Phase Diagram

In a typical volume of soil the three components are arranged in a complex mixture. To visualize the relationships among the components when performing weight-volume or mass-volume calculations, drawing a phase diagram is recommended. Much like checking equilibrium with a free body diagram in statics, completing and balancing a phase diagram with numerical values of all quantities permits cross-checking the solution. Figure 15.4 shows an element of soil and the corresponding phase diagram. All solid particles in the element are taken as an equivalent single mass at the bottom of the diagram. Above the solids is the water—also represented as a single equivalent mass—and above that, the air. On the sides of the diagram are variable names; numerical values are entered here during calculations. On the left side are the total volume V, the volume of solids V_s, the volume of water V_w, and the volume of air V_a. The combined volume of water and air is the volume of voids V_v. On the right side are shown the total weight W, the weight of solids W_s, and the weight of water W_w. The weight of air is negligible. For problems involving masses, mass values are shown on the right instead of weights and given the equivalent designations M, M_s, and M_w.

Volume Relationships

Volume relationships include the void ratio, the porosity, and the degree of saturation. The **void ratio,** denoted e, is the ratio of the volume of voids to the volume of solids:

$$e = \frac{V_v}{V_s} \tag{15.6}$$

The **porosity**, denoted n, is the ratio of the volume of voids to the total volume:

$$n = \frac{V_v}{V} \tag{15.7}$$

The void ratio and porosity are related as follows:

$$n = \frac{e}{1 + e} \tag{15.8}$$

$$e = \frac{n}{1 - n} \tag{15.9}$$

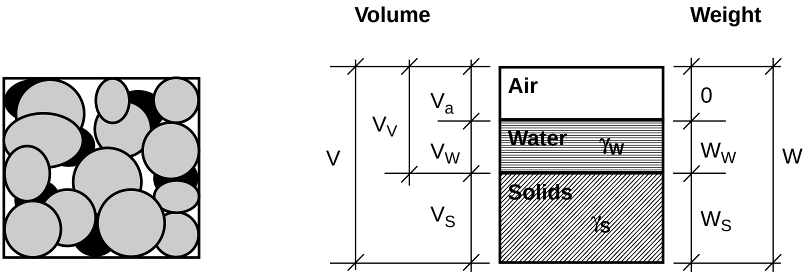

FIGURE 15.4 Soil element and phase diagram.

The **degree of saturation**, denoted S, is the ratio of the volume of water to the volume of voids. It is commonly expressed as a percentage:

$$S = \frac{V_w}{V_v} \times 100\% \tag{15.10}$$

If all the soil voids are filled with water and no undissolved air is present, the soil is said to be **saturated**.

Weight and Mass Relationships

The **water content** (or moisture content), denoted w, is the only relationship involving weights or masses. It is the ratio of the weight of water to the weight of solids or, equivalently, the ratio of the mass of water to the mass of solids:

$$w = \frac{W_w}{W_s} = \frac{M_w}{M_s} \tag{15.11}$$

Unit Weight

The ratio of the weight of a material to its volume is its **unit weight**, sometimes termed *specific weight* or *weight density*. The unit weight of water, γ_w, is 9.81 kN/m^3 in the SI system and 62.4 lb/ft^3 in the English system. The unit weight of solids, γ_s, varies with the mineralogy of the soil particles but is commonly in the range of 26.0 to 27.0 kN/m^3 or 165 to 172 lb/ft^3. The **total unit weight** of a soil (the solids-water-air system), denoted γ, is the ratio of the total weight to the total volume occupied:

$$\gamma = \frac{W}{V} = \frac{W_s + W_w}{V_s + V_w + V_a} \tag{15.12}$$

The **saturated unit weight**, denoted γ_{sat}, is the total unit weight that would be obtained if the air voids were filled with an equal volume of water ($S = 100\%$ and $V_w = V$). The **dry unit weight**, denoted γ_d, is often termed the **dry density** and has particular importance in field control of soil compaction. It is the ratio of the weight of solids to the total volume:

$$\gamma_d = \frac{W_s}{V} = \frac{W_s}{V_s + V_w + V_a} \tag{15.13}$$

Note that the dry unit weight matches the weight of a single component—the solids—with the *entire* volume of solids, water, and air. It does not represent the unit weight of any component or consistent set of components, but rather provides a measure of how much solid material by weight is in the total volume of a container, such as an earthmover or a compaction mold. The **buoyant unit weight** or **effective unit weight**, γ', is equal to the saturated unit weight minus the unit weight of water, γ_w:

$$\gamma' = \gamma_{\text{sat}} - \gamma_w \tag{15.14}$$

The buoyant unit weight is sometimes used to directly calculate vertical effective stresses below the water table instead of calculating total stresses and subtracting pore pressures.

Density

The term **density** is used herein to denote the mass-to-volume ratio of a material. However, some references, particularly older ones, use the term to describe unit weight. Density is denoted by ρ. Because $m = W/g$, the unit weight terms defined above can be converted to mass densities as follows:

$$\rho = \frac{\gamma}{g} = \frac{M}{V} \tag{15.15}$$

$$\rho_{\text{sat}} = \frac{\gamma_{\text{sat}}}{g} = \frac{M_{\text{sat}}}{V} \tag{15.16}$$

$$\rho_d = \frac{\gamma_d}{g} = \frac{M_s}{V} \tag{15.17}$$

$$\rho' = \rho_{\text{sat}} - \rho_w \tag{15.18}$$

In the SI system mass densities are commonly expressed in Mg/m^3, kg/m^3, or g/ml. The mass density of water can therefore be expressed $\rho_w = 1000$ kg/m^3 = 1 Mg/m^3 = 1 g/ml. The mass density of soil solids typically ranges from 2640 to 2750 kg/m^3. Where mass or mass density values (g, kg, or kg/m^3) are given or measured, they must be multiplied by g(9.81 m/s^2) to obtain weights or unit weights before performing stress calculations. In the English system mass density values are virtually never used in geotechnical engineering and all work is performed in terms of unit weights (lb/ft^3).

Specific Gravity

To facilitate working problems across different sets of units, it is convenient to express the unit weight and density of solids as a ratio to the unit weight and density of water. This ratio is termed the **specific gravity** and denoted G_s. For most soil minerals G_s is commonly in the range 2.64 to 2.75. Note that:

$$\gamma_s = G_s \gamma_w \tag{15.19}$$

and

$$\rho_s = G_s \rho_w \tag{15.20}$$

Conversion of Unit Weight and Density

A soil sample has a total unit weight of 125 lb/ft^3. It is desired to find its total unit weight in kN/m^3 and its density in kg/m^3. Although the problem can be worked using a chain of conversion factors, a simpler approach is to consider that the unit weight and density of the soil sample have a constant ratio to the unit weight and density of water. Placing the unit weight or density of water in any system of units in both the numerator and denominator of a fraction forms an equality. (It is assumed that the problem is on the planet Earth and a *mass* of 1000 kg *weighs* 9.81 kN!) Thus,

$$\gamma = (125 \text{ lb/ft}^3)\left(\frac{9.81 \text{ kN/m}^3}{62.4 \text{ lb/ft}^3}\right) = 19.7 \text{ kN/m}^3$$

$$\rho = (125 \text{ lb/ft}^3)\left(\frac{1000 \text{ kg/m}^3}{62.4 \text{ lb/ft}^3}\right) = 2003 \text{ kg/m}^3$$

Weight–Volume Problems Involving Defined Quantities

Weight-volume problems may be divided into two categories: those where there is a defined quantity of soil, and those where the quantity of soil is not defined and it is only desired to make conversions among relationships. The solution to problems of the first category is discussed first; discussion of the second category follows. Problems of the first category can be solved in four steps:

1. A blank phase diagram is sketched and known weights, volumes, and unit weights are entered on the diagram.
2. Known volumes are multiplied by their respective unit weights to obtain weights. Known weights are divided by unit weights to obtain volumes. Where values of some relationships are given,

additional weight and volume values are calculated using the definitions of Eqs. (15.6), (15.7), (15.10), and (15.11). To numerically balance the phase diagram it is recommended that all calculations be carried to at least four significant digits.

3. Multiplication and division horizontally across the diagram and addition and subtraction vertically along the sides is continued until all weights, volumes, and unit weights are determined and found to numerically balance.
4. All desired values and relationships can now be calculated from the completed and checked diagram.

Example 15.4 (English Units)

Assume that a **compaction mold** having a volume of 1/30 ft^3 was filled with moist soil. The total weight of the soil in the mold was found to be 4.10 lb. The soil was oven dried and its weight after drying was 3.53 lb. The specific gravity of solids was known to be 2.70. Water content, void ratio, porosity, degree of saturation, total unit weight, and dry unit weight must be determined. A phase diagram is shown in Fig. 15.5, with the known quantities in bold. The weight and volume of water are calculated as

$$W_w = W - W_s = 4.10 - 3.53 = 0.57 \text{ lb}$$

$$V_w = W_w / \gamma_w = 0.57/62.4 = 0.00913 \text{ ft}^3$$

The volume of solids is

$$V_s = W_s / G_s \gamma_w = 3.53/[(2.70)(62.4)] = 0.02095 \text{ ft}^3$$

The volume of air is

$$V_a = V - V_w - V_s = 0.03333 - 0.00913 - 0.02095 = 0.00325 \text{ ft}^3$$

The volume of voids is

$$V_v = V_w + V_a = 0.00913 + 0.00325 = 0.01238 \text{ ft}^3$$

With all quantities now known, all relationships can be determined.

$$w = W_w / W_s = 0.57/3.53 = 0.161 = 16.1\%$$

$$e = V_v / V_s = 0.01238/0.02095 = 0.590$$

$$n = V_v / V = 0.01238/(1/30) = 0.371$$

$$S = (V_w / V_v)\, 100\% = (0.00913/0.01238)\, 100\% = 73.7\%$$

$$\gamma = W / V = 4.10/(1/30) = 123.0 \text{ lb/ft}^3$$

$$\gamma_d = W_s / V = 3.53/(1/30) = 105.9 \text{ lb/ft}^3$$

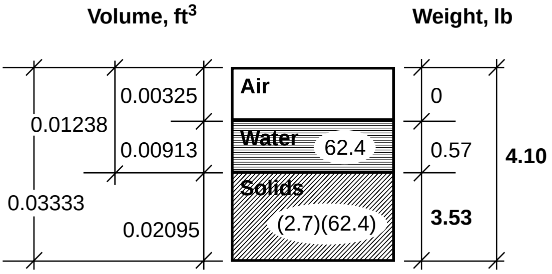

FIGURE 15.5 Phase diagram for Example 15.4.

If the same soil now becomes saturated by the addition of water at constant total volume, the saturated water content and saturated unit weight can be calculated as follows. The new volume of water is the entire void volume, 0.01238 ft^3. Multiplying this value by 62.4 lb/ft^3, the new weight of water is 0.77 lb. The water content at saturation is then

$$w = W_w / W_s = 0.77/3.53 = 0.218 = 21.8\%$$

The total weight is then

$$W = W_s + W_w = 3.53 + 0.77 = 4.30 \text{ lb}$$

The total and dry unit weights are then

$$\gamma = W/V = 4.30/(1/30) = 129.0 \text{ lb/ft}^3$$
$$\gamma_d = W_s/V = 3.53/(1/30) = 105.9 \text{ lb/ft}^3$$

Note that the dry unit weight does not change if water is added without changing total volume.

Example 15.5 (SI units)

A soil sample has a volume of 2.5 liters (2.5×10^{-3} m^3) and a total mass of 4.85 kg. A water content test indicates the water content is 28%. Assuming that the specific gravity of solids is 2.72, it is desired to determine the total density, total unit weight, dry density, dry unit weight, void ratio, porosity, and degree of saturation.

A phase diagram is shown in Fig. 15.6, with known values shown in bold.

$$M = M_s + M_w = 4.85 \text{ kg}$$

From the definition of the water content,

$$\begin{aligned} M_s + 0.28 M_s &= 4.85 \text{ kg} \\ 1.28 M_s &= 4.85 \text{ kg} \\ M_s &= 3.789 \text{ kg} \\ M_w = M - M_s &= 1.061 \text{ kg} \end{aligned}$$

With the mass side of the diagram complete, the masses are divided by density values to obtain volumes:

$$V_s = M_s/\rho_s = 3.789 \text{ kg}/(2720 \text{ kg/m}^3) = 0.00139 \text{ m}^3$$
$$V_w = M_w/\rho_w = 1.061 \text{ kg}/(1000 \text{ kg/m}^3) = 0.00106 \text{ m}^3$$

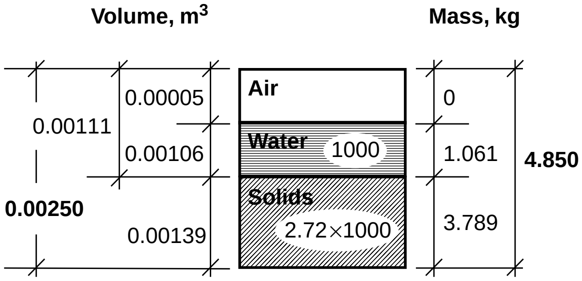

FIGURE 15.6 Phase diagram for Example 15.5.

Then

$$V_a = V - V_s - V_w = 0.00250 - 0.00319 - 0.00106 = 0.00005 \text{ m}^3$$

The total density is

$$\rho = M/V = 4.850 \text{ kg}/0.00250 \text{ m}^3 = 1940 \text{ kg/m}^3$$

The total unit weight is

$$\gamma = \rho g = (1940 \text{ kg/m}^3)(9.81 \text{ m/s}^2) = 19{,}031 \text{ N/m}^3 = 19.03 \text{ kN/m}^3$$

The dry density is

$$\rho_d = M/V = 3.789 \text{ kg}/0.00250 \text{ m}^3 = 1515 \text{ kg/m}^3$$

The dry unit weight is

$$\gamma_d = \rho_d g = (1515 \text{ kg/m}^3)(9.81 \text{ m/s}^2) = 14{,}862 \text{ N/m}^3 = 14.86 \text{ kN/m}^3$$

The void ratio is

$$e = V_v/V_s = 0.00111/0.00139 = 0.799$$

The porosity is

$$n = V_v/V = 0.00139/0.00250 = 0.556$$

The degree of saturation is

$$S = V_w/V_v = 0.00106/0.00111 = 0.955 = 95.5\%$$

Weight–Volume Problems Involving Only Relationships

If only relationships (e.g., void ratio or unit weight) are given, the quantity of soil is indefinite and only other relationships can be calculated. Nevertheless, it is convenient to solve such problems using a phase diagram and assuming one fixed weight or volume value. That quantity of solids, water, or soil is "brought to the paper" and used to calculate corresponding quantities of components. Although any one quantity can be assumed to have any value, the following assumptions simplify calculations:

- If a water content is known, assume $W_s = 1.0$ or 100.0 (lb or kN); then $W_w = w$ or $100w$.
- If a void ratio is known, assume the volume of solids $V_s = 1.0$ ft^3 or 1.0 m^3; then $V_v = e$.
- If a dry unit weight is known, assume the total volume $V = 1.0$ ft^3 or 1.0 m^3; then $W_s = \gamma_d$.
- If a total unit weight is known, assume the total volume $V = 1.0$ ft^3 or 1.0 m^3; then $W = \gamma$.

Example 15.6

Assume that a soil has a water content of 30%, a void ratio of 0.850, and a specific gravity of 2.75. It is desired to find the degree of saturation, porosity, total unit weight, and dry unit weight. Any single fixed quantity of soil or water may be assumed in order to start the calculations; it is assumed that the volume of solids $V_s = 1.000$ m^3. A phase diagram for the problem is shown in Fig. 15.7. The calculations then proceed as follows:

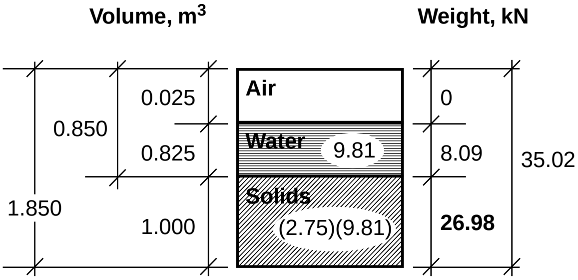

FIGURE 15.7 Phase diagram for Example 15.6.

$$
\begin{aligned}
V_v &= eV_s = (0.850)(1.000) = 0.850 \text{ m}^3\\
V &= V_s + V_v = 1.000 + 0.850 = 1.850 \text{ m}^3\\
W_s &= V_s G_s \gamma_w = (1.000)(2.75)(9.81) = 26.98 \text{ kN}\\
W_w &= wW_s = (0.30)(26.98) = 8.09 \text{ kN}\\
W &= W_s + W_w = 26.98 + 8.09 = 35.02 \text{ kN}\\
V_w &= W_w/\gamma_w = 8.09/9.81 = 0.825 \text{ m}^3\\
V_a &= V_v - V_w = 0.850 - 0.825 = 0.025
\end{aligned}
$$

At this point the weights and volumes of all components are known for the assumed 1.000 m³ of solids and all desired relationships can be calculated as follows:

$$
\begin{aligned}
S &= (V_w/V_v)100\% = (0.825/0.850)100\% = 97.1\%\\
n &= V_v/V = 0.850/1.850 = 0.459\\
\gamma &= 35.02/1.850 = 18.93 \text{ kN/m}^3\\
\gamma_d &= 26.98/1.850 = 14.58 \text{ kN/m}^3
\end{aligned}
$$

These relationships derive from the given relationships regardless of the quantity of soil considered.

Equations among Relationships

Solving weight-volume problems using a phase diagram provides a visual display of whether sufficient information is available to complete the problem, whether additional assumptions must be introduced, or whether the problem is overconstrained by an unwarranted assumption. For example, without completing a phase diagram, it may not be immediately apparent from the information given whether a soil is saturated. Nevertheless, a few additional equations are sometimes very useful for converting from one relationship to another.

Four distinct relationships combine to form the equation

$$Se = wG_s \tag{15.21}$$

For saturated soils $S = 100\%$, and this can be written

$$e = wG_s \tag{15.22}$$

The total unit weight can be obtained using the following:

$$\gamma = \frac{(G_s + Se)\gamma_w}{1 + e} = \frac{(1 + w)\gamma_w}{w/S + 1/G_s} \tag{15.23}$$

The dry unit weight can be obtained using the following:

$$\gamma_d = \frac{G_s \gamma_w}{1 + e} = \frac{G_s \gamma_w}{1 + (wG_s/S)} \tag{15.24}$$

Example 15.6 can be reworked using the equations as follows:

$$\begin{aligned}
Se &= wG_s \\
S &= wG_s/e = (0.30)(2.75)/0.850 = 0.971 = 97.1\% \\
n &= \frac{e}{1 + e} = \frac{0.850}{1 + 0.850} = 0.459 \\
\gamma &= \frac{(1 + w)\gamma_w}{w/S + 1/G_s} = \frac{(1.30)(9.81)}{(0.30/0.971) + (1/2.75)} = 18.96 \text{ kN/m}^3 \\
\gamma_d &= \frac{G_s \gamma_w}{1 + e} = \frac{(2.75)(9.81)}{1.85} = 14.58 \text{ kN/m}^3
\end{aligned}$$

The discrepancy in the fourth decimal place for the unit weight γ is due to rounding and the use of only three significant figures in the input values.

Defining Terms

Section 15.1

AASHTO classification system — A classification system developed by the American Association of State Highway and Transportation Officials that rates soils relative to their suitability for road embankments, subgrades, subbases, and basis.

Atterberg limits — Water contents at which soil changes engineering behavior; the most important ones in classification are the liquid limit and plastic limit.

Boulders — Rock particles larger than 9 to 12 inches or 200 to 300 mm.

Clay — Fine-grained soil that exhibits plasticity.

Coarse grained — Soils that are retained on a No. 200 sieve.

Coarse fraction — In the Unified Soil Classification System, that portion of a soil sample retained on a No. 200 sieve.

Cobbles — Rock particles smaller than a boulder but larger than 3 inches (75 mm).

Coefficient of curvature — A mathematical parameter, $D_{30}^2/(D_{60}D_{10})$, used as a measure of the smoothness of a gradation curve.

Coefficient of uniformity — A mathematical parameter, D_{60}/D_{10}, used as a measure of the slope of a gradation curve.

D_{10} size — The grain size, in mm, for which 10% by weight of a soil sample is finer.

Effective grain size — Another name for the D_{10} size.

Fat clay — Highly plastic clay; clay with a liquid limit greater than 50.

Fine fraction — In the unified soil classification system, that portion of a soil sample passing a No. 200 sieve.

Fine grained — Soil passing a No. 200 sieve.

Grain-size analysis — The determination of the relative proportions of soil particles of each size in a soil sample, performed by passing the sample over a nest of sieves.

Grain-size distribution curve — A plot of percent finer or coarser versus soil-grain size. Grain size is plotted on a logarithmic scale.

Granular material — In the AASHTO classification system, soil with less than 35% passing the No. 200 sieve.

Gravel — Soil or rock particles smaller than 3 inches but retained on a No. 4 sieve (Unified Soil Classification System) or on a No. 10 sieve (AASHTO system).

Lean clay — Clay with low plasticity; clay with a liquid limit less than 50.

Liquid limit — The water content above which soil behavior changes from a plastic solid to a viscous liquid.

Median grain size — The grain size for which one-half of a soil sample, by weight, is larger and half is smaller.

Nest of sieves — A stack of sieves of different sizes, having the largest opening on the top and progressing downward to successively smaller openings.

Peat — A highly organic soil, dark brown to black in color, with noticeable organic odor and visible vegetable matter.

Plastic limit — The water content above which the soil behavior changes from a brittle solid to a plastic solid.

Plasticity — The ability of a soil, when mixed with water, to deform at constant volume.

Plasticity index — The difference between the liquid and plastic limit.

Sand — Soil particles retained on the No. 200 sieve that pass the No. 4 sieve (Unified Soil Classification System) or the No. 10 sieve (AASHTO system).

Shrinkage limit — The water content at which further reduction in water content does not cause a further reduction in volume.

Sieve analysis — A grain-size analysis using a nest of sieves.

Silt — Fine-grained soil having a low plasticity index or not exhibiting plasticity.

Unified Soil Classification System — A descriptive classification system based on Casagrande's airfield system and now standardized by ASTM D 2487-93.

Section 15.2

Buoyant unit weight — The apparent unit weight of a submerged soil, obtained as the total unit weight minus the weight of water.

Compaction mold — A metal mold, typically 1/30 ft^3, used to determine the density of compacted soil.

Degree of saturation — The ratio of the volume of water to the volume of void space in a sample of soil.

Density — The mass per unit volume of a soil or one of its components.

Dry density — The ratio of the mass of solids to the total volume of a soil sample.

Dry unit weight — The ratio of the weight of solids to the total volume of a soil sample.

Effective unit weight — Another term for buoyant unit weight.

Porosity — The ratio of the volume of void spaces to the total volume of a soil sample.

Saturated — The condition in which all of the void spaces in a soil are filled with water and the volume of air is zero.

Saturated unit weight — The unit weight obtained if a soil sample is saturated by adding water at constant total volume.

Specific gravity — The ratio of the density of a material to the density of water; usually refers to the specific gravity of soil solids.

Total unit weight — The total combined weight of solids and water in a unit volume of soil.

Unit weight — The ratio of the weight of a material to its volume.

Void ratio — The ratio of the volume of void space in a soil sample to the volume of solid particles.

Water content — The ratio of the weight of water to the weight of solids of a soil sample.

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AASHTO Standard M 145-87. The classification of soils and soil-aggregate mixtures for highway construction purposes. *AASHTO Materials, Part I, Specifications*. American Association of State Highway and Transportation Officials, Washington, D.C.

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Further Information

The development and underlying philosophy of the Unified Soil Classification System was summarized by Casagrande [1948]. The complete and official rules for classifying soil according to the Unified Soil Classification System are given in ASTM Designation D 2487-92. The complete and official rules for classifying soil according to the AASHTO classification system is given in AASHTO Standard M 145-87. The procedures for performing index tests related to soil classification are specified in the other AASHTO and ASTM standards listed in the references.

A more detailed presentation of soil relationships and classification is given in most introductory geotechnical engineering textbooks. Two notable examples are *An Introduction to Geotechnical Engineering* by Holtz and Kovacs and *Principles of Geotechnical Engineering* by B. M. Das.