

# 48

## Design of Steel Structures

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## 48.1 Materials

### Stress–Strain Behavior of Structural Steel

Structural steel is a construction material that possesses attributes such as *strength*, *stiffness*, *toughness*, and *ductility* that are desirable in modern constructions. Strength is the ability of a material to resist stresses. It is measured in terms of the material's yield strength  $F_y$  and ultimate or tensile strength  $F_u$ . Steel used in ordinary constructions normally has values of  $F_y$  and  $F_u$  that range from 36 to 50 ksi (248 to 345 MPa) and from 58 to 70 ksi (400 to 483 MPa), respectively, although higher strength steels are becoming more common. Stiffness is the ability of a material to resist deformation. It is measured in terms of the modulus of elasticity  $E$  and the modulus of rigidity  $G$ . With reference to Fig. 48.1, in which several uniaxial engineering stress–strain curves obtained from coupon tests for various grades of steels are shown, it is seen that the modulus of elasticity  $E$  does not vary appreciably for the different steel grades. Therefore, a value of 29,000 ksi (200 GPa) is often used for design. Toughness is the ability of a material to absorb energy before failure. It is measured as the area under the material's stress–strain curve. As shown in Fig. 48.1, most (especially the lower grade) steels possess high toughness that make them suitable for both static and seismic applications. Ductility is the ability of a material to undergo large inelastic (or plastic) deformation before failure. It is measured in terms of percent elongation or percent reduction in the area of the specimen tested in uniaxial tension. For steel, percent elongation ranges from around 10 to 40 for a 2-in. (5-cm)-gauge-length specimen. Ductility generally decreases with increasing steel strength. Ductility is a very important attribute of steel. The ability of structural steel to deform considerably before failure by fracture allows an indeterminate structure to undergo stress redistribution. Ductility also enhances the energy absorption characteristic of the structure, which is extremely important in seismic design.

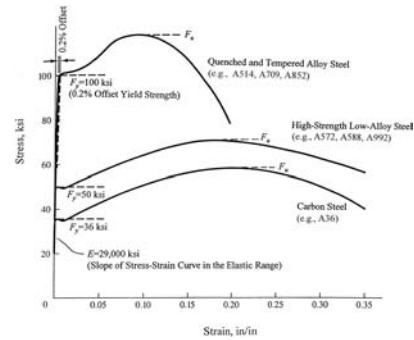


FIGURE 48.1 Uniaxial stress–strain behavior of steel.

### Types of Steel

Structural steels used for construction are designated by the American Society of Testing and Materials (ASTM) (see table on page 48-3).

A summary of the specified minimum yield stresses  $F_y$ , the specified minimum tensile strengths  $F_u$ , and general uses for some commonly used steels is given in Table 48.1.

### High-Performance Steel

High-performance steel (HPS) is a name given to a group of high-strength low-alloy (HSLA) steels that exhibit high strength, a higher yield-to-tensile-strength ratio, enhanced toughness, and improved weldability. Although research is still under way to develop and quantify the properties of a number of HPSSs, one high-performance steel that is currently in use, especially for bridge construction, is HPS 70W. HPS 70W is a derivative of ASTM A709 grade 70W steel (see Table 48.1). Compared to ASTM A709 grade 70W, HPS 70W has improved mechanical properties and is more resistant to postweld cracking, even without preheating before welding.

### Fireproofing of Steel

Although steel is an incombustible material, its strength ( $F_y$ ,  $F_u$ ) and stiffness ( $E$ ) reduce quite noticeably at temperatures normally reached in fires when other materials in a building burn. Exposed steel members

| ASTM<br>Designation <sup>a</sup> | Steel Type   |
|----------------------------------|--|
| A36/A36M                         | Carbon structural steel  |
| A131/A131M                       | Structural steel for ships   |
| A242/A242M                       | High-strength low-alloy structural steel   |
| A283/A283M                       | Low- and intermediate-tensile-strength carbon steel plates   |
| A328/A328M                       | Steel sheet piling   |
| A514/A514M                       | High-yield-strength, quenched and tempered alloy steel plate suitable for welding  |
| A529/A529M                       | High-strength carbon–manganese steel of structural quality   |
| A572/A572M                       | High-strength low-alloy columbium–vanadium steel   |
| A573/A573M                       | Structural carbon steel plates of improved toughness   |
| A588/A588M                       | High-strength low-alloy structural steel with 50-ksi (345-MPa) minimum yield point to 4 in. (100 mm) thick                                       |
| A633/A633M                       | Normalized high-strength low-alloy structural steel plates   |
| A656/A656M                       | Hot-rolled structural steel, high-strength low-alloy plate with improved formability   |
| A678/A678M                       | Quenched and tempered carbon and high-strength low-alloy structural steel plates   |
| A690/A690M                       | High-strength low-alloy steel H piles and sheet piling for use in marine environments  |
| A709/A709M                       | Carbon and high-strength low-alloy structural steel shapes, plates, and bars and quenched and tempered alloy structural steel plates for bridges |
| A710/A710M                       | Age-hardening low-carbon nickel–copper–chromium–molybdenum–columbium alloy structural steel plates   |
| A769/A769M                       | Carbon and high-strength electric resistance welded steel structural shapes  |
| A786/A786M                       | Rolled steel floor plates  |
| A808/A808M                       | High-strength low-alloy carbon–manganese–columbium–vanadium steel of structural quality with improved notch toughness                            |
| A827/A827M                       | Plates, carbon steel, for forging and similar applications   |
| A829/A829M                       | Plates, alloy steel, structural quality  |
| A830/A830M                       | Plates, carbon steel, structural quality, furnished to chemical composition requirements   |
| A852/A852M                       | Quenched and tempered low-alloy structural steel plate with 70-ksi (485-MPa) minimum yield strength to 4 in. (100 mm) thick                      |
| A857/A857M                       | Steel sheet piling, cold formed, light gauge   |
| A871/A871M                       | High-strength low alloy structural steel plate with atmospheric corrosion resistance   |
| A913/A913M                       | High-strength low-alloy steel shapes of structural quality, produced by quenching and self-tempering (QST) process                               |
| A945/A945M                       | High-strength low-alloy structural steel plate with low carbon and restricted sulfur for improved weldability, formability, and toughness        |
| A992/A992M                       | Steel for structural shapes (W sections) for use in building framing   |

<sup>a</sup> The letter M in the designations stands for metric.

that may be subjected to high temperature in a fire should be fireproofed to conform to the fire ratings set forth in city codes. Fire ratings are expressed in units of time (usually hours) beyond which the structural members under a standard ASTM specification (E119) fire test will fail under a specific set of criteria. Various approaches are available for fireproofing steel members. Steel members can be fireproofed by encasement in concrete if a minimum cover of 2 in. (5.1 mm) of concrete is provided. If the use of concrete is undesirable (because it adds weight to the structure), a lath and plaster (gypsum) ceiling placed underneath the structural members supporting the floor deck of an upper story can be used. In lieu of such a ceiling, spray-on materials such as mineral fibers, perlite, vermiculite, gypsum, etc. can also be used for fireproofing. Other means of fireproofing include placing steel members away from the source of heat, circulating liquid coolant inside box or tubular members, and the use of insulative paints. These special paints foam and expand when heated, thus forming a shield for the members [Rains, 1976]. For a more detailed discussion of structural steel design for fire protection, refer to the latest edition of AISI publication FS3, *Fire-Safe Structural Steel: A Design Guide*. Additional information on fire-resistant standards and fire protection can be found in the AISI booklets on *Fire Resistant Steel Frame Construction*, *Designing Fire Protection for Steel Columns*, and *Designing Fire Protection for Steel Trusses*, as well as in the *Uniform Building Code*.

**TABLE 48.1** Steel Types and General Uses

| ASTM Designation    | $F_y$ (ksi) <sup>a</sup> | $F_u$ (ksi) <sup>a</sup>    | Plate Thickness (in.) <sup>b</sup> | General Uses  |
|---------------------|--------------------------|-----------------------------|------------------------------------|---|
| A36/A36M            | 36                       | 58–80                       | To 8                               | Riveted, bolted, and welded buildings and bridges   |
| A529/A529M          | 42                       | 60–85                       | To 0.5                             | Similar to A36; the higher yield stress for A529 steel allows for savings in weight; A529 supersedes A441   |
|                     | 50                       | 70–100                      | To 1.5                             |   |
| A572/A572M          |                          |                             |                                    |   |
| Grade 42            | 42                       | 60                          | To 6                               | Grades 60 and 65 not suitable for welded bridges  |
| Grade 50            | 50                       | 65                          | To 4                               |   |
| Grade 60            | 60                       | 75                          | To 1.25                            |   |
| Grade 65            | 65                       | 80                          | To 1.25                            |   |
| A242/A242M          | 42                       | 63                          | 1.5–5                              | Riveted, bolted, and welded buildings and bridges; used when weight savings and enhanced atmospheric corrosion resistance are desired; specific instructions must be provided for welding |
|                     | 46                       | 67                          | 0.75–1.5                           |   |
|                     | 50                       | 70                          | 0.5–0.75                           |   |
| A588/A588M          | 42                       | 63                          | 5–8                                | Similar to A242; atmospheric corrosion resistance is about four times that of A36 steel   |
|                     | 46                       | 67                          | 4–5                                |   |
|                     | 50                       | 70                          | To 4                               |   |
| A709/A709M          |                          |                             |                                    | Primarily for use in bridges  |
| Grade 36            | 36                       | 58–80                       | To 4                               |   |
| Grade 50            | 50                       | 65                          | To 4                               |   |
| Grade 50W           | 50                       | 70                          | To 4                               |   |
| Grade 70W           | 70                       | 90–110                      | To 4                               |   |
| Grades 100 and 100W | 90                       | 100–130                     | 2.5–4                              |   |
| Grades 100 and 100W | 100                      | 110–130                     | To 2.5                             |   |
| A852/A852M          | 70                       | 90–110                      | To 4                               | Plates for welded and bolted construction where atmospheric corrosion resistance is desired   |
| A514/A514M          | 90–100                   | 100–130<br>110–130          | 2.5–6                              | Primarily for welded bridges; avoid use if ductility is important   |
| A913/A913M          | 50–65                    | 65 (max. $F_y/F_u = 0.85$ ) | To 4                               | Used for seismic applications   |
| A992/A992M          | 50–65                    | 65(max. $F_y/F_u = 0.85$ )  | To 4                               | Hot-rolled wide flange shapes for use in building frames  |

<sup>a</sup> 1 ksi = 6.895 MPa.<sup>b</sup> 1 in. = 25.4 mm.

## Corrosion Protection of Steel

Atmospheric corrosion occurs when steel is exposed to a continuous supply of water and oxygen. The rate of corrosion can be reduced if a barrier is used to keep water and oxygen from contact with the surface of bare steel. Painting is a practical and cost-effective way to protect steel from corrosion. The Steel Structures Painting Council issues specifications for the surface preparation and painting of steel structures for corrosion protection of steel. In lieu of painting, the use of other coating materials such as epoxies or other mineral and polymeric compounds can be considered. The use of corrosion resistance steels such as ASTM A242, A588, or A606 steel or galvanized or stainless steel is another alternative. Corrosion-resistant steels such as A588 retard corrosion by the formation of a layer of deep reddish brown to black patina (an oxidized metallic film) on the steel surface after a few wetting–drying cycles, which usually take place within 1 to 3 years. Galvanized steel has a zinc coating. In addition to acting as a protective cover, zinc is anodic to steel. The steel, being cathodic, is therefore protected from corrosion. Stainless steel is more resistant to rusting and staining than ordinary steel, primarily because of the presence of chromium as an alloying element.

## Structural Steel Shapes

Steel sections used for construction are available in a variety of shapes and sizes. In general, there are three procedures by which steel shapes can be formed: hot rolled, cold formed, and welded. All steel shapes must be manufactured to meet ASTM standards. Commonly used steel shapes include the wide flange (W) sections, the American Standard beam (S) sections, bearing pile (HP) sections, American Standard channel (C) sections, angle (L) sections, and tee (WT) sections, as well as bars, plates, pipes, and hollow structural sections (HSS). I sections that, by dimensions, can not be classified as W or S shapes are designated miscellaneous (M) sections, and C sections that, by dimensions, can not be classified as American Standard channels are designated miscellaneous channel (MC) sections.

Hot-rolled shapes are classified in accordance with their tensile property into five size groups by the American Society of Steel Construction (AISC). The groupings are given in the AISC manuals [AISC, 1989, 2001]. Groups 4 and 5 shapes and group 3 shapes with a flange thickness exceeding 1½ in. are generally used for application as *compression members*. When weldings are used, care must be exercised to minimize the possibility of cracking in regions at the vicinity of the welds by carefully reviewing the material specification and fabrication procedures of the pieces to be joined.

## Structural Fasteners

Steel sections can be fastened together by rivets, bolts, and welds. Although rivets were used quite extensively in the past, their use in modern steel construction has become almost obsolete. Bolts have essentially replaced rivets as the primary means to connect nonwelded structural components.

### Bolts

Four basic types of bolts are commonly in use. They are designated by ASTM as A307, A325, A490, and A449 [ASTM, 2001a, 2001b, 2001c, 2001d]. A307 bolts are called common, unfinished, machine, or rough bolts. They are made from low-carbon steel. Two grades (A and B) are available. They are available in diameters from 1/4 to 4 in. (6.4 to 102 mm) in 1/8-in. (3.2-mm) increments. They are used primarily for low-stress connections and for secondary members. A325 and A490 bolts are called high-strength bolts. A325 bolts are made from a heat-treated medium-carbon steel. They are available in two types: type 1, bolts made of medium-carbon steel; and type 3, bolts having atmospheric corrosion resistance and weathering characteristics comparable to those of A242 and A588 steel. A490 bolts are made from quenched and tempered alloy steel and thus have a higher strength than A325 bolts. Like A325 bolts, two types (types 1 and 3) are available. Both A325 and A490 bolts are available in diameters from 1/2 to 1½ in. (13 to 38 mm) in 1/8-in. (3.2-mm) increments. They are used for general construction purposes. A449 bolts are made from quenched and tempered steel. They are available in diameters from 1/4 to 3 in. (6.4 to 76 mm). Because A449 bolts are not produced to the same quality requirements or same heavy hex head and nut dimensions as A325 or A490 bolts, they are not to be used for slip critical connections. A449 bolts are used primarily when diameters over 1½ in. (38 mm) are needed. They are also used for anchor bolts and threaded rods.

High-strength bolts can be tightened to two conditions of tightness: snug tight and fully tight. Snug-tight conditions can be attained by a few impacts of an impact wrench or the full effort of a worker using an ordinary spud wrench. Snug-tight conditions must be clearly identified on the design drawing and are permitted in bearing-type connections where a slip is permitted or in tension or combined shear and tension applications where loosening or fatigue due to vibration or load fluctuations is not a design consideration. Bolts used in slip-critical conditions (i.e., conditions for which the integrity of the connected parts is dependent on the frictional force developed between the interfaces of the joint) and in conditions where the bolts are subjected to direct tension are required to be tightened to develop a pretension force equal to about 70% of the minimum tensile stress  $F_u$  of the material from which the bolts are made. This can be accomplished by using the turn-of-the-nut method, the calibrated wrench method, alternate design fasteners, or direct tension indicators [RCSC, 2000].

**TABLE 48.2** Electrode Designations

| Welding Processes                 | Electrode Designations | Remarks  |
|-----------------------------------|------------------------|--|
| Shielded metal arc welding (SMAW) | E60XX                  | The E denotes electrode; the first two digits indicate tensile strength in ksi <sup>a</sup> ; the two X's represent numbers indicating the electrode use   |
|                                   | E70XX                  |  |
|                                   | E80XX                  |  |
|                                   | E100XX                 |  |
|                                   | E110XX                 |  |
| Submerged arc welding (SAW)       | F6X-EXXX               | The F designates a granular flux material; the digit(s) following the F indicate the tensile strength in ksi (6 means 60 ksi, 10 means 100 ksi, etc.); the digit before the hyphen gives the Charpy V-notched impact strength; the E and the X's that follow represent numbers relating to the electrode use |
|                                   | F7X-EXXX               |  |
|                                   | F8X-EXXX               |  |
|                                   | F10X-EXXX              |  |
|                                   | F11X-EXXX              |  |
| Gas metal arc welding (GMAW)      | ER70S-X                | The digits following the letters ER represent the tensile strength of the electrode in ksi   |
|                                   | ER80S                  |  |
|                                   | ER100S                 |  |
|                                   | ER110S                 |  |
| Flux cored arc welding (FCAW)     | E6XT-X                 | The digit(s) following the letter E represent the tensile strength of the electrode in ksi (6 means 60 ksi, 10 means 100 ksi, etc.)  |
|                                   | E7XT-X                 |  |
|                                   | E8XT                   |  |
|                                   | E10XT                  |  |
|                                   | E11XT                  |  |

<sup>a</sup> 1 ksi = 6.895 MPa.

## Welds

Welding is a very effective means to connect two or more pieces of material together. The four most commonly used welding processes are *shielded metal arc welding* (SMAW), *submerged arc welding* (SAW), *gas metal arc welding* (GMAW), and *flux core arc welding* (FCAW) [AWS, 2000]. Welding can be done with or without filler materials, although most weldings used for construction utilize filler materials. The filler materials used in modern-day welding processes are electrodes. [Table 48.2](#) summarizes the electrode designations used for the aforementioned four most commonly used welding processes. In general, the strength of the electrode used should equal or exceed the strength of the steel being welded [AWS, 2000].

Finished welds should be inspected to ensure their quality. Inspection should be performed by qualified welding inspectors. A number of inspection methods are available for weld inspections. They include visual methods; the use of liquid penetrants, magnetic particles, and ultrasonic equipment; and radiographic methods. Discussion of these and other welding inspection techniques can be found in the *Welding Handbook* [AWS, 1987].

## Weldability of Steel

Weldability is the capacity of a material to be welded under a specific set of fabrication and design conditions and to perform as expected during its service life. Generally speaking, weldability is considered very good for low-carbon steel (carbon level, <0.15% by weight), good for mild steel (carbon level, 0.15 to 0.30%), fair for medium-carbon steel (carbon level, 0.30 to 0.50%), and questionable for high-carbon steel (carbon level, 0.50 to 1.00%). Because weldability normally decreases with increasing carbon content, special precautions such as preheating, controlling heat input, and postweld heat treating are normally required for steel with a carbon content reaching 0.30%. In addition to carbon content, the presence of other alloying elements will have an effect on weldability. In lieu of more accurate data, the table below can be used as a guide to determine the weldability of steel [Blodgett, undated].

| Element    | Range for Satisfactory Weldability (%) | Level Requiring Special Care (%) |
|------------|--|----------------------------------|
| Carbon     | 0.06–0.25                              | 0.35                             |
| Manganese  | 0.35–0.80                              | 1.40                             |
| Silicon    | 0.10 max.                              | 0.30                             |
| Sulfur     | 0.035 max.                             | 0.050                            |
| Phosphorus | 0.030 max.                             | 0.040                            |

A quantitative approach to determine the weldability of steel is to calculate its *carbon equivalent value*. One definition of the carbon equivalent value  $C_{eq}$  is

$$C_{eq} = \text{Carbon} + \frac{(\text{Manganese} + \text{Silicon})}{6} + \frac{(\text{Copper} + \text{Nickel})}{15} + \frac{(\text{Chromium} + \text{Molybdenum} + \text{Vanadium} + \text{Columbium})}{5} \quad (48.1)$$

A steel is considered weldable if  $C_{eq} \leq 0.50\%$  for steel in which the carbon content does not exceed 0.12%, and if  $C_{eq} \leq 0.45\%$  for steel in which the carbon content exceeds 0.12%.

The above equation indicates that the presence of alloying elements decreases the weldability of steel. An example of high-alloy steels is stainless steel. There are three types of stainless steel: austenitic, martensitic, and ferritic. Austenitic stainless steel is the most weldable, but care must be exercised to prevent thermal distortion, because heat dissipation is only about one third as fast as it is in plain carbon steel. Martensitic steel is also weldable, but prone to cracking because of its high ability to harden. Preheating and the maintaining of an interpass temperature are often needed, especially when the carbon content is above 0.10%. Ferritic steel is weldable, but decreased ductility and toughness in the weld area can present a problem. Preheating and postweld annealing may be required to minimize these undesirable effects.

## 48.2 Design Philosophy and Design Formats

### Design Philosophy

Structural design should be performed to satisfy the criteria for strength, serviceability, and economy. *Strength* pertains to the general integrity and safety of the structure under extreme load conditions. The structure is expected to withstand occasional overloads without severe distress and damage during its lifetime. *Serviceability* refers to the proper functioning of the structure as related to its appearance, maintainability, and durability under normal, or service load, conditions. Deflection, vibration, permanent deformation, cracking, and corrosion are some design considerations associated with serviceability. *Economy* is concerned with the overall material, construction, and labor costs required for the design, fabrication, erection, and maintenance processes of the structure.

### Design Formats

At present, steel design in the U.S. can be performed in accordance with one of the following three formats:

*Allowable stress design* (ASD), which has been in use for decades for the steel design of buildings and bridges. It continues to enjoy popularity among structural engineers engaged in steel building design. In allowable stress (or working stress) design, member stresses computed under service (or working) loads are compared to some predesignated stresses called allowable stresses. The allowable stresses are often expressed as a function of the yield stress ( $F_y$ ) or tensile stress ( $F_u$ ) of



the material divided by a factor of safety. The factor of safety is introduced to account for the effects of overload, understrength, and approximations used in structural analysis. The general format for an allowable stress design has the form

$$\frac{R_n}{F.S.} \geq \sum_{i=1}^m Q_{ni} \quad (48.2)$$

where  $R_n$  = the nominal resistance of the structural component expressed in unit of stress (i.e., the allowable stress)

$Q_{ni}$  = the service, or working, stresses computed from the applied working load of type  $i$

$F.S.$  = the factor of safety,  $i$  is the load type (dead, live, wind, etc.)

$m$  = the number of load type considered in the design

*Plastic design* (PD), which makes use of the fact that steel sections have reserved strength beyond the first yield condition. When a section is under flexure, yielding of the cross section occurs in a progressive manner, commencing with the fibers farthest away from the neutral axis and ending with the fibers nearest the neutral axis. This phenomenon of progressive yielding, referred to as *plastification*, means that the cross section does not fail at first yield. The additional moment that a cross section can carry in excess of the moment that corresponds to first yield varies, depending on the shape of the cross section. To quantify such reserved capacity, a quantity called the *shape factor*, defined as the ratio of the *plastic moment* (moment that causes the entire cross section to yield, resulting in the formation of a *plastic hinge*) to the *yield moment* (moment that causes yielding of the extreme fibers only) is used. The shape factor for hot-rolled I-shaped sections bent about the strong axes has a value of about 1.15. The value is about 1.50 when these sections are bent about their weak axes.

For an indeterminate structure, failure of the structure will not occur after the formation of a plastic hinge. After complete yielding of a cross section, force (or, more precisely, moment) redistribution will occur in which the unyielded portion of the structure continues to carry any additional loadings. Failure will occur only when enough cross sections have yielded, rendering the structure unstable, resulting in the formation of a *plastic collapse mechanism*.

In plastic design, the factor of safety is applied to the applied loads to obtain *factored loads*. A design is said to have satisfied the strength criterion if the load effects (i.e., forces, shears, and moments) computed using these factored loads do not exceed the nominal plastic strength of the structural component. Plastic design has the form

$$R_n \geq \gamma \sum_{i=1}^m Q_{ni} \quad (48.3)$$

where  $R_n$  = the nominal plastic strength of the member

$Q_{ni}$  = the nominal load effect from loads of type  $i$

$\gamma$  = the load factor

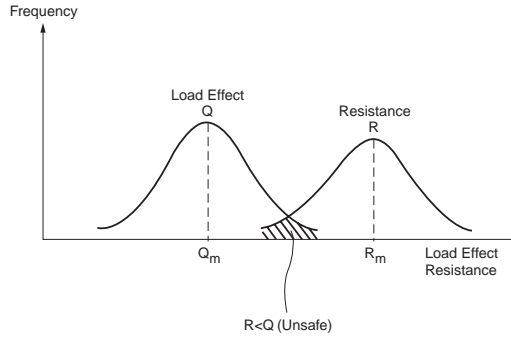
$i$  = the load type

$m$  = the number of load types.

In steel building design, the load factor is given by the AISC specification as 1.7 if  $Q_n$  consists of dead and live gravity loads only, and as 1.3 if  $Q_n$  consists of dead and live gravity loads acting in conjunction with wind or earthquake loads.

*Load and resistance factor design* (LRFD) which is a probability-based limit state design procedure. A *limit state* is defined as a condition in which a structure or structural component becomes unsafe (i.e., a violation of the strength limit state) or unsuitable for its intended function (i.e., a violation of the serviceability limit state). In a limit state design, the structure or structural component is





**FIGURE 48.2** Frequency distribution of load effect and resistance.

designed in accordance to its limits of usefulness, which may be strength related or serviceability related. In developing the LRFD method, both load effects and resistance were treated as random variables. Their variabilities and uncertainties were represented by frequency distribution curves. A design is considered satisfactory according to the strength criterion if the resistance exceeds the load effects by a comfortable margin. The concept of safety is represented schematically in Fig. 48.2. Theoretically, the structure will not fail unless the load effect  $Q$  exceeds the resistance  $R$ , as shown by the shaded portion in the figure. The smaller this shaded area, the less likely that the structure will fail. In actual design, a *resistance factor*  $\phi$  is applied to the nominal resistance of the structural component to account for any uncertainties associated with the determination of its strength, and a *load factor*  $\gamma$  is applied to each load type to account for the uncertainties and difficulties associated with determining its actual load magnitude. Different load factors are used for different load types to reflect the varying degree of uncertainties associated with the determination of load magnitudes. In general, a lower load factor is used for a load that is more predictable, and a higher load factor is used for a load that is less predictable. Mathematically, the LRFD format takes the form

$$\phi R_n \geq \sum_{i=1}^m \gamma_i Q_{ni} \quad (48.4)$$

where  $\phi R_n$  represents the *design* (or usable) *strength* and  $\sum_i Q_{ni}$  represents the required strength or load effect for a given load combination. Table 48.3 shows examples of load combinations [ASCE, 1998] to be used on the right-hand side of Eq. (48.4). For a safe design, all load combinations should be investigated and the design based on the worst-case scenario.

## 48.3 Tension Members

Tension members are designed to resist tensile forces. Examples of tension members are hangers, truss members, and bracing members that are in tension. Cross sections that are used most often for tension members are solid and hollow circular rods, bundled bars and cables, rectangular plates, single and double angles, channels, WT and W sections, and a variety of built-up shapes.

### Tension Member Design

Tension members are to be designed to preclude the following possible failure modes under normal load conditions: yielding in gross section, fracture in effective net section, block shear, shear rupture along a plane through the fasteners, bearing on fastener holes, and prying (for lap or hanger-type joints). In addition, the fasteners' strength must be adequate to prevent failure in the fasteners. Also, except for rods in tension, the slenderness of the tension member obtained by dividing the length of the member by its least radius of gyration should preferably not exceed 300.

**TABLE 48.3** Load Factors and Load Combinations

---

|   |
|---|
| 1.4D  |
| 1.2(D + F + T) + 1.6(L + H) + 0.5(L <sub>r</sub> or S or R) |
| 1.2D + 1.6(L <sub>r</sub> or S or R) + (0.5L or 0.8W)       |
| 1.2D + 1.3W + 0.5L + 0.5(L <sub>r</sub> or S or R)          |
| 1.2D + 1.0E + 0.5L + 0.2S                                   |
| 0.9D + (1.3W or 1.0E)                                       |

---

where D = dead load

E = earthquake load

F = load due to fluids with well-defined pressures and maximum heights

H = load due to the weight and lateral pressure of soil and water in soil

L = live load

L<sub>r</sub> = roof live load

R = rain load

S = snow load

T = self-straining force

W = wind load

---

*Note:* The load factor on L in the third, fourth, and fifth load combinations shown above shall equal 1.0 for garages, areas occupied as places of public assembly, and all areas where the live load is greater than 100 psf (4.79 kN/m<sup>2</sup>).

### Allowable Stress Design

The computed tensile stress  $f_t$  in a tension member shall not exceed the allowable stress for tension  $F_t$ , given by  $0.60F_y$  for yielding on the gross area and by  $0.50F_u$  for fracture on the effective net area. While the gross area is just the nominal cross-sectional area of the member, the *effective net area* is the smallest cross-sectional area accounting for the presence of fastener holes and the effect of *shear lag*. It is calculated using the equation

$$A_e = UA_n = U \left[ A_g - \sum_{i=1}^m d_{m_i} t_i + \sum_{j=1}^k \left( \frac{s^2}{4g} \right)_j t_j \right] \quad (48.5)$$

where  $U$  is a reduction coefficient given by [Munse and Chesson, 1963].

$$U = 1 - \frac{\bar{x}}{l} \leq 0.90 \quad (48.6)$$

in which  $l$  is the length of the connection and  $\bar{x}$  is the larger of the distance measured from the centroid of the cross section to the contact plane of the connected pieces or to the fastener lines. In the event that the cross section has two symmetrically located planes of connection,  $\bar{x}$  is measured from the centroid of the nearest one half the area (Fig. 48.3). This reduction coefficient is introduced to account for the shear lag effect that arises when some component elements of the cross section in a joint are not connected, rendering the connection less effective in transmitting the applied load. The terms in brackets in Eq. (48.5) constitute the so-called net section  $A_n$ . The various terms are defined as follows:

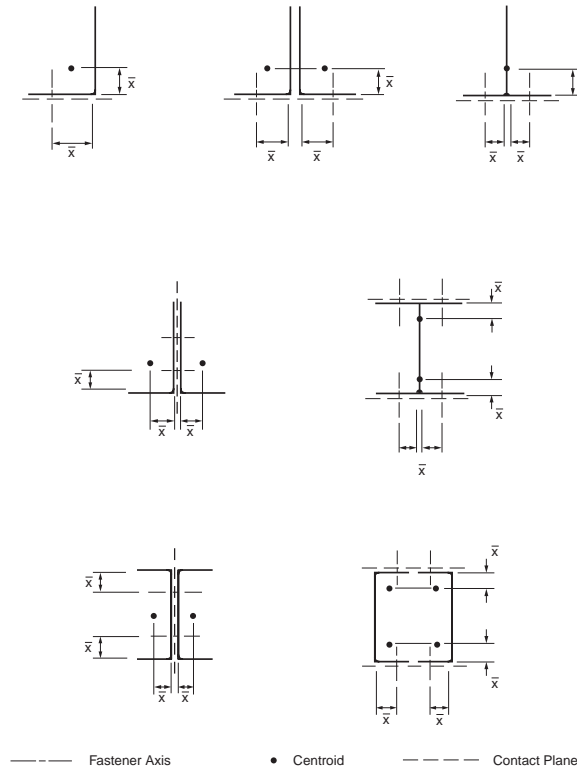
$A_g$  = gross cross-sectional area

$d_n$  = nominal diameter of the hole (bolt cutout), taken as the nominal bolt diameter plus 1/8 in. (3.2 mm)

$t$  = thickness of the component element

$s$  = longitudinal center-to-center spacing (pitch) of any two consecutive fasteners in a chain of staggered holes

$g$  = transverse center-to-center spacing (gauge) between two adjacent fastener gauge lines in a chain of staggered holes



**FIGURE 48.3** Definition of  $\bar{x}$  for selected cross-sections.

The second term inside the brackets of Eq. (48.5) accounts for loss of material due to bolt cutouts; the summation is carried for all bolt cutouts lying on the failure line. The last term inside the brackets of Eq. (48.5) indirectly accounts for the effect of the existence of a combined stress state (tensile and shear) along an inclined failure path associated with staggered holes. The summation is carried for all staggered paths along the failure line. This term vanishes if the holes are not staggered. Normally, it is necessary to investigate different failure paths that may occur in a connection; the critical failure path is the one giving the smallest value for  $A_e$ .

To prevent block shear failure and shear rupture, the allowable strengths for block shear and shear rupture are specified as follows:

Block shear:

$$R_{BS} = 0.30 A_v F_u + 0.50 A_t F_u \quad (48.7)$$

Shear rupture:

$$F_v = 0.30 F_u \quad (48.8)$$

where  $A_v$  = the net area in shear  
 $A_t$  = the net area in tension  
 $F_u$  = the specified minimum tensile strength.

The tension member should also be designed to possess adequate thickness, and the fasteners should be placed within a specific range of spacings and edge distances to prevent failure due to bearing or prying action (see Section 48.11).

## Load and Resistance Factor Design

According to the LRFD specification [AISC, 1999], tension members designed to resist a factored axial force of  $P_u$  calculated using the load combinations shown in Table 48.3 must satisfy the condition of

$$\phi_t P_n \geq P_u \quad (48.9)$$

The design strength  $\phi_t P_n$  is evaluated as follows:

Yielding in gross section:

$$\phi_t P_n = 0.90 [F_y A_g] \quad (48.10)$$

where 0.90 = the resistance factor for tension

$F_y$  = the specified minimum yield stress of the material

$A_g$  = the gross cross-sectional area of the member.

Fracture in effective net section:

$$\phi_t P_n = 0.75 [F_u A_e] \quad (48.11)$$

where 0.75 = the resistance factor for fracture in tension

$F_u$  = the specified minimum tensile strength

$A_e$  = the effective net area given in Eq. (48.5).

Block shear: if  $F_u A_{nt} \geq 0.6 F_u A_{nv}$  (i.e., shear yield–tension fracture),

$$\phi_t P_n = 0.75 [0.60 F_y A_{gv} + F_u A_{nt}] \leq 0.75 [0.6 F_u A_{nv} + F_u A_{nt}] \quad (48.12a)$$

and if  $F_u A_{nt} < 0.6 F_u A_{nv}$  (i.e., shear fracture–tension yield),

$$\phi_t P_n = 0.75 [0.60 F_u A_{nv} + F_y A_{gt}] \leq 0.75 [0.60 F_u A_{nv} + F_u A_{nt}] \quad (48.12b)$$

where 0.75 = the resistance factor for block shear

$F_y$  and  $F_u$  = the specified minimum yield stress and tensile strength, respectively

$A_{gv}$  = the gross shear area

$A_{nt}$  = the net tension area

$A_{nv}$  = the net shear area

$A_{gt}$  = the gross tension area.

### Example 48.1

Using LRFD, select a double-channel tension member, shown in Fig. 48.4a, to carry a dead load  $D$  of 40 kips and a live load  $L$  of 100 kips. The member is 15 feet long. Six 1-in.-diameter A325 bolts in standard-size holes are used to connect the member to a 3/8-in. gusset plate. Use A36 steel ( $F_y = 36$  ksi,  $F_u = 58$  ksi) for all the connected parts.

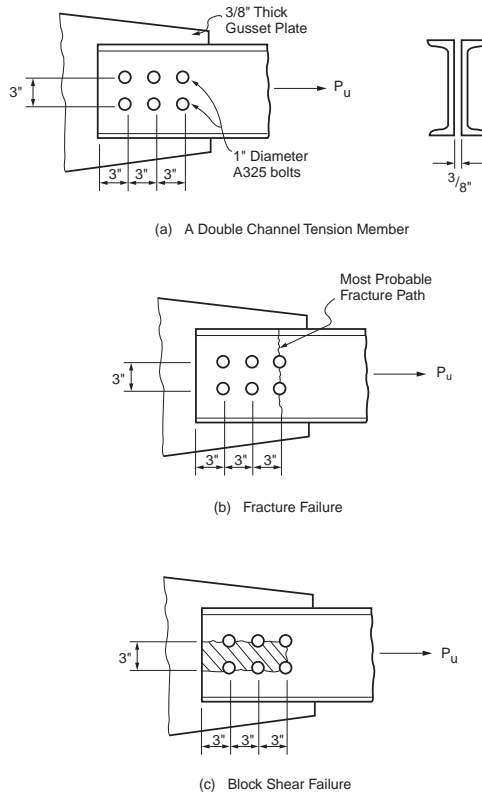
*Load combinations:*

From Table 48.3, the applicable load combinations are:

$$1.4D = 1.4(40) = 56 \text{ kips}$$

$$1.2D + 1.6L = 1.2(40) + 1.6(100) = 208 \text{ kips}$$

The design of the tension member is to be based on the larger of the two, i.e., 208 kips, and so *each* channel is expected to carry 104 kips.



**FIGURE 48.4** Design of a double-channel tension member (1 in. = 25.4 mm).

*Yielding in gross section:*

Using Eqs. (48.9) and (48.10), the gross area required to prevent cross section yielding is

$$0.90[F_y A_g] \geq P_u$$

$$0.90[(36)(A_g)] \geq 104 \text{ kips}$$

$$(A_g)_{reqd} \geq 3.21 \text{ in}^2$$

From the section properties table contained in the AISC-LRFD manual, one can select the following trial sections: C8x11.5 ( $A_g = 3.38 \text{ in}^2$ ), C9x13.4 ( $A_g = 3.94 \text{ in}^2$ ), or C8x13.75 ( $A_g = 4.04 \text{ in}^2$ ).

*Check for the limit state of fracture on the effective net area:*

The above sections are checked for the limiting state of fracture in the following table:

| Section  | $A_g$ (in. <sup>2</sup> ) | $t_{gw}$ (in.) | $\bar{x}$ (in.) | $U^a$ | $A_e^b$ (in. <sup>2</sup> ) | $\phi_t P_n$ (kips) |
|----------|---------------------------|----------------|-----------------|-------|-----------------------------|---------------------|
| C8x11.5  | 3.38                      | 0.220          | 0.571           | 0.90  | 2.6                         | 113.1               |
| C9x13.4  | 3.94                      | 0.233          | 0.601           | 0.90  | 3.07                        | 133.5               |
| C8x13.75 | 4.04                      | 0.303          | 0.553           | 0.90  | 3.02                        | 131.4               |

<sup>a</sup>Eq. (48.6).

<sup>b</sup>Eq. (48.5), [Fig. 48.4b](#).

From the last column of the above table, it can be seen that fracture is not a problem for any of the trial section.

*Check for the limit state of block shear:*

Figure 48.4c shows a possible block shear failure mode. To avoid block shear failure, the required strength of  $P_u = 104$  kips should not exceed the design strength,  $\phi_t P_n$ , calculated using Eq. (48.12a) or (48.12b), whichever is applicable.

For the C8x11.5 section:

$$A_{gv} = 2(9)(0.220) = 3.96 \text{ in}^2$$

$$A_{nv} = A_{gv} - 5(1 + 1/8)(0.220) = 2.72 \text{ in}^2$$

$$A_{gt} = (3)(0.220) = 0.66 \text{ in}^2$$

$$A_{nt} = A_{gt} - 1(1 + 1/8)(0.220) = 0.41 \text{ in}^2$$

Substituting the above into Eq. (48.12b), since  $[F_u A_{nt} = 23.8 \text{ kips}]$  is smaller than  $[0.6 F_u A_{nv} = 94.7 \text{ kips}]$ , we obtain  $\phi_t P_n = 88.8 \text{ kips}$ , which is less than  $P_u = 104 \text{ kips}$ . The C8x11.5 section is therefore not adequate. Significant increase in block shear strength is not expected from the C9x13.4 section because its web thickness  $t_w$  is just slightly over that of the C8x11.5 section. As a result, we shall check the adequacy of the C8x13.75 section instead.

For the C8x13.75 section:

$$A_{gv} = 2(9)(0.303) = 5.45 \text{ in}^2$$

$$A_{nv} = A_{gv} - 5(1 + 1/8)(0.303) = 3.75 \text{ in}^2$$

$$A_{gt} = (3)(0.303) = 0.91 \text{ in}^2$$

$$A_{nt} = A_{gt} - 1(1 + 1/8)(0.303) = 0.57 \text{ in}^2$$

Substituting the above into Eq. (48.12b), since  $[F_u A_{nt} = 33.1 \text{ kips}]$  is smaller than  $[0.6 F_u A_{nv} = 130.5 \text{ kips}]$ , we obtain  $\phi_t P_n = 122 \text{ kips}$ , which exceeds the required strength  $P_u$  of 104 kips. Therefore, block shear will not be a problem for the C8x13.75 section.

*Check for the limiting slenderness ratio:*

Using the parallel axis theorem, the least radius of gyration of the double-channel cross section is calculated to be 0.96 in. Therefore,  $L/r = (15 \text{ ft})(12 \text{ in./ft})/0.96 \text{ in.} = 187.5$ , which is less than the recommended maximum value of 300.

*Check for the adequacy of the connection:*

An example of the calculations is shown in Section 48.11.

*Longitudinal spacing of connectors:*

According to Section J3.5 of the LRFD specification, the maximum spacing of connectors in built-up tension members shall not exceed:

- 24 times the thickness of the thinner plate or 12 in. (305 mm) for painted members or unpainted members not subject to corrosion
- 14 times the thickness of the thinner plate or 7 in. (180 mm) for unpainted members of weathering steel subject to atmospheric corrosion

Assuming the first condition applies, a spacing of 6 inches is to be used.

Use 2C8x13.75 connected intermittently at 6-in. intervals.

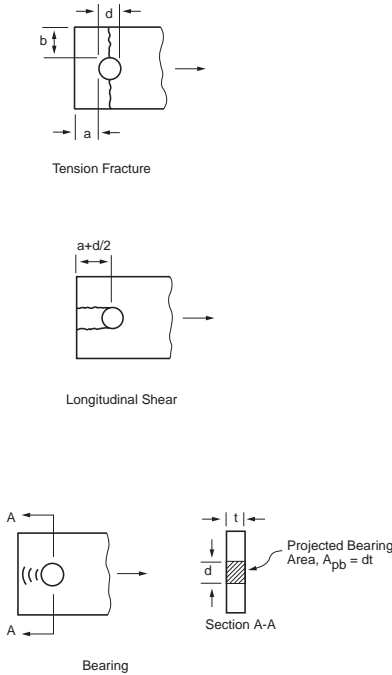


FIGURE 48.5 Failure modes of pin-connected members.

## Pin-Connected Members

Pin-connected members shall be designed to preclude the following failure modes: (1) tension yielding in the gross section, (2) tension fracture on the effective net area, (3) longitudinal shear on the effective area, and (4) bearing on the projected pin area (Fig. 48.5).

### Allowable Stress Design

The allowable stresses for tension yield, tension fracture, and shear rupture are  $0.60F_y$ ,  $0.45F_u$ , and  $0.30F_u$ , respectively. The allowable stresses for bearing are given in Section 48.11.

### Load and Resistance Factor Design

The design tensile strength  $\phi_t P_n$  for pin-connected members is given as follows:

*Tension on gross area:* see Eq. (48.10).

*Tension on effective net area:*

$$\phi_t P_n = 0.75 \left[ 2 t b_{eff} F_u \right] \quad (48.13)$$

*Shear on effective area:*

$$\phi_{sf} P_n = 0.75 \left[ 0.6 A_{sf} F_u \right] \quad (48.14)$$

Bearing on projected pin area: see Section 48.11.

The terms in Fig. 48.5 and the above equations are defined as follows:

$a$  = shortest distance from edge of the pin hole to the edge of the member measured in the direction of the force

$A_{pb}$  = projected bearing area =  $dt$



$$A_{sf} = 2t(a + d/2)$$

$b_{eff} = 2t + 0.63$  in. (or  $2t + 16$  mm), but not more than the actual distance from the edge of the hole to the edge of the part measured in the direction normal to the applied force

$d$  = pin diameter

$t$  = plate thickness

## Threaded Rods

### Allowable Stress Design

Threaded rods under tension are treated as bolts subject to tension in allowable stress design. These allowable stresses are given in the Section 48.11.

### Load and Resistance Factor Design

Threaded rods designed as tension members shall have a gross area  $A_b$  given by

$$A_b \geq \frac{P_u}{\phi 0.75 F_u} \quad (48.15)$$

where  $A_b$  = the gross area of the rod computed using a diameter measured to the outer extremity of the thread

$P_u$  = the factored tensile load

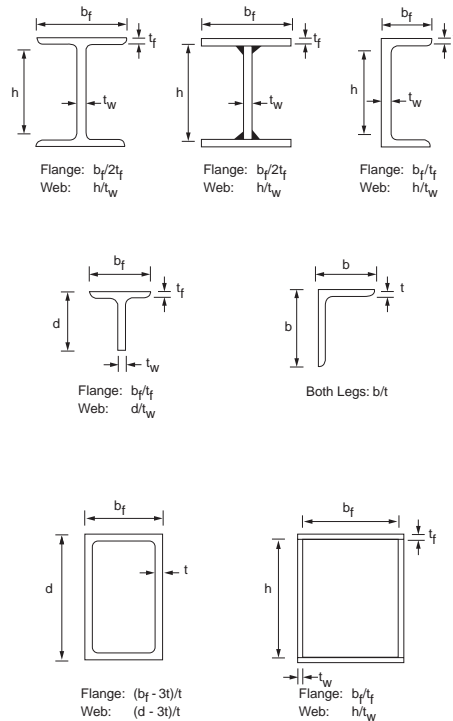
$\phi$  = the resistance factor given as 0.75

$F_u$  = the specified minimum tensile strength.

## 48.4 Compression Members

Members under compression can fail by yielding, inelastic buckling, or elastic buckling, depending on the slenderness ratio of the members. Members with low slenderness ratios tend to fail by yielding, while members with high slenderness ratios tend to fail by elastic buckling. Most compression members used in construction have intermediate slenderness ratios, so the predominant mode of failure is inelastic buckling. Overall member buckling can occur in one of three different modes: flexural, torsional, and flexural-torsional. Flexural buckling occurs in members with doubly symmetric or doubly antisymmetric cross sections (e.g., I or Z sections) and in members with singly symmetric sections (e.g., channel, tee, equal-legged angle, and double angle sections) when such sections are buckled about an axis that is *perpendicular* to the axis of symmetry. Torsional buckling occurs in members with doubly symmetric sections such as cruciform or built-up shapes with very thin walls. Flexural-torsional buckling occurs in members with singly symmetric cross sections (e.g., channel, tee, equal-legged angle, and double-angle sections) when such sections are buckled about the axis of symmetry and in members with unsymmetric cross sections (e.g., unequal-legged L). Normally, torsional buckling of symmetric shapes is not particularly important in the design of hot-rolled compression members. Either it does not govern or its buckling strength does not differ significantly from the corresponding weak-axis flexural buckling strengths. However, torsional buckling may become important for open sections with relatively thin component plates. It should be noted that for a given cross-sectional area, a closed section is much stiffer torsionally than an open section. Therefore, if torsional deformation is of concern, a closed section should be used. Regardless of the mode of buckling, the governing effective slenderness ratio ( $Kl/r$ ) of the compression member preferably should not exceed 200.

In addition to the slenderness ratio and cross-sectional shape, the behavior of compression members is affected by the relative thickness of the component elements that constitute the cross section. The relative thickness of a component element is quantified by the width-thickness ratio ( $b/t$ ) of the element. The width-thickness ratios of some selected steel shapes are shown in [Fig. 48.6](#). If the width-thickness ratio falls within a limiting value (denoted by the LRFD Specification [AISC, 1999] as  $\lambda_r$ ) as shown in [Table 48.4](#), the section will not experience local buckling prior to overall buckling of the member.



**FIGURE 48.6** Definition of width–thickness ratio of selected cross sections.

**TABLE 48.4** Limiting Width–Thickness Ratios for Compression Elements under Pure Compression

| Component Element   | Width–Thickness      |                             |
|---|----------------------|-----------------------------|
|   | Ratio                | Limiting Value, $\lambda_r$ |
| Flanges of I-shaped sections; plates projecting from compression elements; outstanding legs of pairs of angles in continuous contact; flanges of channels               | $b/t$                | $0.56\sqrt{E/F_y}$          |
| Flanges of square and rectangular box and hollow structural sections of uniform thickness; flange cover plates and diaphragm plates between lines of fasteners or welds | $b/t$                | $1.40\sqrt{E/F_y}$          |
| Unsupported width of cover plates perforated with a succession of access holes  | $b/t$                | $1.86\sqrt{E/F_y}$          |
| Legs of single-angle struts; legs of double-angle struts with separators; unstiffened elements (i.e., elements supported along one edge)                                | $b/t$                | $0.45\sqrt{E/F_y}$          |
| Flanges projecting from built-up members  | $b/t$                | $0.64\sqrt{E/(F_y/k_c)}$    |
| Stems of tees   | $d/t$                | $0.75\sqrt{E/F_y}$          |
| All other uniformly compressed stiffened elements (i.e., elements supported along two edges)  | $b/t$                | $1.49\sqrt{E/F_y}$          |
| Circular hollow sections  | $h/t_w^a$<br>$D/t^b$ | $0.11E/F_y$                 |

<sup>a</sup>  $h$  = web depth,  $t_w$  = web thickness.

<sup>b</sup>  $D$  = outside diameter,  $t$  = wall thickness.

<sup>c</sup>  $E$  = modulus of elasticity,  $F_y$  = specified minimum yield stress,  $k_c = 4/\sqrt{(h/t_w)}$ ;  $0.35 \leq k_c \leq 0.763$  for I-shaped sections, and  $k_c = 0.763$  for other sections.

However, if the width–thickness ratio exceeds this limiting width–thickness value, consideration of local buckling in the design of the compression member is required.

To facilitate the design of compression members, column tables for W, tee, double-angle, square and rectangular tubular, and circular pipe sections are available in the AISC manuals for both allowable stress design [AISC, 1989] and load and resistance factor design [AISC, 2001].

## Compression Member Design

### Allowable Stress Design

The computed compressive stress  $f_a$  in a compression member shall not exceed its allowable value given by

$$F_a = \begin{cases} \left[ 1 - \frac{(Kl/r)^2}{2C_c^2} \right] F_y, & \text{if } Kl/r \leq C_c \\ \frac{5}{3} + \frac{3(Kl/r)}{8C_c} - \frac{(Kl/r)^3}{8C_c^3}, & \\ \frac{12\pi^2 E}{23 (Kl/r)^2}, & \text{if } Kl/r > C_c \end{cases} \quad (48.16)$$

where  $Kl/r$  = the slenderness ratio

$K$  = the effective length factor of the compression member in the plane of buckling

$l$  = the unbraced member length in the plane of buckling

$r$  = the radius of gyration of the cross section about the axis of buckling

$E$  = the modulus of elasticity

$C_c = \sqrt{(2\pi^2 E/F_y)}$  is the slenderness ratio that demarcates inelastic from elastic member buckling.  $Kl/r$  should be evaluated for both buckling axes, and the larger value should be used in Eq. (48.16) to compute  $F_a$ .

The first of Eq. (48.16) is the allowable stress for inelastic buckling; the second is the allowable stress for elastic buckling. In ASD, no distinction is made between flexural, torsional, and flexural–torsional buckling.

### Load and Resistance Design

Compression members are to be designed so that the design compressive strength  $\phi_c P_n$  will exceed the required compressive strength  $P_u$ .  $\phi_c P_n$  is to be calculated as follows for the different types of overall buckling modes:

*Flexural buckling* (with a width–thickness ratio of  $\leq \lambda_c$ ):

$$\phi_c P_n = \begin{cases} 0.85 \left[ A_g \left( 0.658^{\lambda_c^2} \right) F_y \right], & \text{if } \lambda_c \leq 1.5 \\ 0.85 \left[ A_g \left( \frac{0.877}{\lambda_c^2} \right) F_y \right], & \text{if } \lambda_c > 1.5 \end{cases} \quad (48.17)$$

where  $\lambda_c = (KL/r\pi)\sqrt{(F_y/E)}$  is the slenderness parameter

$A_g$  = the gross cross-sectional area

$F_y$  = the specified minimum yield stress

$E$  = the modulus of elasticity

$K$  = the effective length factor

$l$  = the unbraced member length in the plane of buckling  
 $r$  = the radius of gyration of the cross section about the axis of buckling

The first of Eq. (48.17) is the design strength for inelastic buckling; the second is the design strength for elastic buckling. The slenderness parameter  $\lambda_c = 1.5$  demarcates inelastic from elastic behavior.

*Torsional buckling* (with a width–thickness ratio of  $\leq \lambda_r$ ):

$\phi_c P_n$  is to be calculated from Eq. (48.17), but with  $\lambda_c$  replaced by  $\lambda_e$  given by

$$\lambda_e = \sqrt{\frac{F_y}{F_e}} \quad (48.18)$$

where

$$F_e = \left[ \frac{\pi^2 E C_w}{(K_z L)^2} + GJ \right] \frac{1}{I_x + I_y} \quad (48.19)$$

in which  $C_w$  = the warping constant

$G$  = the shear modulus, which equals 11,200 ksi (77,200 MPa)

$I_x$  and  $I_y$  = the moments of inertia about the major and minor principal axes, respectively

$J$  = the torsional constant

$K_z$  = the effective length factor for torsional buckling

The warping constant  $C_w$  and the torsional constant  $J$  are tabulated for various steel shapes in the AISC-LRFD manual [AISC, 2001]. Equations for calculating approximate values for these constants for some commonly used steel shapes are shown in [Table 48.5](#).

*Flexural–torsional buckling* (with a width–thickness ratio of  $\leq \lambda_r$ ):

Same as for torsional buckling, except  $F_e$  is now given by:

*For singly symmetric sections:*

$$F_e = \frac{F_{es} + F_{ez}}{2H} \left[ 1 - \sqrt{1 - \frac{4 F_{es} F_{ez} H}{(F_{es} + F_{ez})^2}} \right] \quad (48.20)$$

where  $F_{es} = F_{ex}$  if the x axis is the axis of symmetry of the cross section, or  $= F_{ey}$  if the y axis is the axis of symmetry of the cross section

$F_{ex} = \pi^2 E / (K_x l / r_x)^2$ ;  $F_{ey} = \pi^2 E / (K_y l / r_y)^2$

$H = 1 - (x_o^2 + y_o^2) / r_o^2$ , in which  $K_x$  and  $K_y$  are the effective length factors for buckling about the x and y axes, respectively

$l$  = the unbraced member length in the plane of buckling

$r_x$  and  $r_y$  = the radii of gyration about the x and y axes, respectively

$x_o$  and  $y_o$  = the shear center coordinates with respect to the centroid ([Fig. 48.7](#)),  $r_o^2 = x_o^2 + y_o^2 + r_x^2 + r_y^2$ .

Numerical values for  $r_o$  and  $H$  are given for hot-rolled W, channel, tee, single-angle, and double-angle sections in the AISC-LRFD manual [AISC, 2001].

*For unsymmetric sections:*

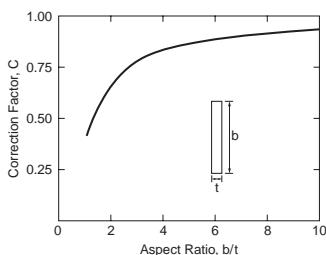
$F_e$  is to be solved from the cubic equation

$$(F_e - F_{ex})(F_e - F_{ey})(F_e - F_{ez}) - F_e^2 (F_e - F_{ey}) \left( \frac{x_o}{r_o} \right)^2 - F_e^2 (F_e - F_{ex}) \left( \frac{y_o}{r_o} \right)^2 = 0 \quad (48.21)$$

The terms in the above equations are defined the same as in Eq. (48.20).

**TABLE 48.5** Approximate Equations for  $C_w$  and  $J$

| Structural Shape | Warping Constant, $C_w$                     | Torsional Constant, $J$      |       |
|------------------|---|------------------------------|-------|
| I                | $h'^2 I_c I_t / (I_c + I_t)$                | $\sum C_i (b_i t_i^3 / 3)^a$ |       |
| C                | $(b' - 3E_o) h'^2 b'^2 t_f / 6 + E_o^2 I_x$ | $b_i / t_i$                  | $C_i$ |
|                  | where                                       | 1.00                         | 0.423 |
|                  | $E_o = b'^2 t_f / (2b' t_f + h' t_w / 3)$   | 1.20                         | 0.500 |
| T                | $(b_f^3 t_f^3 / 4 + h''^3 t_w^3) / 36$      | 1.50                         | 0.588 |
|                  | ( $\approx 0$ for small $t$ )               | 1.75                         | 0.642 |
| L                | $(l_1^3 t_1^3 + l_2^3 t_2^3) / 36$          | 2.00                         | 0.687 |
|                  | ( $\approx 0$ for small $t$ )               | 2.50                         | 0.747 |
|                  |   | 3.00                         | 0.789 |
|                  |   | 4.00                         | 0.843 |
|                  |   | 5.00                         | 0.873 |
|                  |   | 6.00                         | 0.894 |
|                  |   | 8.00                         | 0.921 |
|                  |   | 10.00                        | 0.936 |
|                  |   | $\infty$                     | 1.000 |



Note:

- $b'$  = distance measured from toe of flange to centerline of web
- $h'$  = distance between centerlines of flanges
- $h''$  = distance from centerline of flange to tip of stem
- $l_1, l_2$  = length of the legs of the angle
- $t_1, t_2$  = thickness of the legs of the angle
- $b_f$  = flange width
- $t_f$  = average thickness of flange
- $t_w$  = thickness of web
- $I_c$  = moment of inertia of compression flange taken about the axis of the web
- $I_t$  = moment of inertia of tension flange taken about the axis of the web
- $I_x$  = moment of inertia of the cross section taken about the major principal axis

<sup>a</sup>  $b_i$  = width of component element  $i$ ,  $t_i$  = thickness of component element  $i$ ,  
 $C_i$  = correction factor for component element  $i$ .

*Local buckling* (with a width–thickness ratio of  $\geq \lambda_r$ ):

Local buckling in the component element of the cross section is accounted for in design by introducing a reduction factor  $Q$  in Eq. (48.17) as follows:

$$\phi_c P_n = \begin{cases} 0.85 \left[ A_g Q (0.658^{Q\lambda^2}) F_y \right], & \text{if } \lambda \sqrt{Q} \leq 1.5 \\ 0.85 \left[ A_g \left( \frac{0.877}{\lambda^2} \right) F_y \right], & \text{if } \lambda \sqrt{Q} > 1.5 \end{cases} \quad (48.22)$$

where  $\lambda = \lambda_c$  for flexural buckling and  $\lambda = \lambda_e$  for flexural–torsional buckling.

The  $Q$  factor is given by

$$Q = Q_s Q_a \quad (48.23)$$

where  $Q_s$  = the reduction factor for unstiffened compression elements of the cross section (see [Table 48.6](#))

$Q_a$  = the reduction factor for stiffened compression elements of the cross section (see [Table 48.7](#)).

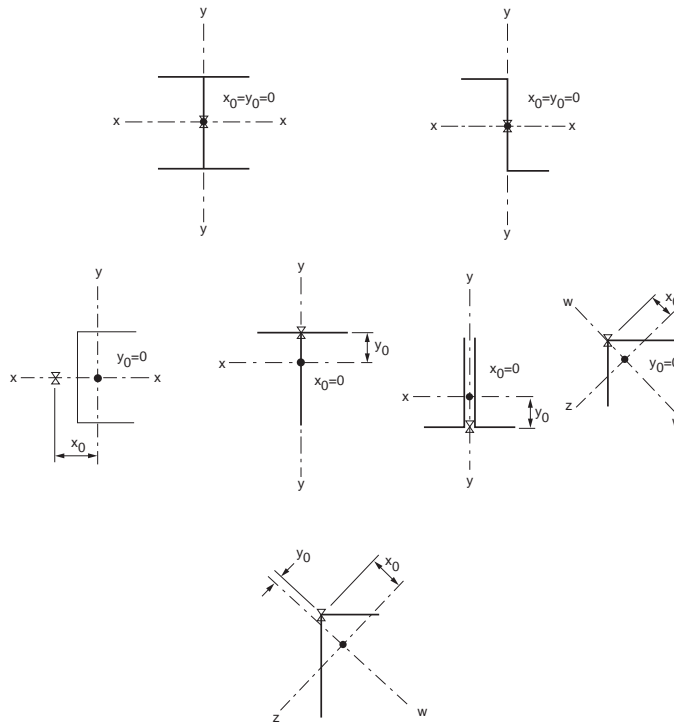


FIGURE 48.7 Location of shear center for selected cross sections.

TABLE 48.6 Formulas for  $Q_s$

| Structural Element  | Range of $b/t$   | $Q_s$   |
|---|--|---|
| Single angles   | $0.45\sqrt{(E/F_y)} < b/t < 0.91\sqrt{(E/F_y)}$<br>$b/t \geq 0.91\sqrt{(E/F_y)}$                   | $1.340 - 0.76(b/t)\sqrt{(F_y/E)}$<br>$0.53E/[F_y(b/t)^2]$         |
| Flanges, angles, and plates projecting from columns or other compression members          | $0.56\sqrt{(E/F_y)} < b/t < 1.03\sqrt{(E/F_y)}$<br>$b/t \geq 1.03\sqrt{(E/F_y)}$                   | $1.415 - 0.74(b/t)\sqrt{(F_y/E)}$<br>$0.69E/[F_y(b/t)^2]$         |
| Flanges, angles, and plates projecting from built-up columns or other compression members | $0.64\sqrt{(E/(F_y/k_c))} < b/t < 1.17\sqrt{[E/(F_y/k_c)]}$<br>$b/t \geq 1.17\sqrt{[E/(F_y/k_c)]}$ | $1.415 - 0.65(b/t)\sqrt{(F_y/k_c E)}$<br>$0.90E/k_c/[F_y(b/t)^2]$ |
| Stems of tees   | $0.75\sqrt{E/F_y} < d/t < 1.03\sqrt{(E/F_y)}$<br>$d/t \geq 1.03\sqrt{(E/F_y)}$                     | $1.908 - 1.22(d/t)\sqrt{(F_y/E)}$<br>$0.69E/[F_y(b/t)^2]$         |

Note:  $k_c$  is defined in the footnote of Table 48.4,  $E$  = modulus of elasticity,  $F_y$  = specified minimum yield stress,  $b$  = width of the component element,  $d$  = depth of tee sections,  $t$  = thickness of the component element.

**TABLE 48.7** Formula for  $Q_a$

$$Q_s = \frac{\text{effective area}}{\text{actual area}}$$

The effective area is equal to the summation of the effective areas of the stiffened elements of the cross section.

The effective area of a stiffened element is equal to the product of its thickness,  $t$ , and its effective width,  $b_e$ , given by

For flanges of square and rectangular sections of uniform thickness, when  $b/t \geq 1.40\sqrt{(E/f)^a}$ :

$$b_e = 1.91t \sqrt{\frac{E}{f}} \left[ 1 - \frac{0.38}{(b/t)} \sqrt{\frac{E}{f}} \right] \leq b$$

For other noncircular uniformly compressed elements, when  $b/t \geq 1.49\sqrt{(E/f)^a}$ :

$$b_e = 1.91t \sqrt{\frac{E}{f}} \left[ 1 - \frac{0.34}{(b/t)} \sqrt{\frac{E}{f}} \right] \leq b$$

For axially loaded circular sections with  $0.11E/F_y < D/t < 0.45E/F_y$ :

$$Q_a = \frac{0.038E}{F_y(D/t)} + \frac{2}{3}$$

Note:  $b$  = actual width of the stiffened element,  $t$  = wall thickness,  $E$  = modulus of elasticity,  $f$  = computed elastic compressive stress in the stiffened elements,  $D$  = outside diameter of circular sections.

<sup>a</sup>  $b_e = b$  otherwise.

## Built-up Compression Members

*Built-up members* are members made by bolting or welding together two or more standard structural shapes. For a built-up member to be fully effective (i.e., if all component structural shapes are to act as one unit, rather than as individual units), the following conditions must be satisfied:

1. Slippage of component elements near the ends of the built-up member must be prevented.
2. Adequate fasteners must be provided along the length of the member.
3. The fasteners must be able to provide sufficient gripping force on all component elements.

Condition 1 is satisfied if all component elements in contact near the ends of the built-up member are connected by a weld having a length not less than the maximum width of the member or by bolts spaced longitudinally not more than four diameters apart for a distance equal to one and a half times the maximum width of the member. Condition 2 is satisfied if continuous welds are used throughout the length of the built-up compression member. Condition 3 is satisfied if either welds or fully tightened bolts are used as the fasteners. Although condition 1 is mandatory, conditions 2 and 3 can be violated in design. If condition 2 or condition 3 is violated, the built-up member is not fully effective and slight slippage among component elements may occur. To account for the decrease in capacity due to slippage, a modified slenderness ratio is used to compute the design compressive strength when buckling of the built-up member is about an axis *coinciding* or *parallel* to at least one plane of contact for the component shapes. The modified slenderness ratio  $(KL/r)_m$  is given as follows:

If condition 2 is violated:

$$\left( \frac{KL}{r} \right)_m = \sqrt{\left( \frac{KL}{r} \right)_o^2 + \frac{0.82 \alpha^2}{(1 + \alpha^2)} \left( \frac{a}{r_{ib}} \right)^2} \quad (48.24)$$



If condition 3 is violated:

$$\left(\frac{KL}{r}\right)_m = \sqrt{\left(\frac{KL}{r}\right)_o^2 + \left(\frac{a}{r_i}\right)^2} \quad (48.25)$$

In the above equations,  $(KL/r)_o = (KL/r)_x$  if the buckling axis is the x axis and at least one plane of contact between component elements is parallel to that axis;  $(KL/r)_o = (KL/r)_y$  if the buckling axis is the y axis and at least one plane of contact is parallel to that axis.  $a$  is the longitudinal spacing of the fasteners,  $r_i$  is the minimum radius of gyration of any component element of the built-up cross section,  $r_{ib}$  is the radius of gyration of an individual component relative to its centroidal axis parallel to the axis of buckling of the member, and  $h$  is the distance between centroids of component elements measured perpendicularly to the buckling axis of the built-up member.

No modification to  $(KL/r)$  is necessary if the buckling axis is perpendicular to the planes of contact of the component shapes. Modifications to both  $(KL/r)_x$  and  $(KL/r)_y$  are required if the built-up member is so constructed that planes of contact exist in both the x and y directions of the cross section.

Once the modified slenderness ratio is computed, it is to be used in the appropriate equation to calculate  $F_a$  in allowable stress design or  $\phi_c P_n$  in load and resistance factor design.

An additional requirement for the design of built-up members is that the effective slenderness ratio,  $Ka/r_i$ , of each component element, where  $K$  is the effective length factor of the component element between adjacent fasteners, does not exceed three fourths of the governing slenderness ratio of the built-up member. This provision is provided to prevent component element buckling between adjacent fasteners from occurring prior to overall buckling of the built-up member.

### Example 48.2

Using LRFD, determine the size of a pair of cover plates to be bolted, using fully tightened bolts, to the flanges of a W24×229 section as shown in Fig. 48.8, so that its design strength,  $\phi_c P_n$ , will be increased by 20%. Also determine the spacing of the bolts along the longitudinal axis of the built-up column. The effective lengths of the section about the major  $(KL)_x$  and minor  $(KL)_y$  axes are both equal to 20 feet. A992 steel is to be used.

*Determine design strength for the W24×229 section:*

Since  $(KL)_x = (KL)_y$  and  $r_x > r_y$ ,  $(KL/r)_y$  will exceed  $(KL/r)_x$  and the design strength will be controlled by flexural buckling about the minor axis. Using section properties,  $r_y = 3.11$  in. and  $A = 67.2$  in.<sup>2</sup>, obtained from the AISC-LRFD manual [AISC, 2001], the slenderness parameter  $\lambda_c$  about the minor axis can be calculated as follows:

$$(\lambda_c)_y = \frac{1}{\pi} \left( \frac{KL}{r} \right)_y \sqrt{\frac{F_y}{E}} = \frac{1}{3.142} \left( \frac{20 \times 12}{3.11} \right) \sqrt{\frac{50}{29,000}} = 1.02$$

Substituting  $\lambda_c = 1.02$  into Eq. (48.17), the design strength of the section is

$$\phi_c P_n = 0.85 \left[ 67.2 \left( 0.658^{1.02^2} \right) 50 \right] = 1848 \text{ kips}$$

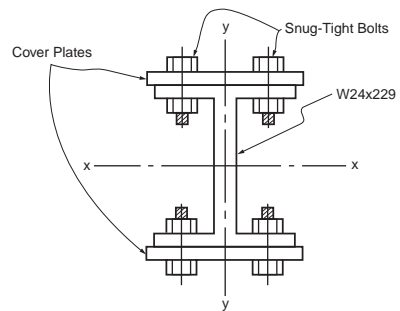


FIGURE 48.8 Design of cover plates for a compression member.

*Determine design strength for the built-up section:*

The built-up section is expected to possess a design strength that is 20% in excess of the design strength of the W24×229 section, so

$$(\phi_c P_n)_{reqd} = (1.20)(1848) = 2218 \text{ kips}$$

*Determine size of the cover plates:*

After cover plates are added, the resulting section is still doubly symmetric. Therefore, the overall failure mode is still flexural buckling. For flexural buckling about the minor axis (y-y), no modification to  $(KL/r)$  is required, since the buckling axis is perpendicular to the plane of contact of the component shapes, so no relative movement between the adjoining parts is expected. However, for flexural buckling about the major (x-x) axis, modification to  $(KL/r)$  is required, since the buckling axis is parallel to the plane of contact of the adjoining structural shapes and slippage between the component pieces will occur. We shall design the cover plates assuming flexural buckling about the minor axis will control and check for flexural buckling about the major axis later.

A W24×229 section has a flange width of 13.11 in.; so, as a trial, use cover plates with widths of 14 in., as shown in Fig. 48.8. Denoting  $t$  as the thickness of the plates, we have

$$(r_y)_{built-up} = \sqrt{\frac{(I_y)_{W-shape} + (I_y)_{plates}}{A_{W-shape} + A_{plates}}} = \sqrt{\frac{651 + 457.3t}{67.2 + 28t}}$$

and

$$(\lambda_c)_{y,built-up} = \frac{1}{\pi} \left( \frac{KL}{r} \right)_{y,built-up} \sqrt{\frac{F_y}{E}} = 3.17 \sqrt{\frac{67.2 + 28t}{651 + 457.3t}}$$

Assuming  $(\lambda_c)_{y,built-up}$  is less than 1.5, one can substitute the above expression for  $\lambda_c$  in Eq. (48.17). With  $\phi_c P_n$  equals 2218, we can solve for  $t$ . The result is  $t \approx 3/8$  in. Backsubstituting  $t = 3/8$  into the above expression, we obtain  $(\lambda_c)_{y,built-up} = 0.975$ , which is indeed  $< 1.5$ . So, try  $14 \times 3/8$  in. cover plates.

*Check for local buckling:*

For the I section:

$$\begin{aligned} \text{Flange: } \left[ \frac{b_f}{2 t_f} = 3.8 \right] &< \left[ 0.56 \sqrt{\frac{E}{F_y}} = 0.56 \sqrt{\frac{29,000}{50}} = 13.5 \right] \\ \text{Web: } \left[ \frac{h_c}{t_w} = 22.5 \right] &< \left[ 1.49 \sqrt{\frac{E}{F_y}} = 1.49 \sqrt{\frac{29,000}{50}} = 35.9 \right] \end{aligned}$$

For the cover plates, if 3/4-in. diameter bolts are used and assuming an edge distance of 2 in., the width of the plate between fasteners will be  $13.11 - 4 = 9.11$  in. Therefore, we have

$$\left[ \frac{b}{t} = \frac{9.11}{3/8} = 24.3 \right] < \left[ 1.40 \sqrt{\frac{E}{F_y}} = 1.40 \sqrt{\frac{29,000}{50}} = 33.7 \right]$$

Since the width–thickness ratios of all component shapes do not exceed the limiting width–thickness ratio for local buckling, local buckling is not a concern.

*Check for flexural buckling about the major (x-x) axis:*

Since the built-up section is doubly symmetric, the governing buckling mode will be flexural buckling regardless of the axes. Flexural buckling will occur about the major axis if the modified slenderness ratio  $(KL/r)_m$  about the major axis exceeds  $(KL/r)_y$ . Therefore, as long as  $(KL/r)_m$  is less than  $(KL/r)_y$ , buckling will occur about the minor axis and flexural buckling about the major axis will not control. In order to arrive at an optimal design, we shall determine the longitudinal fastener spacing,  $a$ , such that the modified slenderness ratio  $(KL/r)_m$  about the major axis will be equal to  $(KL/r)_y$ . That is, we shall solve for  $a$  from the equation

$$\left[ \left( \frac{KL}{r} \right)_m = \sqrt{\left( \frac{KL}{r} \right)_x^2 + \left( \frac{a}{r_i} \right)^2} \right] = \left[ \left( \frac{KL}{r} \right)_y = 73.8 \right]$$

In the above equation,  $(KL/r)_x$  is the slenderness ratio about the major axis of the built-up section and  $r_i$  is the least radius of gyration of the component shapes, which in this case is the cover plate.

Substituting  $(KL/r)_x = 21.7$  and  $r_i = r_{\text{cover plate}} = \sqrt{(I/A)_{\text{cover plate}}} = \sqrt{[(3/8)^2/12]} = 0.108$  into the above equation, we obtain  $a = 7.62$  in. Since  $(KL) = 20$  ft, we shall use  $a = 6$  in. for the longitudinal spacing of the fasteners.

*Check for component element buckling between adjacent fasteners:*

$$\left[ \frac{Ka}{r_i} = \frac{1 \times 6}{0.108} = 55.6 \right] \approx \left[ \frac{3}{4} \left( \frac{KL}{r} \right)_y = \frac{3}{4} (73.8) = 55.4 \right]$$

so the component element buckling criterion is not a concern.

Use  $14 \times 3/8$  in. cover plates bolted to the flanges of the W24×229 section by 3/4-in.-diameter fully tightened bolts spaced 6 in. longitudinally.

## Column Bracing

The design strength of a column can be increased if lateral braces are provided at intermediate points along its length in the buckled direction of the column. The AISC-LRFD specification [AISC, 1999] identifies two types of bracing systems for columns. A relative bracing system is one in which the movement of a braced point with respect to other adjacent braced points is controlled, e.g., the diagonal braces used in buildings. A nodal (or discrete) brace system is one in which the movement of a braced point with respect to some fixed point is controlled, e.g., the guy wires of guyed towers. A bracing system is effective only if the braces are designed to satisfy both stiffness and strength requirements. The following equations give the required stiffness and strength for the two bracing systems:

*Required braced stiffness:*

$$\beta_{cr} = \begin{cases} \frac{2P_u}{\phi L_{br}} & \text{for relative bracing} \\ \frac{8P_u}{\phi L_{br}} & \text{for nodal bracing} \end{cases} \quad (48.26)$$

where  $\phi = 0.75$

$P_u$  = the required compression strength of the column

$L_{br}$  = the distance between braces

If  $L_{br}$  is less than  $L_q$  (the maximum unbraced length for  $P_u$ ),  $L_{br}$  can be replaced by  $L_q$  in the above equations.

*Required braced strength:*

$$P_{br} = \begin{cases} 0.004P_u & \text{for relative bracing} \\ 0.01P_u & \text{for nodal bracing} \end{cases} \quad (48.27)$$

where  $P_u$  is defined as in Eq. (48.26).

## 48.5 Flexural Members

Depending on the width–thickness ratios of the component elements, steel sections used as *flexural members* are classified as compact, noncompact, and slender element sections. Compact sections are sections that can develop the cross section plastic moment ( $M_p$ ) under flexure and sustain that moment through a large hinge rotation without fracture. Noncompact sections are sections that either cannot develop the cross section full plastic strength or cannot sustain a large hinge rotation at  $M_p$ , probably due to local buckling of the flanges or web. Slender elements are sections that fail by local buckling of component elements long before  $M_p$  is reached. A section is considered compact if all its component elements have width–thickness ratios less than a limiting value (denoted as  $\lambda_p$  in LRFD). A section is considered noncompact if one or more of its component elements have width–thickness ratios that fall in between  $\lambda_p$  and  $\lambda_r$ . A section is considered a slender element if one or more of its component elements have width–thickness ratios that exceed  $\lambda_r$ . Expressions for  $\lambda_p$  and  $\lambda_r$  are given in the [Table 48.8](#).

In addition to the compactness of the steel section, another important consideration for beam design is the lateral unsupported (unbraced) length of the member. For beams bent about their strong axes, the failure modes, or limit states, vary depending on the number and spacing of lateral supports provided to brace the compression flange of the beam. The compression flange of a beam behaves somewhat like a compression member. It buckles if adequate lateral supports are not provided in a phenomenon called *lateral torsional buckling*. Lateral torsional buckling may or may not be accompanied by yielding, depending on the lateral unsupported length of the beam. Thus, lateral torsional buckling can be inelastic or elastic. If the lateral unsupported length is large, the limit state is elastic lateral torsional buckling. If the lateral unsupported length is smaller, the limit state is inelastic lateral torsional buckling. For compact section beams with adequate lateral supports, the limit state is full yielding of the cross section (i.e., plastic hinge formation). For noncompact section beams with adequate lateral supports, the limit state is flange or web local buckling. For beams bent about their weak axes, lateral torsional buckling will not occur, so the lateral unsupported length has no bearing on the design. The limit states for such beams will be formation of a plastic hinge if the section is compact and flange or web local buckling if the section is noncompact.

Beams subjected to high shear must be checked for possible web shear failure. Depending on the width–thickness ratio of the web, failure by shear yielding or web shear buckling may occur. Short, deep beams with thin webs are particularly susceptible to web shear failure. If web shear is of concern, the use of thicker webs or web reinforcements such as stiffeners is required.

Beams subjected to concentrated loads applied in the plane of the web must be checked for a variety of possible flange and web failures. Failure modes associated with concentrated loads include local flange bending (for a tensile concentrated load), local web yielding (for a compressive concentrated load), web crippling (for a compressive load), sidesway web buckling (for a compressive load), and compression buckling of the web (for a compressive load pair). If one or more of these conditions is critical, transverse stiffeners extending at least one half of the beam depth (use full depth for compressive buckling of the web) must be provided adjacent to the concentrated loads.

**TABLE 48.8**  $\lambda_p$  and  $\lambda_r$  for Members under Flexural Compression

| Component Element   | Width–Thickness Ratio <sup>a</sup> | $\lambda_p$  | $\lambda_r$                                  |
|---|------------------------------------|--|--|
| Flanges of I-shaped rolled beams and channels   | $b/t$                              | $0.38\sqrt{(E/F_y)}$   | $0.83\sqrt{(E/F_L)}^b$                       |
| Flanges of I-shaped hybrid or welded beams  | $b/t$                              | $0.38\sqrt{(E/F_{yf})}$ for nonseismic application<br>$0.31\sqrt{(E/F_{yf})}$ for seismic application  | $0.95\sqrt{[E/(F_L/k_c)]}^c$                 |
| Flanges of square and rectangular box and hollow structural sections of uniform thickness; flange cover plates and diaphragm plates between lines of fasteners or welds | $b/t$                              | $0.939\sqrt{(E/F_y)}$ for plastic analysis   | $1.40/\sqrt{(E/F_y)}$                        |
| Unsupported width of cover plates perforated with a succession of access holes  | $b/t$                              | NA   | $1.86\sqrt{(E/F_y)}$                         |
| Legs of single-angle struts; legs of double-angle struts with separators; unstiffened elements  | $b/t$                              | NA   | $0.45/\sqrt{(E/F_y)}$                        |
| Stems of tees   | $d/t$                              | NA   | $0.75\sqrt{(E/F_y)}$                         |
| Webs in flexural compression  | $h_c/t_w$                          | $3.76\sqrt{(E/F_y)}$ for nonseismic application<br>$3.05\sqrt{(E/F_y)}$ for seismic application  | $5.70\sqrt{(E/F_y)}^d$                       |
| Webs in combined flexural and axial compression   | $h_c/t_w$                          | For $P_u/\phi_b P_y \leq 0.125$ :<br>$3.76(1 - 2.75P_u/\phi_b P_y)\sqrt{(E/F_y)}$ for nonseismic application<br>$3.05(1 - 1.54P_u/\phi_b P_y)\sqrt{(E/F_y)}$ for seismic application<br><br>For $P_u/\phi_b P_y > 0.125$ :<br>$1.12(2.33 - P_u/\phi_b P_y)\sqrt{(E/F_y)}$<br>$\geq 1.49\sqrt{(E/F_y)}$ | $5.70(1 - 0.74P_u/\phi_b P_y)\sqrt{(E/F_y)}$ |
| Circular hollow sections  | $D/t$                              | $0.07E/F_y$  | $0.31E/F_y$                                  |

Note: NA = not applicable,  $E$  = modulus of elasticity,  $F_y$  = minimum specified yield strength,  $F_L$  = smaller of  $(F_{yf} - F_r)$  or  $F_{yw}$ ,  $F_{yf}$  = flange yield strength,  $F_{yw}$  = web yield strength,  $F_r$  = flange compressive residual stress (10 ksi for rolled shapes, 16.5 ksi for welded shapes),  $k_c$  is as defined in the footnote of Table 48.4,  $\phi_b = 0.90$ ,  $P_u$  = factored axial force,  $P_y = A_g F_y$ ,  $D$  = outside diameter,  $t$  = wall thickness.

<sup>a</sup> See Fig. 48.5 for definitions of  $b$ ,  $h_c$ , and  $t$ .

<sup>b</sup> For ASD, this limit is  $0.56\sqrt{(E/F_y)}$ .

<sup>c</sup> For ASD, this limit is  $0.56\sqrt{E/(F_{yf}/k_c)}$ , where  $k_c = 4.05/(h/t)^{0.46}$  if  $h/t > 70$ ; otherwise,  $k_c = 1.0$ .

<sup>d</sup> For ASD, this limit is  $4.46\sqrt{(E/F_b)}$ ;  $F_b$  = allowable bending stress.

Long beams can have deflections that may be too excessive, leading to problems in serviceability. If deflection is excessive, the use of intermediate supports or beams with higher flexural rigidity is required.

The design of flexural members should satisfy, at the minimum, the following criteria: (1) flexural strength criterion, (2) shear strength criterion, (3) criteria for concentrated loads, and (4) deflection criterion. To facilitate beam design, a number of beam tables and charts are given in the AISC manuals [AISC, 1989, 2001] for both allowable stress and load and resistance factor design.

## Flexural Member Design

### Allowable Stress Design

#### Flexural Strength Criterion

The computed flexural stress,  $f_b$ , shall not exceed the allowable flexural stress,  $F_b$ , given as follows. In all equations, the minimum specified yield stress,  $F_y$ , can not exceed 65 ksi.

Compact-Section Members Bent about Their Major Axes — For  $L_b \leq L_c$ ,

$$F_b = 0.66 F_y \quad (48.28)$$

where  $L_c$  is the smaller of  $\{76b_f/\sqrt{F_y}, 20,000/(d/A_f)F_y\}$  for I and channel shapes and equal to  $[1950 + 1200(M_1/M_2)](b/F_y) \geq 1200(b/F_y)$  for box sections and rectangular and circular tubes in which  $b_f$  is the flange width (in.),  $d$  is the overall depth of section (ksi),  $A_f$  is the area of compression flange (in.<sup>2</sup>),  $b$  is the width of cross section (in.), and  $M_1/M_2$  is the ratio of the smaller to larger moments at the ends of the unbraced length of the beam ( $M_1/M_2$  is positive for reverse curvature bending and negative for single curvature bending).

For the above sections to be considered as compact, in addition to having the width–thickness ratios of their component elements falling within the limiting value of  $\lambda_p$ , shown in Table 48.8, the flanges of the sections must be continuously connected to the webs. For box-shaped sections, the following requirements must also be satisfied: the depth-to-width ratio should not exceed 6 and the flange-to-web thickness ratio should not exceed 2.

For  $L_b > L_c$ , the allowable flexural stress in tension is given by

$$F_b = 0.60 F_y \quad (48.29)$$

and the allowable flexural stress in compression is given by the larger value calculated from Eqs. (48.30) and (48.31). Equation (48.30) normally controls for deep, thin-flanged sections where warping restraint torsional resistance dominates, and Eq. (48.31) normally controls for shallow, thick-flanged sections where St. Venant torsional resistance dominates.

$$F_b = \begin{cases} \left[ \frac{2}{3} - \frac{F_y (l/r_T)^2}{1530 \times 10^3 C_b} \right] F_y \leq 0.60 F_y, & \text{if } \sqrt{\frac{102,000 C_b}{F_y}} \leq \frac{l}{r_T} < \sqrt{\frac{510,000 C_b}{F_y}} \\ \frac{170,000 C_b}{(l/r_T)^2} \leq 0.60 F_y, & \text{if } \frac{l}{r_T} \geq \sqrt{\frac{510,000 C_b}{F_y}} \end{cases} \quad (48.30)$$

$$F_b = \frac{12,000 C_b}{ld/A_f} \leq 0.60 F_y \quad (48.31)$$

where  $l$  = the distance between cross sections braced against twist or lateral displacement of the compression flange (in.)  
 $r_T$  = the radius of gyration of a section comprising the compression flange plus one third of the compression web area, taken about an axis in the plane of the web (in.)  
 $A_f$  = the compression flange area (in.<sup>2</sup>)  
 $d$  = the depth of cross section (in.)  
 $C_b = 12.5M_{max}/(2.5M_{max} + 3M_A + 4M_B + 3M_C)$   
 $M_{max}$ ,  $M_A$ ,  $M_B$ , and  $M_C$  = the absolute values of the maximum moment, quarter-point moment, midpoint moment, and three-quarter point moment, respectively, along the unbraced length of the member.

(For simplicity in design,  $C_b$  can conservatively be taken as unity.)

It should be cautioned that Eqs. (48.30) and (48.31) are applicable only to I and channel shapes with an axis of symmetry in and loaded in the plane of the web. In addition, Eq. (48.31) is applicable only if the compression flange is solid and approximately rectangular in shape, and its area is not less than the tension flange.

*Compact Section Members Bent about Their Minor Axes* — Since lateral torsional buckling will not occur for bending about the minor axes, regardless of the value of  $L_b$ , the allowable flexural stress is

$$F_b = 0.75F_y \quad (48.32)$$

*Noncompact Section Members Bent about Their Major Axes* — For  $L_b \leq L_c$ ,

$$F_b = 0.60F_y \quad (48.33)$$

where  $L_c$  is defined as in Eq. (48.28).

For  $L_b > L_c$ ,  $F_b$  is given in Eqs. (48.29) to (48.31).

*Noncompact Section Members Bent about Their Minor Axes* — Regardless of the value of  $L_b$ ,

$$F_b = 0.60F_y \quad (48.34)$$

*Slender Element Sections* — Refer to the Section 48.10.

### **Shear Strength Criterion**

For practically all structural shapes commonly used in constructions, the shear resistance from the flanges is small compared to the webs. As a result, the shear resistance for flexural members is normally determined on the basis of the webs only. The amount of web shear resistance is dependent on the width–thickness ratio  $h/t_w$  of the webs. If  $h/t_w$  is small, the failure mode is web yielding. If  $h/t_w$  is large, the failure mode is web buckling. To avoid web shear failure, the computed shear stress,  $f_v$ , shall not exceed the allowable shear stress,  $F_v$ , given by

$$F_v = \begin{cases} 0.40 F_y & \text{if } \frac{h}{t_w} \leq \frac{380}{\sqrt{F_y}} \\ \frac{C_v}{2.89} F_y \leq 0.40 F_y & \text{if } \frac{h}{t_w} > \frac{380}{\sqrt{F_y}} \end{cases} \quad (48.35)$$

where  $C_v = 45,000k_v/[F_y(h/t_w)^2]$  if  $C_v \leq 0.8$  and  $[190/(h/t_w)]\sqrt{(k_v/F_y)}$  if  $C_v > 0.8$

$k_v = 4.00 + 5.34/(a/h)^2$  if  $a/h \leq 1.0$  and  $5.34 + 4.00/(a/h)^2$  if  $a/h > 1.0$

$t_w$  = the web thickness (in.)

$a$  = the clear distance between transverse stiffeners (in.)

$h$  = the clear distance between flanges at the section under investigation (in.)

### **Criteria for Concentrated Loads**

*Local Flange Bending* — If the concentrated force that acts on the beam flange is tensile, the beam flange may experience excessive bending, leading to failure by fracture. To preclude this type of failure, transverse stiffeners are to be provided opposite the tension flange, unless the length of the load when measured across the beam flange is less than 0.15 times the flange width, or if the flange thickness,  $t_f$ , exceeds

$$0.4 \sqrt{\frac{P_{bf}}{F_y}} \quad (48.36)$$

where  $P_{bf}$  = the computed tensile force multiplied by 5/3 if the force is due to live and dead loads only or by 4/3 if the force is due to live and dead loads in conjunction with wind or earthquake loads (kips)

$F_y$  = the specified minimum yield stress (ksi).



*Local Web Yielding* — To prevent local web yielding, the concentrated compressive force,  $R$ , should not exceed  $0.66R_n$ , where  $R_n$  is the web yielding resistance given in Eq. (48.54) or (48.55), whichever applies.

*Web Crippling* — To prevent web crippling, the concentrated compressive force,  $R$ , should not exceed  $0.50R_n$ , where  $R_n$  is the web crippling resistance given in Eq. (48.56), (48.57), or (48.58), whichever applies.

*Sidesway Web Buckling* — To prevent sidesway web buckling, the concentrated compressive force,  $R$ , should not exceed  $R_n$ , where  $R_n$  is the sidesway web buckling resistance given in Eq. (48.59) or (48.60), whichever applies, except the term  $C_r t_w^3 t_f / h^2$  is replaced by  $6,800 t_w^3 / h$ .

*Compression Buckling of the Web* — When the web is subjected to a pair of concentrated forces acting on both flanges, buckling of the web may occur if the web depth clear of fillet,  $d_c$ , is greater than

$$\frac{4100 t_w^3 \sqrt{F_y}}{P_{bf}} \quad (48.37)$$

where  $t_w$  = the web thickness  
 $F_y$  = the minimum specified yield stress  
 $P_{bf}$  = as defined in Eq. (48.36)

### Deflection Criterion

Deflection is a serviceability consideration. Since most beams are fabricated with a camber that somewhat offsets the dead load deflection, consideration is often given to deflection due to live load only. For beams supporting plastered ceilings, the service live load deflection preferably should not exceed  $L/360$ , where  $L$  is the beam span. A larger deflection limit can be used if due considerations are given to ensure the proper functioning of the structure.

### Example 48.3

Using ASD, determine the amount of increase in flexural capacity of a W24×55 section bent about its major axis if two  $7 \times 1/2$  in. ( $178 \times 13$  mm) cover plates are bolted to its flanges, as shown in Fig. 48.9. The beam is laterally supported at every 5-ft (1.52-m) interval. Use A36 steel. Specify the type, diameter, and longitudinal spacing of the bolts used if the maximum shear to be resisted by the cross section is 100 kips (445 kN).

*Section properties:*

A W24×55 section has the following section properties:  $b_f = 7.005$  in.,  $t_f = 0.505$  in.,  $d = 23.57$  in.,  $t_w = 0.395$  in.,  $I_x = 1350$  in.<sup>4</sup>, and  $S_x = 114$  in.<sup>3</sup>

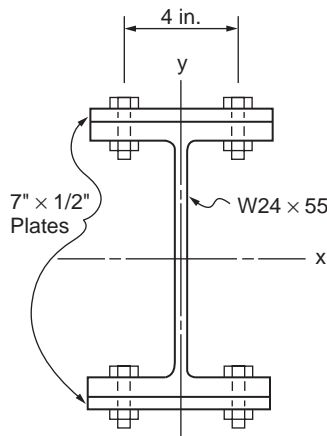


FIGURE 48.9 Beam section with cover plates.

Check compactness:

Refer to Table 48.8; assuming that the transverse distance between the two bolt lines is 4 in., we have

$$\text{Beam flanges} \quad \left[ \frac{b_f}{2 t_f} = 6.94 \right] < \left[ 0.38 \sqrt{\frac{E}{F_y}} = 10.8 \right]$$

$$\text{Beam web} \quad \left[ \frac{d}{t_w} = 59.7 \right] < \left[ 3.76 \sqrt{\frac{E}{F_y}} = 107 \right]$$

$$\text{Cover plates} \quad \left[ \frac{4}{1/2} = 8 \right] < \left[ 0.939 \sqrt{\frac{E}{F_y}} = 26.7 \right]$$

Therefore, the section is compact.

Determine the allowable flexural stress,  $F_b$ :

Since the section is compact and the lateral unbraced length,  $L_b = 60$  in., is less than  $L_c = 83.4$  in., the allowable bending stress from Eq. (48.28) is  $0.66F_y = 24$  ksi.

Determine section modulus of the beam with cover plates:

$$S_{x, \text{combination section}} = \frac{I_{x, \text{combination section}}}{c} = \frac{1350 + 2 \left[ \left( \frac{1}{12} \right) (7)(1/2)^3 + (7)(1/2)(12.035)^2 \right]}{\left( \frac{23.57}{2} + \frac{1}{2} \right)} = 192 \text{ in}^3$$

Determine flexural capacity of the beam with cover plates:

$$M_{x, \text{combination section}} = S_{x, \text{combination section}} F_b = (192)(24) = 4608 \text{ k-in}$$

Since the flexural capacity of the beam without cover plates is

$$M_x = S_x F_b = (114)(24) = 2736 \text{ k-in}$$

the increase in flexural capacity is 68.4%.

Determine diameter and longitudinal spacing of bolts:

From Chapter 46, "Mechanics of Materials," the relationship between the shear flow,  $q$ , the number of bolts per shear plane,  $n$ , the allowable bolt shear stress,  $F_v$ , the cross-sectional bolt area,  $A_b$ , and the longitudinal bolt spacing,  $s$ , at the interface of two component elements of a combination section is given by

$$\frac{n F_v A_b}{s} = q$$

Substituting  $n = 2$  and  $q = VQ/I = (100)[(7)(1/2)(12.035)]/2364 = 1.78$  k/in. into the above equation, we have

$$\frac{F_v A_b}{s} = 0.9 \text{ k/in.}$$

If 1/2-in.-diameter A325-N bolts are used, we have  $A_b = \pi(1/2)^2/4 = 0.196 \text{ in.}^2$  and  $F_v = 21 \text{ ksi}$  (from Table 48.12), from which  $s$  can be solved from the above equation to be 4.57 in. However, for ease of installation, use  $s = 4.5 \text{ in.}$

In calculating the section properties of the combination section, no deduction is made for the bolt holes in the beam flanges or cover plates; this is allowed provided that the following condition is satisfied

$$0.5 F_u A_{fn} \geq 0.6 F_y A_{fg}$$

where  $F_y$  and  $F_u$  = the minimum specified yield strength and tensile strength, respectively

$A_{fn}$  = the net flange area

$A_{fg}$  = the gross flange area. For this problem,

#### *Beam Flanges*

$$\begin{aligned} [0.5 F_u A_{fn} &= 0.5(58)(7.005 - 2 \times 1/2)(0.505) = 87.9 \text{ kips}] \\ &> [0.6 F_y A_{fg} = 0.6(36)(7.005)(0.505) = 76.4 \text{ kips}] \end{aligned}$$

#### *Cover Plates*

$$\begin{aligned} [0.5 F_u A_{fn} &= 0.5(58)(7 - 2 - 1/2)(1/2) = 87 \text{ kips}] \\ &> [0.6 F_y A_{fg} &= 0.6(36)(7)(1/2) = 75.6 \text{ kips}] \end{aligned}$$

so the use of the gross cross-sectional area to compute section properties is justified. In the event that the condition is violated, cross-sectional properties should be evaluated using an effective tension flange area  $A_{fe}$  given by

$$A_{fe} = \frac{5}{6} \frac{F_u}{F_y} A_{fn}$$

So, use 1/2-in.-diameter A325-N bolts spaced 4.5 in. apart longitudinally in two lines 4 in. apart to connect the cover plates to the beam flanges.

### **Load and Resistance Factor Design**

#### ***Flexural Strength Criterion***

Flexural members must be designed to satisfy the flexural strength criterion of

$$\phi_b M_n \geq M_u \quad (48.38)$$

where  $\phi_b M_n$  is the design flexural strength and  $M_u$  is the required strength. The design flexural strength is determined as follows:

*Compact Section Members Bent about Their Major Axes* — For  $L_b \leq L_p$  (plastic hinge formation),

$$\phi_b M_n = 0.90 M_p \quad (48.39)$$

For  $L_p \leq L_b \leq L_r$  (inelastic lateral torsional buckling),

$$\phi_b M_n = 0.90 C_b \left[ M_p - (M_p - M_r) \left( \frac{L_b - L_p}{L_r - L_p} \right) \right] \leq 0.90 M_p \quad (48.40)$$

For  $L_b > L_r$  (elastic lateral torsional buckling),

For I-shaped members and channels:

$$\phi_b M_n = 0.90 C_b \left[ \frac{\pi}{L_b} \sqrt{EI_y GJ + \left( \frac{\pi E}{L_b} \right)^2 I_y C_w} \right] \leq 0.90 M_p \quad (48.41)$$

For solid rectangular bars and symmetric box sections:

$$\phi_b M_n = 0.90 C_b \frac{57,000 \sqrt{JA}}{L_b / r_y} \leq 0.90 M_p \quad (48.42)$$

The variables used in the above equations are defined as follows:  $L_b$  is the lateral unsupported length of the member and  $L_p$  and  $L_r$  are the limiting lateral unsupported lengths given in the following table:

| Structural Shape                                  | $L_p$  | $L_r$  |
|---|--|--|
| I-shaped sections and channels                    | $1.76 r_y \sqrt{(E/F_{yf})}$<br>where<br>$r_y$ = radius of gyration about minor axis<br>$E$ = modulus of elasticity<br>$F_{yf}$ = flange yield strength  | $[r_y X_1 / F_L] \{ \sqrt{[1 + \sqrt{(1 + X_2 F_L^2)}]} \}$<br>where<br>$r_y$ = radius of gyration about minor axis (in.)<br>$X_1 = (\pi / S_x) \sqrt{(EGJA/2)}$<br>$X_2 = (4C_w / I_y) (S_x / GJ)^2$<br>$F_L$ = smaller of $(F_{yf} - F_r)$ or $F_{yw}$<br>$F_{yf}$ = flange yield stress (ksi)<br>$F_{yw}$ = web yield stress (ksi)<br>$F_r$ = 10 ksi for rolled shapes and 16.5 ksi for welded shapes<br>$S_x$ = elastic section modulus about the major axis (in. <sup>3</sup> ) (use $S_{xc}$ , the elastic section modulus about the major axis with respect to the compression flange if the compression flange is larger than the tension flange)<br>$I_y$ = moment of inertia about the minor axis (in. <sup>4</sup> )<br>$J$ = torsional constant (in. <sup>4</sup> )<br>$C_w$ = warping constant (in. <sup>6</sup> )<br>$E$ = modulus of elasticity (ksi) |
| Solid rectangular bars and symmetric box sections | $[0.13 r_y E \sqrt{(JA)}] / M_p$<br>where<br>$r_y$ = radius of gyration about minor axis<br>$E$ = modulus of elasticity<br>$J$ = torsional constant<br>$A$ = cross-sectional area<br>$M_p$ = plastic moment capacity = $F_y Z_x$<br>$F_y$ = yield stress<br>$Z_x$ = plastic section modulus about the major axis | $[2 r_y E \sqrt{(JA)}] / M_r$<br>where<br>$r_y$ = radius of gyration about minor axis<br>$J$ = torsional constant<br>$A$ = cross-sectional area<br>$M_r = F_{yf} S_x$<br>$F_y$ = yield stress<br>$F_{yf}$ = flange yield strength<br>$S_x$ = elastic section modulus about the major axis  |

*Note:* The  $L_p$  values given in this table are valid only if the bending coefficient  $C_b$  is equal to unity. If  $C_b > 1$ , the value of  $L_p$  can be increased. However, using the  $L_p$  expressions given above for  $C_b > 1$  will give a conservative value for the flexural design strength.

The remaining variables in the above equations are further defined as follows:

- $M_p = F_y Z_x$   
 $M_r = F_L S_x$  for I-shaped sections and channels and  $F_{yf} S_x$  for solid rectangular bars and box sections  
 $F_L$  = the smaller of  $(F_{yf} - F_r)$  or  $F_{yw}$   
 $F_{yf}$  = flange yield stress (ksi)  
 $F_{yw}$  = web yield stress (ksi)  
 $F_r$  = 10 ksi for rolled sections and 16.5 ksi for welded sections  
 $F_y$  = specified minimum yield stress  
 $S_x$  = elastic section modulus about the major axis  
 $Z_x$  = plastic section modulus about the major axis  
 $I_y$  = moment of inertia about the minor axis  
 $J$  = torsional constant  
 $C_w$  = warping constant  
 $E$  = modulus of elasticity  
 $G$  = shear modulus  
 $C_b = 12.5 M_{max} / (2.5 M_{max} + 3 M_A + 4 M_B + 3 M_C)$   
 $M_{max}$ ,  $M_A$ ,  $M_B$ , and  $M_C$  = absolute values of maximum, quarter-point, midpoint, and three-quarter point moments, respectively, along the unbraced length of the member.

$C_b$  is a factor that accounts for the effect of moment gradient on the lateral torsional buckling strength of the beam. The lateral torsional buckling strength increases for a steep moment gradient. The worst loading case as far as lateral torsional buckling is concerned is when the beam is subjected to a uniform moment resulting in single curvature bending. For this case,  $C_b = 1$ . Therefore, the use of  $C_b = 1$  is conservative for the design of beams.

*Compact Section Members Bent about Their Minor Axes* — Regardless of  $L_b$ , the limit state will be plastic hinge formation:

$$\phi_b M_n = 0.90 M_{py} = 0.90 F_y Z_y \quad (48.43)$$

*Noncompact Section Members Bent about Their Major Axes* — For  $L_b \leq L'_p$  (flange or web local buckling),

$$\phi_b M_n = \phi_b M_{n'} = 0.90 \left[ M_p - (M_p - M_r) \left( \frac{\lambda - \lambda_p}{\lambda_r - \lambda_p} \right) \right] \quad (48.44)$$

where

$$L_{p'} = L_p + (L_r - L_p) \left( \frac{M_p - M_{n'}}{M_p - M_r} \right) \quad (48.45)$$

$L_p$ ,  $L_r$ ,  $M_p$ , and  $M_r$  are defined as before for compact section members.

For flange local buckling,  $\lambda = b_f / 2t_f$  for I-shaped members and  $b_f / t_f$  for channels.  $\lambda_p$  and  $\lambda_r$  are defined in Table 48.8.

For web local buckling,  $\lambda = h_c / t_w$ .  $\lambda_p$  and  $\lambda_r$  are defined in Table 48.8, in which  $b_f$  is the flange width,  $t_f$  is the flange thickness,  $h_c$  is twice the distance from the neutral axis to the inside face of the compression flange less the fillet or corner radius, and  $t_w$  is the web thickness.

For  $L'_p < L_b \leq L_r$  (inelastic lateral torsional buckling),  $\phi_b M_n$  is given by Eq. (48.40), except that the limit  $0.90 M_p$  is to be replaced by the limit  $0.90 M_{n'}$ .

For  $L_b > L_r$  (elastic lateral torsional buckling),  $\phi_b M_n$  is the same as for compact section members, as given in Eq. (48.41) or (48.42).

*Noncompact Section Members Bent about Their Minor Axes* — Regardless of the value of  $L_b$ , the limit state will be either flange or web local buckling, and  $\phi_b M_n$  is given by Eq. (48.42).

*Slender Element Sections* — Refer to Section 48.10.

*Tees and Double Angles Bent about Their Major Axes* — The design flexural strength for tees and double-angle beams with flange and web slenderness ratios less than the corresponding limiting slenderness ratios  $\lambda_r$  shown in Table 48.8 is given by

$$\phi_b M_n = 0.90 \left[ \frac{\pi \sqrt{EI_y GJ}}{L_b} \left( B + \sqrt{1 + B^2} \right) \right] \leq 0.90 (\beta M_y) \quad (48.46)$$

where

$$B = \pm 2.3 \left( \frac{d}{L_b} \right) \sqrt{\frac{I_y}{J}} \quad (48.47)$$

Use the plus sign for  $B$  if the entire length of the stem along the unbraced length of the member is in tension. Otherwise, use the minus sign.  $\beta$  equals 1.5 for stems in tension and 1.0 for stems in compression. The other variables in Eq. (48.46) are defined as before, in Eq. (48.41).

### Shear Strength Criterion

For a satisfactory design, the design shear strength of the webs must exceed the factored shear acting on the cross section, i.e.,

$$\phi_v V_n \geq V_u \quad (48.48)$$

Depending on the slenderness ratios of the webs, three limit states can be identified: shear yielding, inelastic shear buckling, and elastic shear buckling. The design shear strengths that correspond to each of these limit states are given as follows:

For  $h/t_w \leq 2.45 \sqrt{E/F_{yw}}$  (shear yielding of web),

$$\phi_v V_n = 0.90 [0.60 F_{yw} A_w] \quad (48.49)$$

For  $2.45 \sqrt{E/F_{yw}} < h/t_w \leq 3.07 \sqrt{E/F_{yw}}$  (inelastic shear buckling of web),

$$\phi_v V_n = 0.90 \left[ 0.60 F_{yw} A_w \frac{2.45 \sqrt{E/F_{yw}}}{h/t_w} \right] \quad (48.50)$$

For  $3.07 \sqrt{E/F_{yw}} < h/t_w \leq 260$  (elastic shear buckling of web),

$$\phi_v V_n = 0.90 A_w \left[ \frac{4.52 E}{(h/t_w)^2} \right] \quad (48.51)$$

The variables used in the above equations are defined as follows:  $h$  is the clear distance between flanges less the fillet or corner radius,  $t_w$  is the web thickness,  $F_{yw}$  is the yield stress of the web,  $A_w = dt_w$ , and  $d$  is the overall depth of the section.

### Criteria for Concentrated Loads

When concentrated loads are applied normal to the flanges in planes parallel to the webs of flexural members, the flanges and webs must be checked to ensure that they have sufficient strengths  $\phi R_n$  to withstand the concentrated forces  $R_u$ , i.e.,

$$\phi R_n \geq R_u \quad (48.52)$$

The design strengths for a variety of limit states are given below.

*Local Flange Bending* — The design strength for local flange bending is given by

$$\phi R_n \geq 0.90 \left[ 6.25 t_f^2 F_{yf} \right] \quad (48.53)$$

where  $t_f$  = the flange thickness of the loaded flange  
 $F_{yf}$  = the flange yield stress

The design strength in Eq. (48.53) is applicable only if the length of load across the member flange exceeds  $0.15b$ , where  $b$  is the member flange width. If the length of load is less than  $0.15b$ , the limit state of local flange bending need not be checked. Also, Eq. (48.53) shall be reduced by a factor of half if the concentrated force is applied less than  $10t_f$  from the beam end.

*Local Web Yielding* — The design strengths for yielding of beam web at the toe of the fillet under tensile or compressive loads acting on one or both flanges are as follows:

If the load acts at a distance from the beam end that exceeds the depth of the member,

$$\phi R_n = 1.00 \left[ (5k + N) F_{yw} t_w \right] \quad (48.54)$$

If the load acts at a distance from the beam end that does not exceed the depth of the member,

$$\phi R_n = 1.00 \left[ (2.5k + N) F_{yw} t_w \right] \quad (48.55)$$

where  $k$  = the distance from the outer face of the flange to the web toe of fillet  
 $N$  = the length of bearing on the beam flange  
 $F_{yw}$  = the web yield stress  
 $t_w$  = the web thickness

*Web Crippling* — The design strengths for crippling of the beam web under compressive loads acting on one or both flanges are as follows:

If the load acts at a distance from the beam end that exceeds half the depth of the beam,

$$\phi R_n = 0.75 \left\{ 0.80 t_w^2 \left[ 1 + 3 \left( \frac{N}{d} \right) \left( \frac{t_w}{t_f} \right)^{1.5} \right] \sqrt{\frac{EF_{yw} t_f}{t_w}} \right\} \quad (48.56)$$

If the load acts at a distance from the beam end that does not exceed half the depth of the beam and if  $N/d \leq 0.2$ ,

$$\phi R_n = 0.75 \left\{ 0.40 t_w^2 \left[ 1 + 3 \left( \frac{N}{d} \right) \left( \frac{t_w}{t_f} \right)^{1.5} \right] \sqrt{\frac{EF_{yw} t_f}{t_w}} \right\} \quad (48.57)$$

If the load acts at a distance from the beam end that does not exceed half the depth of the beam and if  $N/d > 0.2$ ,



$$\phi R_n = 0.75 \left\{ 0.40 t_w^2 \left[ 1 + \left( \frac{4N}{d} - 0.2 \right) \left( \frac{t_w}{t_f} \right)^{1.5} \right] \sqrt{\frac{EF_{yw} t_f}{t_w}} \right\} \quad (48.58)$$

where  $d$  is the overall depth of the section and  $t_f$  is the flange thickness. The other variables are the same as those defined in Eqs. (48.54) and (48.55).

*Side Sway Web Buckling* — Side sway web buckling may occur in the web of a member if a compressive concentrated load is applied to a flange not restrained against relative movement by stiffeners or lateral bracings. The side sway web buckling design strength for the member is as follows:

If the loaded flange is restrained against rotation about the longitudinal member axis and  $(h/t_w)(l/b_f)$  is less than 2.3,

$$\phi R_n = 0.85 \left\{ \frac{C_r t_w^3 t_f}{h^2} \left[ 1 + 0.4 \left( \frac{h/t_w}{l/b_f} \right)^3 \right] \right\} \quad (48.59)$$

If the loaded flange is not restrained against rotation about the longitudinal member axis and  $(d_c/t_w)(l/b_f)$  is less than 1.7,

$$\phi R_n = 0.85 \left\{ \frac{C_r t_w^3 t_f}{h^2} \left[ 0.4 \left( \frac{h/t_w}{l/b_f} \right)^3 \right] \right\} \quad (48.60)$$

where  $t_f$  = the flange thickness (in.)

$t_w$  = the web thickness (in.)

$h$  = the clear distance between flanges less the fillet or corner radius for rolled shapes (the distance between adjacent lines of fasteners or the clear distance between flanges when welds are used for built-up shapes) (in.)

$b_f$  = the flange width (in.)

$l$  = the largest laterally unbraced length along either flange at the point of load (in.)

$C_r$  = 960,000 ksi if  $M_u/M_y < 1$  at the point of load or 480,000 ksi if  $M_u/M_y \geq 1$  at the point of load ( $M_y$  is the yield moment).

*Compression Buckling of the Web* — This limit state may occur in members with unstiffened webs when both flanges are subjected to compressive forces. The design strength for this limit state is

$$\phi R_n = 0.90 \left[ \frac{24 t_w^3 \sqrt{EF_{yw}}}{h} \right] \quad (48.61)$$

This design strength shall be reduced by a factor of half if the concentrated forces are acting at a distance less than half the beam depth from the beam end. The variables in Eq. (48.61) are the same as those defined in Eqs. (48.58) to (48.60).

Stiffeners shall be provided in pairs if any one of the above strength criteria is violated. If the local flange bending or the local web yielding criterion is violated, the stiffener pair to be provided to carry the excess  $R_u$  need not extend more than one half the web depth. The stiffeners shall be welded to the loaded flange if the applied force is tensile. They shall either bear on or be welded to the loaded flange if the applied force is compressive. If the web crippling or the compression web buckling criterion is violated, the stiffener pair to be provided shall extend the full height of the web. They shall be designed as axially loaded compression members (see Section 48.4) with an effective length factor of  $K = 0.75$  and

a cross section  $A_g$  composed of the cross-sectional areas of the stiffeners, plus  $25t_w^2$  for interior stiffeners and  $12t_w^2$  for stiffeners at member ends.

### Deflection Criterion

The deflection criterion is the same as that for ASD. Since deflection is a serviceability limit state, service (rather than factored) loads is used in deflection computations.

## Continuous Beams

Continuous beams shall be designed in accordance with the criteria for flexural members given in the preceding section. However, a 10% reduction in negative moments due to gravity loads is permitted at the supports provided that:

1. The maximum positive moment between supports is increased by one tenth the average of the negative moments at the supports.
2. The section is compact.
3. The lateral unbraced length does not exceed  $L_c$  (for ASD) or  $L_{pd}$  (for LRFD),

where  $L_c$  is as defined in Eq. (48.26) and  $L_{pd}$  is given by

$$L_{pd} = \begin{cases} \left[ 0.12 + 0.076 \left( \frac{M_1}{M_2} \right) \right] \left( \frac{E}{F_y} \right) r_y, & \text{for I-shaped members} \\ \left[ 0.17 + 0.10 \left( \frac{M_1}{M_2} \right) \right] \left( \frac{E}{F_y} \right) r_y \geq 0.10 \left( \frac{E}{F_y} \right) r_y, & \text{for solid rectangular and box sections} \end{cases} \quad (48.62)$$

in which  $F_y$  is the specified minimum yield stress of the compression flange;  $M_1/M_2$  is the ratio of smaller to larger moments within the unbraced length, taken as positive if the moments cause reverse curvature and negative if the moments cause single curvature; and  $r_y$  is the radius of gyration about the minor axis.

4. The beam is not a hybrid member.
5. The beam is not made of high-strength steel.
6. The beam is continuous over the supports (i.e., not cantilevered).

### Example 48.4

Using LRFD, select the lightest W section for the three-span continuous beam shown in Fig. 48.10a to support a uniformly distributed dead load of 1.5 k/ft (22 kN/m) and a uniformly distributed live load of 3 k/ft (44 kN/m). The beam is laterally braced at the supports A, B, C, and D. Use A36 steel.

#### Load combinations:

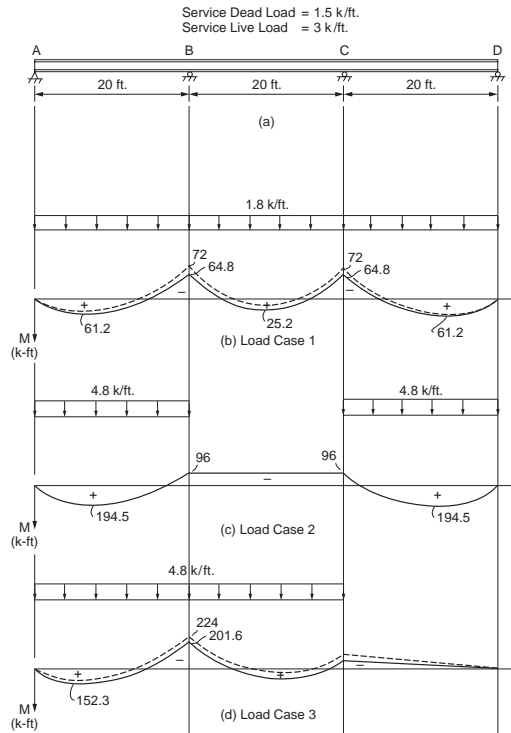
The beam is to be designed based on the worst load combination of Table 48.3. By inspection, the load combination  $1.2D + 1.6L$  will control the design. Thus, the beam will be designed to support a factored uniformly distributed dead load of  $1.2 \times 1.5 = 1.8$  k/ft and a factored uniformly distributed live load of  $1.6 \times 3 = 4.8$  k/ft.

#### Placement of loads:

The uniform dead load is to be applied over the entire length of the beam, as shown in Fig. 48.10b. The uniform live load is to be applied to spans AB and CD, as shown in Fig. 48.10c, to obtain the maximum positive moment, and it is to be applied to spans AB and BC, as shown in Fig. 48.10d, to obtain the maximum negative moment.

#### Reduction of negative moment at supports:

Assuming the beam is compact and  $L_b < L_{pd}$  (we shall check these assumptions later), a 10% reduction in support moment due to gravity load is allowed, provided that the maximum moment is increased by



**FIGURE 48.10** Design of a three-span continuous beam (1 k = 4.45 kN, 1 ft = 0.305 m).

one tenth the average of the negative support moments. This reduction is shown in the moment diagrams as solid lines in Fig. 48.10b and 48.10d. (The dotted lines in these figure parts represent the unadjusted moment diagrams.) This provision for support moment reduction takes into consideration the beneficial effect of moment redistribution in continuous beams, and it allows for the selection of a lighter section if the design is governed by negative moments. Note that no reduction in negative moments is made to the case when only spans AB and CD are loaded. This is because for this load case the negative support moments are less than the positive in-span moments.

#### *Determination of the required flexural strength, $M_u$ :*

Combining load cases 1 and 2, the maximum positive moment is found to be 256 kip-ft. Combining load cases 1 and 3, the maximum negative moment is found to be 266 kip-ft. Thus, the design will be controlled by the negative moment and  $M_u = 266$  kip-ft.

#### *Beam selection:*

A beam section is to be selected based on Eq. (48.38). The critical segment of the beam is span AB. For this span, the lateral unsupported length,  $L_b$ , is equal to 20 ft. For simplicity, the bending coefficient,  $C_b$ , is conservatively taken as 1. The selection of a beam section is facilitated by the use of a series of beam charts contained in the AISC-LRFD manual [AISC, 2001]. Beam charts are plots of flexural design strength  $\phi_b M_n$  of beams as a function of the lateral unsupported length  $L_b$ , based on Eqs. (48.39) to (48.41). A beam is considered satisfactory for the limit state of flexure if the beam strength curve envelopes the required flexural strength for a given  $L_b$ .

For the present example,  $L_b = 20$  ft and  $M_u = 266$  kip-ft; the lightest section (the first solid curve that envelopes  $M_u = 266$  kip-ft for  $L_b = 20$  ft) obtained from the chart is a W16x67 section. Upon adding the factored dead weight of this W16x67 section to the specified loads, the required flexural strength increases from 266 to 269 kip-ft. Nevertheless, the beam strength curve still envelopes this required strength for  $L_b = 20$  ft; therefore, the section is adequate.

*Check for compactness:*

For the W16×67 section,

$$\text{Flange: } \left[ \frac{b_f}{2 t_f} = 7.7 \right] < \left[ 0.38 \sqrt{\frac{E}{F_y}} = 10.8 \right]$$

$$\text{Web: } \left[ \frac{h_c}{t_w} = 35.9 \right] < \left[ 3.76 \sqrt{\frac{E}{F_y}} = 106.7 \right]$$

Therefore, the section is compact.

*Check whether  $L_b < L_{pd}$ :*

Using Eq. (48.62), with  $M_1/M_2 = 0$ ,  $r_y = 2.46$  in., and  $F_y = 36$  ksi, we have  $L_{pd} = 246$  in. (or 20.5 ft). Since  $L_b = 20$  ft is less than  $L_{pd} = 20.5$  ft, the assumption made earlier is validated.

*Check for the limit state of shear:*

The selected section must satisfy the shear strength criterion of Eq. (48.48). From structural analysis, it can be shown that maximum shear occurs just to the left of support B under load case 1 (for dead load) and load case 3 (for live load). It has a magnitude of 81.8 kips. For the W16×67 section,  $h/t_w = 35.9$ , which is less than  $2.45\sqrt{E/F_{yw}} = 69.5$ , so the design shear strength is given by Eq. (48.47). We have, for  $F_{yw} = 36$  ksi and  $A_w = dt_w = (16.33)(0.395)$ ,

$$\left[ \phi_v V_n = 0.90(0.60 F_{yw} A_w) = 125 \text{ kips} \right] > \left[ V_u = 81.8 \text{ kips} \right]$$

Therefore, shear is not a concern. Normally, the limit state of shear will not control, unless for short beams subjected to heavy loads.

*Check for limit state of deflection:*

Deflection is a serviceability limit state. As a result, a designer should use service (not factored) loads for deflection calculations. In addition, most beams are cambered to offset deflection caused by dead loads, so only live loads are considered in deflection calculations. From structural analysis, it can be shown that maximum deflection occurs in spans AB and CD when (service) live loads are placed on those two spans. The magnitude of the deflection is 0.297 in. Assuming the maximum allowable deflection is  $L/360$  where  $L$  is the span length between supports, we have an allowable deflection of  $20 \times 12/360 = 0.667$  in. Since the calculated deflection is less than the allowable deflection, deflection is not a problem.

*Check for the limit state of web yielding and web crippling at points of concentrated loads:*

From structural analysis, it can be shown that a maximum support reaction occurs at support B when the beam is subjected to the loads shown as load case 1 (for dead load) and load case 3 (for live load). The magnitude of the reaction  $R_u$  is 157 kips. Assuming point bearing, i.e.,  $N = 0$ , we have, for  $d = 16.33$  in.,  $k = 1.375$  in.,  $t_f = 0.665$  in., and  $t_w = 0.395$  in.,

$$\text{Web Yielding: } \left[ \phi R_n = \text{Eq. (46.54)} = 97.8 \text{ kips} \right] < \left[ R_u = 157 \text{ kips} \right]$$

$$\text{Web Crippling: } \left[ \phi R_n = \text{Eq. (46.56)} = 123 \text{ kips} \right] < \left[ R_u = 157 \text{ kips} \right]$$

Thus, both the web yielding and web crippling criteria are violated. As a result, we need to provide web stiffeners or a bearing plate at support B. Suppose we choose the latter, the size of the bearing plate can be determined by solving Eqs. (48.54) and (48.56) for  $N$  using  $R_u = 157$  kips. The result is  $N = 4.2$  and 3.3 in., respectively. So, use  $N = 4.25$  in. The width of the plate,  $B$ , should conform with the flange width,  $b_f$ , of the W-section. The W16×67 section has a flange width of 10.235 in., so use  $B = 10.5$  in. The thickness of the bearing plate is to be calculated from the following equation [AISC, 2001]:

$$t = \sqrt{\frac{2.22R_u[B - 2k/2]^2}{AF_y}}$$

where  $R_u$  = the factored concentrated load at the support (kips)  
 $B$  = the width of the bearing plate (in.)  
 $k$  = the distance from the web toe of the fillet to the outer surface of the flange (in.)  
 $A$  = the area of bearing plate (in.<sup>2</sup>)  
 $F_y$  = the yield strength of the bearing plate

Substituting  $R_u = 157$  kips,  $B = 10.5$  in.,  $k = 1.375$  in.,  $A = 42$  in.<sup>2</sup>, and  $F_y = 36$  ksi into the above equation, we obtain  $t = 1.86$  in. Therefore, use a 1 $\frac{7}{8}$ -in. plate.

For uniformity, use the same size plate at all the supports. The bearing plates are to be welded to the supporting flange of the W section.

Use a W16×67 section. Provide bearing plates of size 1 $\frac{7}{8}$  × 4 × 10 $\frac{1}{2}$ –1/2 in. at the supports.

## Beam Bracing

The design strength of beams that bend about their major axes depends on their lateral unsupported length  $L_b$ . The manner a beam is braced against out-of-plane deformation affects its design. Bracing can be provided by various means, such as cross frames, cross beams, or diaphragms, or encasement of the beam flange in the floor slab [Yura, 2001]. Two types of bracing systems are identified in the AISC-LRFD specification: relative and nodal. A relative brace controls the movement of a braced point with respect to adjacent braced points along the span of the beam. A nodal (or discrete) brace controls the movement of a braced point without regard to the movement of adjacent braced points. Regardless of the type of bracing system used, braces must be designed with sufficient strength and stiffness to prevent out-of-plane movement of the beam at the braced points. Out-of-plane movement consists of lateral deformation of the beam and twisting of cross sections. Lateral stability of beams can be achieved by lateral bracing, torsional bracing, or a combination of the two. For lateral bracing, bracing shall be attached near the compression flange for members bent in single curvature (except cantilevers). For cantilevers, bracing shall be attached to the tension flange at the free end. For members bent in double curvature, bracing shall be attached to both flanges near the inflection point. For torsional bracing, bracing can be attached at any cross-sectional location.

### Stiffness Requirement for Lateral Bracing

The required brace stiffness of the bracing assembly in a direction perpendicular to the longitudinal axis of the braced member, in the plane of buckling, is given by

$$\beta_{br} = \begin{cases} \frac{4M_u C_d}{\phi L_{br} h_o} & \text{for relative bracing} \\ \frac{10M_u C_d}{\phi L_{br} h_o} & \text{for nodal bracing} \end{cases} \quad (48.63)$$

where  $\phi = 0.75$ ,  $M_u$  is the required flexural strength  
 $C_d = 1.0$  for single curvature bending and 2.0 for double curvature bending near the inflection point  
 $L_{br}$  = the distance between braces  
 $h_o$  = the distance between flange centroids.  $L_{br}$  can be replaced by  $L_q$  (the maximum unbraced length for  $M_u$ ) if  $L_{br} < L_q$

### Strength Requirement for Lateral Bracing

In addition to the stiffness requirement as stipulated above, braces must be designed for a required brace strength given by

$$P_{br} = \begin{cases} 0.008 \frac{M_u C_d}{h_o} & \text{for relative bracing} \\ 0.02 \frac{M_u C_d}{h_o} & \text{for nodal bracing} \end{cases} \quad (48.64)$$

The terms in Eq. (48.64) are defined as in Eq. (48.63).

### Stiffness Requirement for Torsional Bracing

The required bracing stiffness is

$$\beta_{Tbr} = \frac{\beta_T}{\left(1 - \frac{\beta_T}{\beta_{sec}}\right)} \geq 0 \quad (48.65)$$

where

$$\beta_T = \begin{cases} \frac{2.4LM_u^2}{\phi nEI_y C_b^2} & \text{for nodal bracing} \\ \frac{2.4M_u^2}{\phi EI_y C_b^2} & \text{for continuous bracing} \end{cases} \quad (48.66)$$

and

$$\beta_{sec} = \begin{cases} \frac{3.3E}{h_o} \left( \frac{1.5h_o t_w^3}{12} + \frac{t_s b_s^3}{12} \right) & \text{for nodal bracing} \\ \frac{3.3Et_w^2}{12h_o} & \text{for continuous bracing} \end{cases} \quad (48.67)$$

in which  $\phi = 0.75$

$L$  = the span length

$M_u$  = the required moment

$n$  = the number of brace points within the span

$E$  = the modulus of elasticity

$I_y$  = the moment of inertia of the minor axis

$C_b$  = the bending coefficient as defined in Section 48.5

$h_o$  = the distance between flange centroids

$t_w$  = the thickness of the beam web

$t_s$  = the thickness of the web stiffener

$b_s$  = the width of stiffener (or, for pairs of stiffeners,  $b_s$  is the total width of stiffeners)

### Strength Requirement for Torsional Bracing

The connection between a torsional brace and the beam being braced must be able to withstand a moment given by

$$M_{Tbr} = \frac{0.024M_u L}{nC_b L_{br}} \quad (48.68)$$

where  $L_{br}$  is the distance between braces (if  $L_{br} < L_q$ ; where  $L_q$  is the maximum unbraced length for  $M_u$ , use  $L_q$ ). The other terms in Eq. (48.68) are defined in Eq. (48.66).

### Example 48.5

Design an I-shaped cross beam 12 ft (3.7 m) in length to be used as lateral braces to brace a 30-ft (9.1-m)-long simply supported W30×90 girder at every third point. The girder was designed to carry a moment of 8000 kip-in. (904 kN-m). A992 steel is used.

Because a brace is provided at every third point,  $L_{br} = 10 \text{ ft} = 120 \text{ in.}$   $M_u = 8000 \text{ kip-in.}$ , as stated.  $C_d = 1$  for single curvature bending.  $h_o = d - t_f = 29.53 - 0.610 = 28.92 \text{ in.}$  for the W30×90 section. Substituting these values into Eqs. (48.63) and (48.64) for nodal bracing, we obtain  $\beta_{br} = 30.7 \text{ kips/in.}$  and  $P_{br} = 5.53 \text{ kips.}$

Because the cross beam will be subject to compression, its slenderness ratio,  $l/r$ , should not exceed 200. Let us try the smallest-size W section, a W4×13 section with  $A = 3.83 \text{ in.}^2$ ,  $r_y = 1.00 \text{ in.}$ , and  $\phi_c P_n = 25 \text{ kips.}$

$$\text{Stiffness, } \frac{EA}{l} = \frac{(29000)(3.83)}{12 \times 12} = 771 \text{ kips/in.} > 30.7 \text{ kips/in.}$$

$$\text{Strength, } \phi_c P_n = 25 \text{ kips} > 5.53 \text{ kips}$$

$$\text{Slenderness, } \frac{l}{r_y} = \frac{12 \times 12}{1.00} = 144 < 200$$

Since all criteria are satisfied, the W4×13 section is adequate.

Use W4×13 as cross beams to brace the girder.

## 48.6 Combined Flexure and Axial Force

When a member is subject to the combined action of bending and axial force, it must be designed to resist stresses and forces arising from both bending and axial actions. While a tensile axial force may induce a stiffening effect on the member, a compressive axial force tends to destabilize the member, and the instability effects due to member instability ( $P$ - $\delta$  effect) and frame instability ( $P$ - $\Delta$  effect) must be properly accounted for. The  $P$ - $\delta$  effect arises when the axial force acts through the lateral deflection of the member relative to its chord. The  $P$ - $\Delta$  effect arises when the axial force acts through the relative displacements of the two ends of the member. Both effects tend to increase member deflection and moment; so they must be considered in the design. A number of approaches are available in the literature to handle these so-called  $P$ -delta effects (see, for example, Galambos [1998] and Chen and Lui [1991]). The design of members subject to combined bending and axial force is facilitated by the use of interaction equations. In these equations the effects of bending and axial actions are combined in a certain manner to reflect the capacity demand on the member.

### Design for Combined Flexure and Axial Force

#### Allowable Stress Design

The interaction equations are as follows:

*If the axial force is tensile,*

$$\frac{f_a}{F_t} + \frac{f_{bx}}{F_{bx}} + \frac{f_{by}}{F_{by}} \leq 1.0 \quad (48.69)$$

where  $f_a$  = the computed axial tensile stress  
 $f_{bx}$  and  $f_{by}$  = the computed bending tensile stresses about the major and minor axes, respectively  
 $F_t$  = the allowable tensile stress (see Section 48.3)  
 $F_{bx}$  and  $F_{by}$  = the allowable bending stresses about the major and minor axes, respectively (see Section 48.5).

If the axial force is compressive,

Stability requirement:

$$\frac{f_a}{F_a} + \left[ \frac{C_{mx}}{(1 - \frac{f_a}{F'_{ex}})} \right] \frac{f_{bx}}{F_{bx}} + \left[ \frac{C_{my}}{(1 - \frac{f_a}{F'_{ey}})} \right] \frac{f_{by}}{F_{by}} \leq 1.0 \quad (48.70)$$

Yield requirement:

$$\frac{f_a}{0.66 F_y} + \frac{f_{bx}}{F_{bx}} + \frac{f_{by}}{F_{by}} \leq 1.0 \quad (48.71)$$

However, if the axial force is small (when  $f_a/F_a \leq 0.15$ ), the following interaction equation can be used in lieu of the above equations.

$$\frac{f_a}{F_a} + \frac{f_{bx}}{F_{bx}} + \frac{f_{by}}{F_{by}} \leq 1.0 \quad (48.72)$$

The terms in the Eqs. (48.70) to (48.72) are defined as follows:  $f_a$ ,  $f_{bx}$ , and  $f_{by}$  are the computed axial compressive stress, the computed bending stress about the major axis, and the computer bending stress about the minor axis, respectively. These stresses are to be computed based on a *first-order analysis*:  $F_y$  is the minimum specified yield stress;  $F'_{ex}$  and  $F'_{ey}$  are the Euler stresses about the major and minor axes ( $\pi^2 E / (Kl/r)_x$  and  $\pi^2 E / (Kl/r)_y$ ), respectively, divided by a factor of safety of 23/12; and  $C_m$  is a coefficient to account for the effect of moment gradient on member and frame instabilities ( $C_m$  is defined in the following section). The other terms are defined as in Eq. (48.69)

The terms in brackets in Eq. (48.70) are moment magnification factors. The computed bending stresses  $f_{bx}$  and  $f_{by}$  are magnified by these magnification factors to account for the P-delta effects in the member.

### Load and Resistance Factor Design

Doubly or singly symmetric members subject to combined flexure and axial force shall be designed in accordance with the following interaction equations:

For  $P_u / \phi P_n \geq 0.2$ ,

$$\frac{P_u}{\phi P_n} + \frac{8}{9} \left( \frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}} \right) \leq 1.0 \quad (48.73)$$

For  $P_u / \phi P_n < 0.2$ ,

$$\frac{P_u}{2 \phi P_n} + \left( \frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}} \right) \leq 1.0 \quad (48.74)$$

where, if  $P$  is tensile,  $P_u$  is the factored tensile axial force,  $P_n$  is the design tensile strength (see Section 48.3),  $M_u$  is the factored moment (preferably obtained from a second-order analysis),  $M_n$  is the design flexural



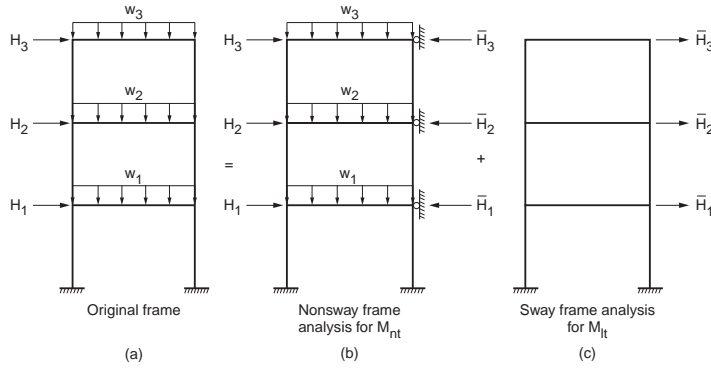


FIGURE 48.11 Calculation of  $M_{nt}$  and  $M_{lt}$ .

strength (see Section 48.5),  $\phi = \phi_t$  = resistance factor for tension = 0.90, and  $\phi_b$  = resistance factor for flexure = 0.90. If  $P$  is compressive,  $P_u$  is the factored compressive axial force,  $P_n$  is the design compressive strength (see Section 48.4),  $M_u$  is the required flexural strength (see discussion below),  $M_n$  is the design flexural strength (see Section 48.5),  $\phi = \phi_c$  = resistance factor for compression = 0.85, and  $\phi_b$  = resistance factor for flexure = 0.90.

The required flexural strength  $M_u$  shall be determined from a second-order elastic analysis. In lieu of such an analysis, the following equation may be used

$$M_u = B_1 M_{nt} + B_2 M_{lt} \quad (48.75)$$

where  $M_{nt}$  = the factored moment in a member, assuming the frame does not undergo lateral translation (see Fig. 48.11)

$M_{lt}$  = the factored moment in a member as a result of lateral translation (see Fig. 48.11)

$B_1 = C_m / (1 - P_u / P_e) \geq 1.0$  and is the  $P$ - $\delta$  moment magnification factor

$P_e = \pi^2 EI / (KL)^2$ , with  $K \leq 1.0$  in the plane of bending

$C_m$  = a coefficient to account for moment gradient (see discussion below)

$B_2 = 1 / [1 - (\Sigma P_u \Delta_{oh} / \Sigma HL)]$  or  $B_2 = 1 / [1 - (\Sigma P_u / \Sigma P_e)]$

$\Sigma P_u$  = the sum of all factored loads acting on and above the story under consideration

$\Delta_{oh}$  = the first-order interstory translation

$\Sigma H$  = the sum of all lateral loads acting on and above the story under consideration

$L$  = the story height

$P_e = \pi^2 EI / (KL)^2$

For end-restrained members that do not undergo relative joint translation and are not subject to transverse loading between their supports in the plane of bending,  $C_m$  is given by

$$C_m = 0.6 - 0.4 \left( \frac{M_1}{M_2} \right)$$

where  $M_1/M_2$  is the ratio of the smaller to larger member end moments. The ratio is positive if the member bends in reverse curvature and negative if the member bends in single curvature.

For end-restrained members that do not undergo relative joint translation and are subject to transverse loading between their supports in the plane of bending

$$C_m = 0.85$$

For unrestrained members that do not undergo relative joint translation and are subject to transverse loading between their supports in the plane of bending

$$C_m = 1.00$$

Although Eqs. (48.73) and (48.74) can be used directly for the design of *beam-columns* by using trial and error or by the use of design aids [Aminmansour, 2000], the selection of trial sections is facilitated by rewriting the interaction equations of Eqs. (48.73) and (48.74) into the so-called equivalent axial load form:

For  $P_u/\phi_c P_n > 0.2$ ,

$$bP_u + mM_{ux} + nM_{uy} \leq 1.0 \quad (48.76)$$

For  $P_u/\phi_c P_n \leq 0.2$ ,

$$b\frac{P_u}{2} + \frac{9}{8}mM_{ux} + \frac{9}{8}nM_{uy} \leq 1.0 \quad (48.77)$$

where

$$b = 1/\phi_c P_n$$

$$m = 8/9 \phi_b M_{nx}$$

$$n = 8/9 \phi_b M_{ny}$$

Numerical values for  $b$ ,  $m$ , and  $n$  are provided in the AISC manual [AISC, 2001]. The advantage of using Eqs. (48.76) and (48.77) for preliminary design is that the terms on the left-hand side of the inequality can be regarded as an equivalent axial load,  $(P_u)_{eff}$ , thus allowing the designer to take advantage of the column tables provided in the manual for selecting trial sections.

## 48.7 Biaxial Bending

Members subjected to bending about both principal axes (e.g., purlins on an inclined roof) should be design for *biaxial bending*. Since both the moment about the major axis  $M_{ux}$  and the moment about the minor axis  $M_{uy}$  create flexural stresses over the cross section of the member, the design must take into consideration these stress combinations.

### Design for Biaxial Bending

#### Allowable Stress Design

The following interaction equation is often used for the design of beams subject to biaxial bending:

$$f_{bx} + f_{by} \leq 0.60F_y \quad \text{or} \quad \frac{M_x}{S_x} + \frac{M_y}{S_y} \leq 0.60F_y \quad (48.78)$$

where  $M_x$  and  $M_y$  = the service load moments about the major and minor beam axes, respectively  
 $S_x$  and  $S_y$  = the elastic section moduli about the major and minor axes, respectively  
 $F_y$  = the specified minimum yield stress.

#### Example 48.6

Using ASD, select a W section to carry dead load moments  $M_x = 20$  k-ft (27 kN-m) and  $M_y = 5$  k-ft (6.8 kN-m) and live load moments  $M_x = 50$  k-ft (68 kN-m) and  $M_y = 15$  k-ft (20 kN-m). Use A992 steel.

Calculate service load moments:

$$M_x = M_{x, dead} + M_{x, live} = 20 + 50 = 70 \text{ k-ft}$$

$$M_y = M_{y, dead} + M_{y, live} = 5 + 15 = 20 \text{ k-ft}$$

Select section:

Substituting the above service load moments into Eq. (48.78), we have

$$\frac{70 \times 12}{S_x} + \frac{20 \times 12}{S_y} \leq 0.60(50) \quad \text{or,} \quad 840 + 240 \frac{S_x}{S_y} \leq 30 S_x$$

For W sections with a depth below 14 in. the value of  $S_x/S_y$  normally falls in the range of 3 to 8, and for W sections with a depth above 14 in. the value of  $S_x/S_y$  normally falls in the range of 5 to 12. Assuming  $S_x/S_y = 10$ , we have from the above equation,  $S_x \geq 108 \text{ in.}^3$ . Using the Allowable Stress Design Selection Table in the AISC-ASD manual, let's try a W24×55 section ( $S_x = 114 \text{ in.}^3$ ,  $S_y = 8.30 \text{ in.}^3$ ). For the W24×55 section,

$$\left[ 840 + 240 \frac{114}{8.30} = 4136 \right] > [30 S_x = 30(114) = 3420] \therefore \text{NG}$$

The next lightest section is W21×62 ( $S_x = 127 \text{ in.}^3$ ,  $S_y = 13.9 \text{ in.}^3$ ). For this section,

$$\left[ 840 + 240 \frac{127}{13.9} = 3033 \right] < [30 S_x = 30(127) = 3810] \therefore \text{OK}$$

Therefore, use a W21×62 section.

### Load and Resistance Factor Design

To avoid distress at the most severely stressed point, the following equation for the limit state of yielding must be satisfied:

$$f_{un} \leq \phi_b F_y \quad (48.79)$$

where  $f_{un} = M_{ux}/S_x + M_{uy}/S_y$  is the flexural stress under factored loads

$S_x$  and  $S_y$  = the elastic section moduli about the major and minor axes, respectively

$$\phi_b = 0.90$$

$F_y$  = the specified minimum yield stress

In addition, the limit state for lateral torsional buckling about the major axis should also be checked, i.e.,

$$\phi_b M_{nx} \geq M_{ux} \quad (48.80)$$

$\phi_b M_{nx}$  is the design flexural strength about the major axis (see Section 48.5).

To facilitate design for biaxial bending, Eq. (48.79) can be rearranged to give

$$S_x \geq \frac{M_{ux}}{\phi_b F_y} + \frac{M_{uy}}{\phi_b F_y} \left( \frac{S_x}{S_y} \right) \approx \frac{M_{ux}}{\phi_b F_y} + \frac{M_{uy}}{\phi_b F_y} \left( 3.5 \frac{d}{b_f} \right) \quad (48.81)$$

In the above equation,  $d$  is the overall depth and  $b_f$  is the flange width of the section. The approximation ( $S_x/S_y \approx (3.5d/b_f)$ ) was suggested by Gaylord et al. [1992] for doubly symmetric I-shaped sections.

## 48.8 Combined Bending, Torsion, and Axial Force

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Members subjected to the combined effect of bending, torsion, and axial force should be designed to satisfy the following limit states:

*Yielding under normal stress:*

$$\phi F_y \geq f_{un} \quad (48.82)$$

where  $\phi = 0.90$

$F_y$  = the specified minimum yield stress

$f_{un}$  = the maximum normal stress determined from an elastic analysis under factored loads

*Yielding under shear stress:*

$$\phi (0.6F_y) \geq f_{uv} \quad (48.83)$$

where  $\phi = 0.90$

$F_y$  = the specified minimum yield stress

$f_{uv}$  = the maximum shear stress determined from an elastic analysis under factored loads

*Buckling:*

$$\phi_c F_{cr} \geq f_{un} \quad \text{or} \quad \phi_c F_{cr} \geq f_{uv}, \text{ whichever is applicable} \quad (48.84)$$

where  $\phi_c F_{cr} = \phi_c P_n / A_g$ , in which  $\phi_c P_n$  is the design compressive strength of the member (see Section on 48.4) and  $A_g$  is the gross cross-section area

$f_{un}$  and  $f_{uv}$  = the normal and shear stresses, respectively, as defined in Eqs. (48.82) and (48.83).

## 48.9 Frames

---

Frames are designed as a collection of structural components such as beams, beam-columns (columns), and connections. According to the restraint characteristics of the connections used in the construction, frames can be designed as type I (rigid framing), type II (simple framing), or type III (semirigid framing) in ASD or as fully restrained (rigid) and partially restrained (semirigid) in LRFD:

The design of rigid frames necessitates the use of connections capable of transmitting the full or a significant portion of the moment developed between the connecting members. The rigidity of the connections must be such that the angles between intersecting members should remain virtually unchanged under factored loads.

The design of simple frames is based on the assumption that the connections provide no moment restraint to the beam insofar as gravity loads are concerned, but these connections should have an adequate capacity to resist wind moments.

The design of semirigid frames is permitted upon evidence of the connections to deliver a predictable amount of moment restraint. Over the past two decades, a tremendous amount of work has been published in the literature on semirigid connections and frame behavior (see, for example, Chen [1987, 2000], Council on Tall Buildings and Urban Habitat [1993], Chen et al. [1996], and Faella et al. [2000]). However, because of the vast number of semirigid connections that can exhibit an appreciable difference in joint behavior, no particular approach has been recommended by any building specifications as of this writing. The design of semirigid or partially restrained frames relies on sound engineering judgment by the designer.

Semirigid and simple framings often incur inelastic deformation in the connections. The connections used in these constructions must be proportioned to possess sufficient ductility to avoid overstress of the fasteners or welds.

Regardless of the types of constructions used, due consideration must be given to account for member and frame instability ( $P$ - $\delta$  and  $P$ - $\Delta$ ) effects either by the use of a second-order analysis or by other means, such as moment magnification factors or notional loads [ASCE, 1997]. The end-restrained effect on a member should also be accounted for by the use of the effective length factor  $K$ .

## Frame Design

Frames can be designed as *side sway inhibited* (braced) or *side sway uninhibited* (unbraced). In side sway inhibited frames, frame *drift* is controlled by the presence of a bracing system (e.g., shear walls, diagonal, cross, or K braces, etc.). In side sway uninhibited frames, frame drift is limited by the flexural rigidity of the connected members and diaphragm action of the floors. Most side sway uninhibited frames are designed as type I or type FR frames using moment connections. Under normal circumstances, the amount of interstory drift under service loads should not exceed  $h/500$  to  $h/300$ , where  $h$  is the story height. A higher value of interstory drift is allowed only if it does not create serviceability concerns.

Beams in side sway inhibited frames are often subject to high axial forces. As a result, they should be designed as beam-columns using beam-column interaction equations. Furthermore, vertical bracing systems should be provided for braced multistory frames to prevent vertical buckling of the frames under gravity loads.

When designing members of a frame, a designer should consider a variety of loading combinations and load patterns, and the members are designed for the most severe load cases. Preliminary sizing of members can be achieved by the use of simple behavioral models such as the simple beam model, cantilever column model, and portal and cantilever method of frame analysis (see, for example, Rossow [1996]).

## Frame Bracing

The subject of frame bracing is discussed in a number of references (see, for example, SSRC [1993] and Galambos [1998]). According to the LRFD specification [AISC, 1999] the required story or panel bracing shear stiffness in side sway inhibited frames is

$$\beta_{cr} = \frac{2 \sum P_u}{\phi L} \quad (48.85)$$

where  $\phi = 0.75$ ,  $\sum P_u$  is the sum of all factored gravity load acting on and above the story or panel supported by the bracing

$L$  = the story height or panel spacing

The required story or panel bracing force is

$$P_{br} = 0.004 \sum P_u \quad (48.86)$$

## 48.10 Plate Girders

Plate girders are built-up beams. They are used as flexural members to carry extremely large lateral loads. A flexural member is considered as a plate girder if the width–thickness ratio of the web,  $h_c/t_w$ , exceeds  $760/\sqrt{F_b}$  ( $F_b$  is the allowable flexural stress) according to ASD or  $\lambda_r$  (see Table 48.8) according to LRFD. Because of the large web slenderness, plate girders are often designed with transverse stiffeners to reinforce the web and to allow for postbuckling (shear) strength (i.e., *tension field action*) to develop. Table 48.9

**TABLE 48.9** Web Stiffeners Requirements

| Range of Web Slenderness   | Stiffener Requirements  |
|--|---|
| $\frac{h}{t_w} \leq 260$   | Plate girder can be designed without web stiffeners   |
| $260 < \frac{h}{t_w} \leq \frac{0.48E}{\sqrt{F_{yf}(F_{yf} + 16.5)}}$                          | Plate girder must be designed with web stiffeners; the spacing of stiffeners, $a$ , can exceed $1.5h$ ; the actual spacing is determined by the shear criterion |
| $\frac{0.48E}{\sqrt{F_{yf}(F_{yf} + 16.5)}} < \frac{h}{t_w} \leq 11.7 \sqrt{\frac{E}{F_{yf}}}$ | Plate girder must be designed with web stiffeners; the spacing of stiffeners, $a$ , cannot exceed $1.5h$  |

*Note:*  $a$  = clear distance between stiffeners,  $h$  = clear distance between flanges when welds are used or the distance between adjacent lines of fasteners when bolts are used,  $t_w$  = web thickness,  $F_{yf}$  = compression flange yield stress (ksi).

summarizes the requirements for transverse stiffeners for plate girders based on the web slenderness ratio  $h/t_w$ . Two types of transverse stiffeners are used for plate girders: bearing stiffeners and intermediate stiffeners. Bearing stiffeners are used at unframed girder ends and at concentrated load points where the web yielding or web crippling criterion is violated. Bearing stiffeners extend the full depth of the web from the bottom of the top flange to the top of the bottom flange. Intermediate stiffeners are used when the width–thickness ratio of the web,  $h/t_w$ , exceeds 260, when the shear criterion is violated, or when tension field action is considered in the design. Intermediate stiffeners need not extend the full depth of the web, but they must be in contact with the compression flange of the girder.

Normally, the depths of plate girder sections are so large that the simple beam theory, postulating that plane sections before bending remain plane after bending, does not apply. As a result, a different set of design formulas for plate girders is required.

## Plate Girder Design

### Allowable Stress Design

#### Allowable Bending Stress

The maximum bending stress in the compression flange of the girder computed using the flexure formula shall not exceed the allowable value,  $F'_b$ , given by

$$F'_b = F_b R_{PG} R_e \quad (48.87)$$

where  $F_b$  = the applicable allowable bending stress, as discussed in Section 48.5 (ksi)

$R_{PG}$  = the plate girder stress reduction factor,  $1 - 0.0005(A_w/A_f)(h/t_w - 760/\sqrt{F_b}) \leq 1.0$

$R_e$  = the hybrid girder factor,  $[12 + (A_w/A_f)(3\alpha - \alpha^3)]/[12 + 2(A_w/A_f)] \leq 1.0$  ( $R_e = 1$  for nonhybrid girders)

$A_w$  = the area of web

$A_f$  = the area of compression flange

$\alpha = 0.60F_{yw}/F_b \leq 1.0$

$F_{yw}$  = the yield stress of web

#### Allowable Shear Stress

The allowable shear stress without tension field action is the same as that for beams given in Eq. (48.35).

The allowable shear stress with tension field action is given by

$$F_v = \frac{F_y}{2.89} \left[ C_v + \frac{1 - C_v}{1.15 \sqrt{1 + (a/h)^2}} \right] \leq 0.40F_y \quad (48.88)$$

Note that tension field action can be considered in the design only for nonhybrid girders. If tension field action is considered, transverse stiffeners must be provided and spaced at a distance so that the computed average web shear stress,  $f_v$ , obtained by dividing the total shear by the web area, does not exceed the allowable shear stress,  $F_v$ , given by Eq. (48.88). In addition, the computed bending tensile stress in the panel where tension field action is considered can not exceed  $0.60F_y$ , or  $(0.825 - 0.375f_v/F_v)F_y$  where  $f_v$  is the computed average web shear stress, and  $F_v$  is the allowable web shear stress given in Eq. (48.88). The shear transfer criterion given by Eq. (48.91) must also be satisfied.

### **Transverse Stiffeners**

Transverse stiffeners must be designed to satisfy the following criteria:

#### *Moment of inertia criterion:*

With reference to an axis in the plane of the web, the moment of inertia of the stiffeners (in square inches) shall satisfy the condition

$$I_{st} \geq \left( \frac{h}{50} \right)^4 \quad (48.89)$$

where  $h$  is the clear distance between flanges (in inches).

#### *Area criterion:*

The total area of the stiffeners (in square inches) shall satisfy the condition

$$A_{st} \geq \frac{1 - C_v}{2} \left[ \frac{a}{h} - \frac{(a/h)^2}{\sqrt{1 + (a/h)^2}} \right] Y D h t_w \quad (48.90)$$

where  $C_v$  = the shear buckling coefficient as defined in Eq. (48.35)

$a$  = the stiffeners' spacing

$h$  = the clear distance between flanges

$t_w$  = the web thickness

$Y$  = the ratio of web yield stress to stiffener yield stress

$D$  = 1.0 for stiffeners furnished in pairs, 1.8 for single-angle stiffeners, and 2.4 for single-plate stiffeners.

#### *Shear transfer criterion:*

If tension field action is considered, the total shear transfer (in kips/in.) of the stiffeners shall not be less than

$$f_{vs} = h \sqrt{\left( \frac{F_{yw}}{340} \right)^3} \quad (48.91)$$

where  $F_{yw}$  = the web yield stress (ksi)

$h$  = the clear distance between flanges (in.)

The value of  $f_{vs}$  can be reduced proportionally if the computed average web shear stress,  $f_v$ , is less than  $F_v$  given in Eq. (48.88).

## **Load and Resistance Factor Design**

### **Flexural Strength Criterion**

Doubly or singly symmetric single-web plate girders loaded in the plane of the web should satisfy the flexural strength criterion of Eq. (48.38). The plate girder design flexural strength is given by:

For the limit state of tension flange yielding,

$$\phi_b M_n = 0.90 [S_{xt} R_e F_{yt}] \quad (48.92)$$

For the limit state of compression flange buckling,

$$\phi_b M_n = 0.90 [S_{xc} R_{PG} R_e F_{cr}] \quad (48.93)$$

where  $S_{xt}$  = the section modulus referred to the tension flange,  $I_x/c_t$   
 $S_{xc}$  = the section modulus referred to the compression flange,  $I_x/c_c$   
 $I_x$  = the moment of inertia about the major axis  
 $c_t$  = the distance from the neutral axis to the extreme fiber of the tension flange  
 $c_c$  = the distance from the neutral axis to the extreme fiber of the compression flange  
 $R_{PG}$  = the plate girder bending strength reduction factor,  $1 - a_r[h_c/t_w - 5.70\sqrt{(E/F_{cr})}]/[1200 + 300a_r] \leq 1.0$   
 $R_e$  = the hybrid girder factor,  $[12 + a_r(3m - m^3)]/[12 + 2a_r] \leq 1.0$  ( $R_e = 1$  for nonhybrid girders)  
 $a_r$  = the ratio of web area to compression flange area  
 $m$  = the ratio of web yield stress to flange yield stress or ratio of web yield stress to  $F_{cr}$   
 $F_{yt}$  = the tension flange yield stress; and  $F_{cr}$  is the critical compression flange stress calculated as follows:

| Limit State                | Range of Slenderness  | $F_{cr}$ (ksi)   |
|----------------------------|---|--|
| Flange local buckling      | $\frac{b_f}{2t_f} \leq 0.38 \sqrt{\frac{E}{F_{yf}}}$                                    | $F_{yf}$   |
|                            | $0.38 \sqrt{\frac{E}{F_{yf}}} < \frac{b_f}{2t_f} \leq 1.35 \sqrt{\frac{E}{F_{yf}/k_c}}$ | $F_{yf} \left[ 1 - \frac{1}{2} \left( \frac{\frac{b_f}{2t_f} - 0.38 \sqrt{\frac{E}{F_{yf}}}}{1.35 \sqrt{\frac{E}{F_{yf}/k_c}} - 0.38 \sqrt{\frac{E}{F_{yf}}}} \right) \right] \leq F_{yf}$ |
|                            | $\frac{b_f}{2t_f} > 1.35 \sqrt{\frac{E}{F_{yf}/k_c}}$                                   | $\frac{26,000 k_c}{\left( \frac{b_f}{2t_f} \right)^2}$   |
| Lateral torsional buckling | $\frac{L_b}{r_t} \leq 1.76 \sqrt{\frac{E}{F_{yf}}}$                                     | $F_{yf}$   |
|                            | $1.76 \sqrt{\frac{E}{F_{yf}}} < \frac{L_b}{r_t} \leq 4.44 \sqrt{\frac{E}{F_{yf}}}$      | $C_b F_{yf} \left[ 1 - \frac{1}{2} \left( \frac{\frac{L_b}{r_t} - 1.76 \sqrt{\frac{E}{F_{yf}}}}{4.44 \sqrt{\frac{E}{F_{yf}}} - 1.76 \sqrt{\frac{E}{F_{yf}}}} \right) \right] \leq F_{yf}$  |
|                            | $\frac{L_b}{r_t} > 4.44 \sqrt{\frac{E}{F_{yf}}}$  | $\frac{286,000 C_b}{\left( \frac{L_b}{r_t} \right)^2}$   |

Note:  $k_c = 4/\sqrt{(h/t_w)}$ ,  $0.35 \leq k_c \leq 0.763$ ,  $b_f$  = compression flange width,  $t_f$  = compression flange thickness,  $L_b$  = lateral unbraced length of the girder,  $r_t = \sqrt{[(t_f b_f^3/12 + h_c t_w^3/72)/(b_f t_f + h_c t_w/6)]}$ ,  $h_c$  = twice the distance from the neutral axis to the inside face of the compression flange less the fillet,  $t_w$  = web thickness,  $F_{yf}$  = yield stress of compression flange (ksi),  $C_b$  = bending coefficient (see Section 48.5).



$F_{cr}$  must be calculated for both flange local buckling and lateral torsional buckling. The smaller value of  $F_{cr}$  is used in Eq. (48.93).

The plate girder bending strength reduction factor  $R_{PG}$  is a factor to account for the nonlinear flexural stress distribution along the depth of the girder. The hybrid girder factor is a reduction factor to account for the lower yield strength of the web when the nominal moment capacity is computed assuming a homogeneous section made entirely of the higher yield stress of the flange.

### Shear Strength Criterion

Plate girders can be designed with or without the consideration of tension field action. If tension field action is considered, intermediate web stiffeners must be provided and spaced at a distance  $a$  such that  $a/h$  is smaller than 3 or  $[260/(h/t_w)]^2$ , whichever is smaller. Also, one must check the flexure–shear interaction of Eq. (48.96), if appropriate. Consideration of tension field action is not allowed if (1) the panel is an end panel, (2) the plate girder is a hybrid girder, (3) the plate girder is a web-tapered girder, or (4)  $a/h$  exceeds 3 or  $[260/(h/t_w)]^2$ , whichever is smaller.

The design shear strength,  $\phi_v V_n$ , of a plate girder is determined as follows:

If tension field action is not considered:

$\phi_v V_n$  is the same as those for beams given in Eqs. (48.49) to (48.51).

If tension field action is considered and  $h/t_w \leq 1.10\sqrt{(k_v E/F_{yw})}$ :

$$\phi_v V_n = 0.90 \left[ 0.60 A_w F_{yw} \right] \quad (48.94)$$

If  $h/t_w > 1.10\sqrt{(k_v E/F_{yw})}$ :

$$\phi_v V_n = 0.90 \left[ 0.60 A_w F_{yw} \left( C_v + \frac{1 - C_v}{1.15 \sqrt{1 + (a/h)^2}} \right) \right] \quad (48.95)$$

where  $k_v = 5 + 5/(a/h)^2$  ( $k_v$  shall be taken as 5.0 if  $a/h$  exceeds 3.0 or  $[260/(h/t_w)]^2$ , whichever is smaller)

$A_w = dt_w$  (where  $d$  is the section depth and  $t_w$  is the web thickness)

$F_{yw}$  = the web yield stress

$C_v$  = the shear coefficient, calculated as follows:

| Range of $h/t_w$  | $C_v$                                    |
|---|--|
| $1.10 \sqrt{\frac{k_v E}{F_{yw}}} \leq \frac{h}{t_w} \leq 1.37 \sqrt{\frac{k_v E}{F_{yw}}}$ | $\frac{1.10 \sqrt{k_v E/F_{yw}}}{h/t_w}$ |
| $\frac{h}{t_w} > 1.37 \sqrt{\frac{k_v E}{F_{yw}}}$  | $\frac{1.51 k_v E}{(h/t_w)^2 F_{yw}}$    |

### Flexure–Shear Interaction

Plate girders designed for tension field action must satisfy the flexure–shear interaction criterion in regions where  $0.60\phi V_n \leq V_u \leq \phi V_n$  and  $0.75\phi M_n \leq M_u \leq \phi M_n$ :

$$\frac{M_u}{\phi M_n} + 0.625 \frac{V_u}{\phi V_n} \leq 1.375 \quad (48.96)$$

where  $\phi = 0.90$ .

### **Bearing Stiffeners**

Bearing stiffeners must be provided for a plate girder at unframed girder ends and at points of concentrated loads where the web yielding or the web crippling criterion is violated (see Section 48.5 under Criteria for Concentrated Loads). Bearing stiffeners shall be provided in pairs and extend from the upper flange to the lower flange of the girder. Denoting  $b_{st}$  as the width of one stiffener and  $t_{st}$  as its thickness, bearing stiffeners shall be portioned to satisfy the limit states as follows.

For the limit state of local buckling,

$$\frac{b_{st}}{t_{st}} \leq 0.56 \sqrt{\frac{E}{F_y}} \quad (48.97)$$

For the limit state of compression, the design compressive strength,  $\phi_c P_n$ , must exceed the required compressive force acting on the stiffeners.  $\phi_c P_n$  is to be determined based on an effective length factor  $K$  of 0.75 and an effective area,  $A_{eff}$ , equal to the area of the bearing stiffeners plus a portion of the web. For end bearing, this effective area is equal to  $2(b_{st}t_{st}) + 12t_w^2$ , and for interior bearing, this effective area is equal to  $2(b_{st}t_{st}) + 25t_w^2$ , where  $t_w$  is the web thickness. The slenderness parameter,  $\lambda_c$ , is to be calculated using a radius of gyration  $r = \sqrt{(I_{stt}/A_{eff})}$ , where  $I_{st} = t_{st}(2b_{st} + t_w)^3/12$ .

For the limit state of bearing, the bearing strength,  $\phi R_n$ , must exceed the required compression force acting on the stiffeners.  $\phi R_n$  is given by

$$\phi R_n \geq 0.75 \left[ 1.8 F_y A_{pb} \right] \quad (48.98)$$

where  $F_y$  = the yield stress  
 $A_{pb}$  = the bearing area.

### **Intermediate Stiffeners**

Intermediate stiffeners shall be provided if (1) the shear strength capacity is calculated based on tension field action, (2) the shear criterion is violated (i.e., when the  $V_u$  exceeds  $\phi_v V_n$ ), or (3) the web slenderness  $h/t_w$  exceeds  $2.45\sqrt{(E/F_{yw})}$ . Intermediate stiffeners can be provided in pairs or on one side of the web only in the form of plates or angles. They should be welded to the compression flange and the web, but they may be stopped short of the tension flange. The following requirements apply to the design of intermediate stiffeners.

**Local Buckling** — The width–thickness ratio of the stiffener must be proportioned so that Eq. (48.97) is satisfied to prevent failure by local buckling.

**Stiffener Area** — The cross-section area of the stiffener must satisfy the following criterion:

$$A_{st} \geq \frac{F_{yw}}{F_y} \left[ 0.15 D h t_w (1 - C_v) \frac{V_u}{\phi_v V_n} - 18 t_w^2 \right] \geq 0 \quad (48.99)$$

where  $F_y$  is the yield stress of stiffeners and  $D$  is 1.0 for stiffeners in pairs, 1.8 for single-angle stiffeners, and 2.4 for single-plate stiffeners. The other terms in Eq. (48.99) are defined as before, in Eqs. (48.94) and (48.95).

**Stiffener Moment of Inertia** — The moment of inertia for stiffener pairs taken about an axis in the web center or for single stiffeners taken about the face of contact with the web plate must satisfy the following criterion:

$$I_{st} \geq a t_w^3 \left[ \frac{2.5}{(a/h)^2} - 2 \right] \geq 0.5 a t_w^3 \quad (48.100)$$

**Stiffener Length** — The length of the stiffeners,  $l_{st}$ , should fall within the range

$$h - 6 t_w < l_{st} < h - 6 t_w \quad (48.101)$$

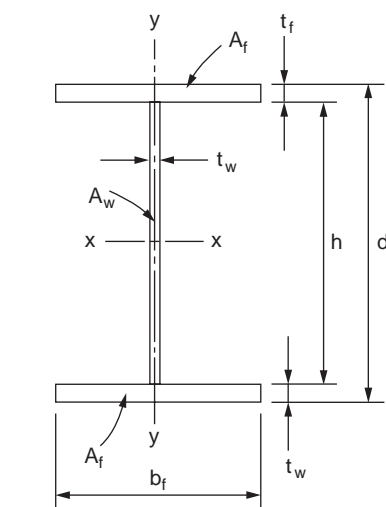
where  $h$  = the clear distance between the flanges less the widths of the flange-to-web welds  
 $t_w$  = the web thickness

If intermittent welds are used to connect the stiffeners to the girder web, the clear distance between welds shall not exceed  $16t_w$  or 10 in. (25.4 cm). If bolts are used, their spacing shall not exceed 12 in. (30.5 cm).

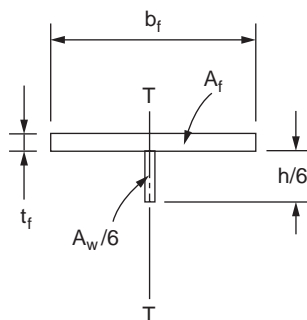
**Stiffener Spacing** — The spacing of the stiffeners,  $a$ , shall be determined from the shear criterion  $\phi_v V_n \geq V_u$ . This spacing shall not exceed the smaller of  $3h$  and  $[260/(h/t_w)]^2 h$ .

### Example 48.7

Using LRFD, design the cross section of an I-shaped plate girder, shown in Fig. 48.12a, to support a factored moment  $M_u$  of 4600 kip-ft (6240 kN-m); the dead weight of the girder is included. The girder is a 60-ft (18.3-m)-long simply supported girder. It is laterally supported at every 20-ft (6.1-m) interval. Use A36 steel.



(a) Plate Girder Nomenclature



(b) Calculation of  $r_T$

**FIGURE 48.12** Design of a plate girder cross section.

*Proportion of the girder web:*

Ordinarily, the overall depth-to-span ratio  $d/L$  of a building girder is in the range of 1/12 to 1/10. So, let us try  $h = 70$  in.

Also, because  $h/t_w$  of a plate girder is normally in the range of  $5.70\sqrt{(E/F_{yf})}$  to  $11.7\sqrt{(E/F_{yf})}$ , using  $E = 29,000$  ksi and  $F_{yf} = 36$  ksi, let's try  $t_w = 5/16$  in.

*Proportion of the girder flanges:*

For a preliminary design, the required area of the flange can be determined using the flange area method:

$$A_f \approx \frac{M_u}{F_y h} = \frac{4600 \text{ kip-ft} \times 12 \text{ in/ft}}{(36 \text{ ksi})(70 \text{ in})} = 21.7 \text{ in}^2$$

So, let  $b_f = 20$  in. and  $t_f = 1\frac{1}{8}$  in., giving  $A_f = 22.5 \text{ in}^2$

*Determine the design flexural strength  $\phi_b M_n$  of the girder:*

Calculate  $I_x$ :

$$\begin{aligned} I_x &= \sum [I_i + A_i y_i^2] \\ &= [8932 + (21.88)(0)^2] + 2 [2.37 + (22.5)(35.56)^2] \\ &= 65840 \text{ in}^4 \end{aligned}$$

Calculate  $S_{xt}$  and  $S_{xc}$ :

$$S_{xt} = S_{xc} = \frac{I_x}{c_t} = \frac{I_x}{c_c} = \frac{65840}{35 + 1.125} = 1823 \text{ in}^3$$

Calculate  $r_T$  (refer to [Fig. 48.12b](#)):

$$r_T = \sqrt{\frac{I_T}{A_f + \frac{1}{6} A_w}} = \sqrt{\frac{(1.125)(20)^3/12 + (11.667)(5/16)^3/12}{22.5 + \frac{1}{6}(21.88)}} = 5.36 \text{ in.}$$

Calculate  $F_{cr}$ :

For flange local buckling (FLB),

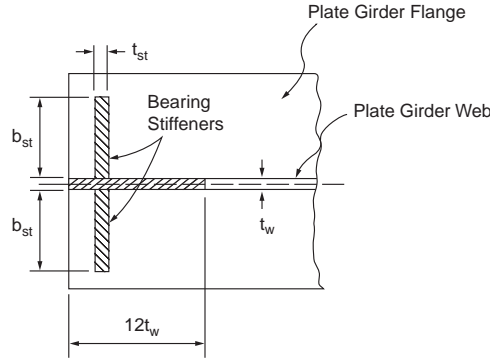
$$\left[ \frac{b_f}{2t_f} = \frac{20}{2(1.125)} = 8.89 \right] < \left[ 0.38 \sqrt{\frac{E}{F_{yf}}} = 10.8 \right] \quad \text{so, } F_{cr} = F_{yf} = 36 \text{ ksi}$$

For lateral torsional buckling (LTB),

$$\left[ \frac{L_b}{r_T} = \frac{20 \times 12}{5.36} = 44.8 \right] < \left[ 1.76 \sqrt{\frac{E}{F_{yf}}} = 50 \right] \quad \text{so, } F_{cr} = F_{yf} = 36 \text{ ksi}$$

Calculate  $R_{PG}$ :

$$R_{PG} = 1 - \frac{a_r \left( h_c/t_w - 5.70\sqrt{(E/F_{cr})} \right)}{(1,200 + 300 a_r)} = 1 - \frac{0.972 \left[ 70/(5/16) - 5.70\sqrt{(29,000/36)} \right]}{[1200 + 300(0.972)]} = 0.96$$



**FIGURE 48.13** Design of bearing stiffeners.

Calculate  $\phi_b M_n$ :

$$\begin{aligned}\phi_b M_n &= \text{smaller of} \begin{cases} 0.90 S_{xt} R_e F_{yt} = (0.90)(1823)(1)(36) = 59,065 \text{ kip-in.} \\ 0.90 S_x R_{PG} R_e F_{cr} = (0.90)(1823)(0.96)(1)(36) = 56,700 \text{ kip-in.} \end{cases} \\ &= 56,700 \text{ kip-in.} \\ &= 4725 \text{ kip-ft.}\end{aligned}$$

Since  $[\phi_b M_n = 4725 \text{ kip-ft}] > [M_u = 4600 \text{ kip-ft}]$ , the cross section is acceptable.

Use web plate 5/16 x 70 in. and two flange plates 1½ x 20 in. for the girder cross section.

### Example 48.8

Design bearing stiffeners for the plate girder of the preceding example for a factored end reaction of 260 kips.

Since the girder end is unframed, bearing stiffeners are required at the supports. The size of the stiffeners must be selected to ensure that the limit states of local buckling, compression, and bearing are not violated.

*Limit state of local buckling:*

Refer to Fig. 48.13; try  $b_{st} = 8$  in. To avoid problems with local buckling,  $b_{st}/2t_{st}$  must not exceed  $0.56\sqrt{(E/F_y)} = 15.8$ . Therefore, try  $t_{st} = 1/2$  in. So,  $b_{st}/2t_{st} = 8$ , which is less than 15.8.

*Limit state of compression:*

$$A_{eff} = 2(b_{st}t_{st}) + 12t_w^2 = 2(8)(0.5) + 12(5/16)^2 = 9.17 \text{ in.}^2$$

$$I_{st} = t_{st}(2b_{st} + t_w)^3/12 = 0.5[2(8) + 5/16]^3/12 = 181 \text{ in.}^4$$

$$r_{st} = \sqrt{(I_{st}/A_{eff})} = \sqrt{(181/9.17)} = 4.44 \text{ in.}$$

$$Kh/r_{st} = 0.75(70)/4.44 = 11.8$$

$$\lambda_c = (Kh/\pi r_{st})\sqrt{(F_y/E)} = (11.8/3.142)\sqrt{(36/29,000)} = 0.132$$

and from Eq. (48.17),

$$\phi_c P_n = 0.85(0.658^{\lambda_c^2})F_y A_{st} = 0.85(0.658)^{0.132^2}(36)(9.17) = 279 \text{ kips}$$

Since  $\phi_b P_n > 260$  kips, the design is satisfactory for compression.

### *Limit state of bearing:*

Assuming there is a 1/4-in. weld cutout at the corners of the bearing stiffeners at the junction of the stiffeners and the girder flanges, the bearing area for the stiffener pairs is  $A_{pb} = (8 - 0.25)(0.5)(2) = 7.75 \text{ in.}^2$ . Substituting this into Eq. (48.98), we have  $\phi R_n = 0.75(1.8)(36)(7.75) = 377 \text{ kips}$ , which exceeds the factored reaction of 260 kips. So bearing is not a problem.

Use two  $1/2 \times 8 \text{ in.}$  plates for bearing stiffeners.

## 48.11 Connections

---

Connections are structural elements used for joining different members of a framework. Connections can be classified according to:

1. the type of connecting medium used: bolted connections, welded connections, bolted–welded connections, riveted connections
2. the type of internal forces the connections are expected to transmit: shear (semirigid, simple) connections, moment (rigid) connections
3. the type of structural elements that made up the connections: single-plate-angle connections, double-web-angle connections, top- and seated-angle connections, seated beam connections, etc.
4. the type of members the connections are joining: beam-to-beam connections (beam splices), column-to-column connections (column splices), beam-to-column connections, hanger connections, etc.

To properly design a connection, a designer must have a thorough understanding of the behavior of the joint under loads. Different modes of failure can occur depending on the geometry of the connection and the relative strengths and stiffnesses of the various components of the connection. To ensure that the connection can carry the applied loads, a designer must check for all perceivable modes of failure pertinent to each component of the connection and the connection as a whole.

### Bolted Connections

Bolted connections are connections whose components are fastened together primarily by bolts. The four basic types of bolts are discussed in Section 48.1 under Structural Fasteners. Depending on the direction and line of action of the loads relative to the orientation and location of the bolts, the bolts may be loaded in tension, shear, or a combination of tension and shear. For bolts subjected to shear forces, the design shear strength of the bolts also depends on whether or not the threads of the bolts are excluded from the shear planes. A letter X or N is placed at the end of the ASTM designation of the bolts to indicate whether the threads are excluded or not excluded, respectively, from the shear planes. Thus, A325-X denotes A325 bolts whose threads are excluded from the shear planes, and A490-N denotes A490 bolts whose threads are not excluded from the shear planes. Because of the reduced shear areas for bolts whose threads are not excluded from the shear planes, these bolts have lower design shear strengths than their counterparts whose threads are excluded from the shear planes.

Bolts can be used in both bearing-type connections and slip-critical connections. Bearing-type connections rely on the bearing between the bolt shanks and the connecting parts to transmit forces. Some slippage between the connected parts is expected to occur for this type of connection. Slip-critical connections rely on the frictional force that develops between the connecting parts to transmit forces. No slippage between connecting elements is expected for this type of connection. Slip-critical connections are used for structures designed for vibratory or dynamic loads, such as bridges, industrial buildings, and buildings in regions of high seismicity. Bolts used in slip-critical connections are denoted by the letter F after their ASTM designation, e.g., A325-F, A490-F.

Holes made in the connected parts for bolts may be standard size, oversize, short slotted, or long slotted. [Table 48.10](#) gives the maximum hole dimension for ordinary construction usage.

**TABLE 48.10** Nominal Hole Dimensions (in.)

| Bolt<br>Diameter, $d$<br>(in.) | Hole Dimensions        |                        |                                       |                                      |
|--------------------------------|------------------------|------------------------|---------------------------------------|--------------------------------------|
|                                | Standard<br>(Diameter) | Oversize<br>(Diameter) | Short Slot<br>(Width $\times$ Length) | Long Slot<br>(Width $\times$ Length) |
| 1/2                            | 9/16                   | 5/8                    | 9/16 $\times$ 11/16                   | 9/16 $\times$ 1 1/4                  |
| 5/8                            | 11/16                  | 13/16                  | 11/16 $\times$ 7/8                    | 11/16 $\times$ 1 1/8                 |
| 3/4                            | 13/16                  | 15/16                  | 13/16 $\times$ 1                      | 13/16 $\times$ 1 1/8                 |
| 7/8                            | 15/16                  | 1 1/16                 | 15/16 $\times$ 1 1/8                  | 15/16 $\times$ 2 3/16                |
| 1                              | 1 1/16                 | 1 1/4                  | 1 1/16 $\times$ 1 1/2                 | 1 1/16 $\times$ 2 1/2                |
| $\geq 1 1/8$                   | $d + 1/16$             | $d + 5/16$             | $(d + 1/16) \times (d + 3/8)$         | $(d + 1/16) \times (2.5d)$           |

*Note:* 1 in. = 25.4 mm

Standard holes can be used for both bearing-type and slip-critical connections. Oversize holes shall be used only for slip-critical connections, and hardened washers shall be installed over these holes in an outer ply. Short-slotted and long-slotted holes can be used for both bearing-type and slip-critical connections, provided that when such holes are used for bearing, the direction of slot is transverse to the direction of loading. While oversize and short-slotted holes are allowed in any or all plies of the connection, long-slotted holes are allowed only in one of the connected parts. In addition, if long-slotted holes are used in an outer ply, plate washers, or a continuous bar with standard holes having a size sufficient to cover, the slot shall be provided.

**Bolts Loaded in Tension**

If a tensile force is applied to the connection such that the direction of load is parallel to the longitudinal axes of the bolts, the bolts will be subjected to tension. The following conditions must be satisfied for bolts under tensile stresses.

**Allowable Stress Design**

For ASD the condition is

$$f_t \leq F_t \tag{48.102}$$

where  $f_t$  = the computed tensile stress in the bolt  
 $F_t$  = the allowable tensile stress in the bolt (see [Table 48.11](#))

**Load and Resistance Factor Design**

For LRFD the condition is

$$\phi_t F_t \geq f_t \tag{48.103}$$

where  $\phi_t = 0.75$ ,  $f_t$  is the tensile stress produced by factored loads (ksi)  
 $F_t$  = the nominal tensile strength given in [Table 48.11](#)

**Bolts Loaded in Shear**

When the direction of load is perpendicular to the longitudinal axes of the bolts, the bolts will be subjected to shear. The conditions that need to be satisfied for bolts under shear stresses are as follows.

**Allowable Stress Design**

For bearing-type and slip-critical connections, the condition is

$$f_v \leq F_v \tag{48.104}$$

where  $f_v$  = the computed shear stress in the bolt (ksi)  
 $F_v$  = the allowable shear stress in the bolt (see [Table 48.12](#))

**TABLE 48.11**  $F_t$  of Bolts (ksi) (1 ksi = 6.895 MPa)

| Bolt Type | ASD                       |                                    | LRFD                      |   |
|-----------|---------------------------|------------------------------------|---------------------------|---|
|           | $F_t$<br>(Static Loading) | $F_t$<br>(Fatigue Loading)         | $F_t$<br>(Static Loading) | $F_t$<br>(Fatigue Loading)                                    |
| A307      | 20                        | Not allowed                        | 45                        | Not allowed   |
| A325      | 44                        | If $N \leq 20,000$ :               | 90                        | $F_t$ is not defined, but the condition $F_t < F_{SR}$        |
| A490      | 54                        | $F_t$ = same as for static loading | 113                       | needs to be satisfied   |
|           |                           | If $20,000 < N \leq 500,000$ :     |                           | where   |
|           |                           | $F_t = 40$ (A325)                  |                           | $f_t$ = tensile stress caused by service loads                |
|           |                           | = 49 (A490)                        |                           | calculated using a net tensile area given by                  |
|           |                           | If $N > 500,000$ :                 |                           | $A_t = \frac{\pi}{4} \left( d_b - \frac{0.9743}{n} \right)^2$ |
|           |                           | $F_t = 31$ (A325)                  |                           | $F_{SR}$ is the design stress range given by                  |
|           |                           | = 38 (A490)                        |                           | $F_{SR} = \left( \frac{3.9 \times 10^8}{N} \right)^{1/3}$     |
|           |                           | where                              |                           | in which  |
|           |                           | $N$ = number of stress range       |                           | $d_b$ = nominal bolt diameter                                 |
|           |                           | fluctuations in the design         |                           | $n$ = threads per inch  |
|           |                           | life                               |                           | $N$ = number of stress range fluctuations in the              |
|           |                           | $F_u$ = minimum specified          |                           | design life   |
|           |                           | tensile strength (ksi)             |                           |   |

**TABLE 48.12**  $F_v$  or  $F_n$  of Bolts (ksi) (1 ksi = 6.895 MPa)

| Bolt Type           | $F_v$ for ASD   | $F_v$ for LRFD  |
|---------------------|---|---|
| A307                | 10.0 <sup>a</sup> (regardless of whether or not threads are excluded from shear planes) | 24.0 <sup>a</sup> (regardless of whether or not threads are excluded from shear planes) |
| A325-N              | 21.0 <sup>a</sup>   | 48.0 <sup>a</sup>   |
| A325-X              | 30.0 <sup>a</sup>   | 60.0 <sup>a</sup>   |
| A325-F <sup>b</sup> | 17.0 for standard size holes  | 17.0 for standard size holes  |
|                     | 15.0 for oversize and short-slotted holes   | 15.0 for oversize and short-slotted holes   |
|                     | 12.0 for long-slotted holes when direction of load is transverse to the slots           | 12.0 for long-slotted holes when direction of load is transverse to the slots           |
|                     | 10.0 for long-slotted holes when direction of load is parallel to the slots             | 10.0 for long-slotted holes when direction of load is parallel to the slots             |
| A490-N              | 28.0 <sup>a</sup>   | 60.0 <sup>a</sup>   |
| A490-X              | 40.0 <sup>a</sup>   | 75.0 <sup>a</sup>   |
| A490-F <sup>b</sup> | 21.0 for standard size holes  | 21.0 for standard size holes  |
|                     | 18.0 for oversize and short-slotted holes   | 18.0 for oversize and short-slotted holes   |
|                     | 15.0 for long-slotted holes when direction of load is transverse to the slots           | 15.0 for long-slotted holes when direction of load is transverse to the slots           |
|                     | 13.0 for long-slotted holes when direction of load is parallel to the slots             | 13.0 for long-slotted holes when direction of load is parallel to the slots             |

<sup>a</sup> Tabulated values shall be reduced by 20% if the bolts are used to splice tension members having a fastener pattern whose length, measured parallel to the line of action of the force, exceeds 50 in. (127 cm).

<sup>b</sup> Tabulated values are applicable only to class A surfaces, i.e., unpainted clean mill surfaces and blast-cleaned surfaces with class A coatings (with slip coefficient = 0.33). For design strengths with other coatings, see *Load and Resistance Factor Design Specification for Structural Joints Using ASTM A325 or A490 Bolts* [RCSC, 2000].

### Load and Resistance Factor Design

For bearing-type connections designed at factored loads and for slip-critical connections designed at service loads, the condition is



**TABLE 48.13** Minimum Fastener Tension (kips)

| Bolt Diameter (in.) | A325 Bolts | A490 Bolts |
|---------------------|------------|------------|
| 1/2                 | 12         | 15         |
| 5/8                 | 19         | 24         |
| 3/4                 | 28         | 35         |
| 7/8                 | 39         | 49         |
| 1                   | 51         | 64         |
| 1 1/8               | 56         | 80         |
| 1 1/4               | 71         | 102        |
| 1 3/8               | 85         | 121        |
| 1 1/2               | 103        | 148        |

$$\phi_v F_v \geq f_v \quad (48.105)$$

where  $\phi_v = 0.75$  for bearing-type connections design for factored loads and 1.00 for slip-critical connections designed at service loads

$f_v$  = the shear stress produced by factored loads for bearing-type connections and by service loads for slip-critical connections (ksi)

$F_v$  = the nominal shear strength given in [Table 48.12](#)

For slip-critical connections designed at factored loads, the condition is

$$\phi r_{str} \geq r_u \quad (48.106)$$

where  $\phi = 1.0$  for standard holes, 0.85 for oversize and short-slotted holes, 0.70 for long-slotted holes transverse to the direction of load, and 0.60 for long-slotted holes parallel to the direction of load

$r_{str}$  = the design slip resistance per bolt,  $1.13\mu T_b N_s$

$\mu = 0.33$  for class A surfaces (i.e., unpainted clean mill surfaces or blast-cleaned surfaces with class A coatings), 0.50 for class B surfaces (i.e., unpainted blast-cleaned surfaces or blast-cleaned surfaces with class B coatings), and 0.35 for class C surfaces (i.e., hot-dip galvanized and roughened surfaces)

$T_b$  = the minimum fastener tension given in [Table 48.13](#)

$N_s$  = the number of slip planes

$r_u$  = the required force per bolt due to factored loads

### **Bolts Loaded in Combined Tension and Shear**

If a tensile force is applied to a connection such that its line of action is at an angle with the longitudinal axes of the bolts, the bolts will be subjected to combined tension and shear. The conditions that need to be satisfied are given below.

#### **Allowable Stress Design**

The conditions are

$$f_v \leq F_v \quad \text{and} \quad f_t \leq F_t \quad (48.107)$$

where  $f_v$  and  $F_v$  = as defined in Eq. (48.104)

$f_t$  = the computed tensile stress in the bolt (ksi)

$F_t$  = the allowable tensile stress given in [Table 48.14](#)

#### **Load and Resistance Factor Design**

For bearing-type connections designed at factored loads and slip-critical connections designed at service loads, the conditions are

**TABLE 48.14**  $F_t$  for Bolts under Combined Tension and Shear (ksi) (1 ksi = 6.895 MPa)

Bearing-Type Connections

| Bolt Type | ASD                                       |                                       | LRFD                                      |                                       |
|-----------|---|---------------------------------------|---|---------------------------------------|
|           | Threads Not Excluded from the Shear Plane | Threads Excluded from the Shear Plane | Threads Not Excluded from the Shear Plane | Threads Excluded from the Shear Plane |
| A307      | $26 - 1.8f_v \leq 20$                     |                                       | $59 - 2.5f_v \leq 45$                     |                                       |
| A325      | $\sqrt{(44^2 - 4.39f_v^2)}$               | $\sqrt{(44^2 - 2.15f_v^2)}$           | $117 - 2.5f_v \leq 90$                    | $117 - 2.0f_v \leq 90$                |
| A490      | $\sqrt{(54^2 - 3.75f_v^2)}$               | $\sqrt{(54^2 - 1.82f_v^2)}$           | $147 - 2.5f_v \leq 113$                   | $147 - 2.0f_v \leq 113$               |

Slip-Critical Connections

For ASD:

Only  $f_v \leq F_v$  needs to be checked

where

 $f_v$  = computed shear stress in the bolt (ksi) $F_v = [1 - (f_t A_b / T_b)] \times (\text{values of } F_v \text{ given in Table 48.12})$  $f_t$  = computed tensile stress in the bolt (ksi) $A_b$  = nominal cross-sectional area of bolt (in.<sup>2</sup>) $T_b$  = minimum pretension load given in Table 48.13

For LRFD:

Only  $\phi_v F_v \geq f_v$  needs to be checked

where

 $\phi_v = 1.0$  $f_v$  = shear stress produced by service load $F_v = [1 - (T_u / 0.8 T_b N_b)] \times (\text{values of } F_v \text{ given in Table 48.12})$  $T$  = service tensile force in the bolt (kips) $T_b$  = minimum pretension load given in Table 48.13 $N_b$  = number of bolts carrying the service load tension  $T$ 

$$\phi_v F_v \geq f_v \quad \text{and} \quad \phi_t F_t \geq f_t \quad (48.108)$$

where  $\phi_v$ ,  $F_v$ , and  $f_v$  = as defined in Eq. (48.105), $\phi_t = 0.75$  $f_t$  = the tensile stress due to factored loads for bearing-type connections and to service loads for slip-critical connections (ksi) $F_t$  = the nominal tension stress limit for combined tension and shear given in Table 48.14

For slip-critical connections designed at factored loads, the condition is given in Eq. (48.106), except that the design slip resistance per bolt  $\phi r_{str}$  shall be multiplied by a reduction factor given by  $1 - T_u / (1.13 T_b N_b)$ , where  $T_u$  is the factored tensile load on the connection,  $T_b$  is given in Table 48.13, and  $N_b$  is the number of bolts carrying the factored load tension  $T_u$ .

### Bearing Strength at Fastener Holes

Connections designed on the basis of bearing rely on the bearing force developed between the fasteners and the holes to transmit forces and moments. The limit state for bearing must therefore be checked to ensure that bearing failure will not occur. Bearing strength is independent of the type of fastener. This is because the bearing stress is more critical on the parts being connected than on the fastener itself. The AISC specification provisions for bearing strength are based on preventing excessive hole deformation. As a result, bearing capacity is expressed as a function of the type of holes (standard, oversize, slotted), bearing area (bolt diameter times the thickness of the connected parts), bolt spacing, edge distance ( $L_e$ ), strength of the connected parts ( $F_u$ ), and the number of fasteners in the direction of the bearing force.

TABLE 48.15 Bearing Capacity

| Conditions   | ASD   | LRFD  |
|--|---|---|
|  | Allowable<br>Bearing Stress, $F_p$<br>(ksi) | Design Bearing<br>Strength, $\phi R_n$<br>(kips)      |
| 1. For standard, oversize, or short-slotted holes loaded in any direction                  | $L_e F_u / 2d \leq 1.2 F_u$                 | $0.75 [1.2 L_e t F_u]$<br>$\leq 0.75 [2.4 d t F_u]^a$ |
| 2. For long-slotted holes with direction of slot perpendicular to the direction of bearing | $L_e F_u / 2d \leq 1.0 F_u$                 | $0.75 [1.0 L_e t F_u]$<br>$\leq 0.75 [2.0 d t F_u]$   |
| 3. If hole deformation at service load is not a design consideration                       | $L_e F_u / 2d \leq 1.5 F_u$                 | $0.75 [1.5 L_e t F_u]$<br>$\leq 0.75 [3.0 d t F_u]$   |

Note:  $L_e$  = distance from free edge to center of the bolt;  $L_c$  = clear distance, in the direction of force, between the edge of the hole and the edge of the adjacent hole or edge of the material;  $d$  = nominal bolt diameter;  $t$  = thickness of the connected part;  $F_u$  = specified minimum tensile strength of the connected part.

<sup>a</sup> This equation is also applicable to long-slotted holes when the direction of the slot is parallel to the direction of the bearing force.

Table 48.15 summarizes the expressions and conditions used in ASD and LRFD for calculating the bearing strength of both bearing-type and slip-critical connections.

Minimum Fastener Spacing

To ensure safety and efficiency and to maintain clearances between bolt nuts, as well as to provide room for wrench sockets, the fastener spacing,  $s$ , should not be less than  $3d$ , where  $d$  is the nominal fastener diameter.

Minimum Edge Distance

To prevent excessive deformation and shear rupture at the edge of the connected part, a minimum edge distance  $L_e$  must be provided in accordance with the values given in Table 48.16 for standard holes. For oversize and slotted holes, the values shown must be incremented by  $C_2$ , given in Table 48.17.

Maximum Fastener Spacing

A limit is placed on the maximum value for the spacing between adjacent fasteners to prevent the possibility of gaps forming or buckling from occurring in between fasteners when the load to be transmitted by the connection is compressive. The maximum fastener spacing measured in the direction of the force is given as follows.

For painted members or unpainted members not subject to corrosion, the maximum fastener spacing is the smaller of  $24t$ , where  $t$  is the thickness of the thinner plate and 12 in. (305 mm).

For unpainted members of weathering steel subject to atmospheric corrosion, the maximum fastener spacing is the smaller of  $14t$ , where  $t$  is the thickness of the thinner plate and 7 in. (178 mm).

TABLE 48.16 Minimum Edge Distance for Standard Holes (in.)

| Nominal Fastener<br>Diameter | At Rolled Edges of<br>Plates, Shapes, and Bars<br>or Gas-Cut Edges |                                  |
|------------------------------|--|----------------------------------|
|                              | At Sheared Edges   |                                  |
| 1/2                          | 7/8  | 3/4                              |
| 5/8                          | 1 1/8  | 7/8                              |
| 3/4                          | 1 1/4  | 1                                |
| 7/8                          | 1 1/2  | 1 1/8                            |
| 1                            | 1 3/4  | 1 1/4                            |
| 1 1/8                        | 2  | 1 1/2                            |
| 1 1/4                        | 2 1/4  | 1 5/8                            |
| Over 1 1/4                   | 1 3/4 $\times$ fastener diameter                                   | 1 1/4 $\times$ fastener diameter |

Note: 1 in. = 25.4 mm

**TABLE 48.17** Values of Edge Distance Increment,  $C_2$  (in.)

| Nominal Diameter<br>of Fastener | Oversize<br>Holes | Slotted Holes           |                        |                          |
|---------------------------------|-------------------|-------------------------|------------------------|--------------------------|
|                                 |                   | Slot Transverse to Edge |                        | Slot Parallel<br>to Edge |
|                                 |                   | Short Slot              | Long Slot <sup>a</sup> |                          |
| $\geq 7/8$                      | 1/16              | 1/8                     | $3d/4$                 | 0                        |
| 1                               | 1/8               | 1/8                     |                        |                          |
| $\geq 1\frac{1}{8}$             | 1/8               | 3/16                    |                        |                          |

*Note:* 1 in. = 25.4 mm

<sup>a</sup> If the length of the slot is less than the maximum shown in [Table 48.10](#), the value shown may be reduced by one half the difference between the maximum and the actual slot lengths.

### Maximum Edge Distance

A limit is placed on the maximum value for edge distance to prevent prying action from occurring. The maximum edge distance shall not exceed the smaller of  $12t$ , where  $t$  is the thickness of the connected part and 6 in. (15 cm).

#### Example 48.9

Check the adequacy of the connection shown in [Fig. 48.4a](#). The bolts are 1-in.-diameter A325-N bolts in standard holes. The connection is a bearing-type connection.

*Check bolt capacity:*

All bolts are subjected to double shear. Therefore, the design shear strength of the bolts will be twice that shown in [Table 48.12](#). Assuming each bolt carries an equal share of the factored applied load, we have from Eq. (48.105)

$$\left[ \phi_v F_v = 0.75(2 \times 48) = 72 \text{ ksi} \right] > \left[ f_v = \frac{208}{(6) \left( \frac{\pi 1^2}{4} \right)} = 44.1 \text{ ksi} \right]$$

The shear capacity of the bolt is therefore adequate.

*Check bearing capacity of the connected parts:*

With reference to [Table 48.15](#), it can be seen that condition 1 applies for the present problem. Therefore, we have

$$\left[ \phi R_n = 0.75(1.2L_c t F_u) = 0.75(1.2) \left( 3 - 1\frac{1}{8} \right) \left( \frac{3}{8} \right) (58) = 36.7 \text{ kips} \right]$$

$$< 0.75(2.4d t F_u) = 0.75(2.4) \left( 1 \right) \left( \frac{3}{8} \right) (58) = 39.2 \text{ kips} \right] > \left[ R_u = \frac{208}{6} = 34.7 \text{ kips} \right]$$

and so bearing is not a problem. Note that bearing on the gusset plate is more critical than bearing on the webs of the channels, because the thickness of the gusset plate is less than the combined thickness of the double channels.

*Check bolt spacing:*

The minimum bolt spacing is  $3d = 3(1) = 3$  in. The maximum bolt spacing is the smaller of  $14t = 14(0.303) = 4.24$  in. or 7 in. The actual spacing is 3 in., which falls within the range of 3 to 4.24 in., so bolt spacing is adequate.

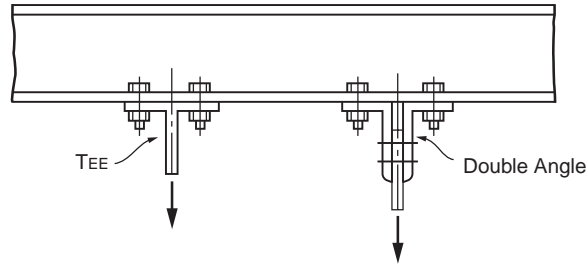


FIGURE 48.14 Hanger connections.

*Check edge distance:*

From Table 48.16, it can be determined that the minimum edge distance is 1.25 in. The maximum edge distance allowed is the smaller of  $12t = 12(0.303) = 3.64$  in. or 6 in. The actual edge distance is 3 in., which falls within the range of 1.25 to 3.64 in., so the edge distance is adequate.

The connection is therefore adequate.

### Bolted Hanger-Type Connections

A typical hanger connection is shown in Fig. 48.14. In the design of such connections, the designer must take into account the effect of *prying action*. Prying action results when flexural deformation occurs in the tee flange or angle leg of the connection (Fig. 48.15). The prying action tends to increase the tensile force, called prying force, in the bolts. To minimize the effect of prying, the fasteners should be placed as close to the tee stem or outstanding angle leg as the wrench clearance will permit (see tables on entering and tightening clearances in Volume II, Connections, of the AISC-LRFD manual [AISC, 1992]). In addition, the flange and angle thicknesses should be proportioned so that the full tensile capacities of the bolts can be developed.

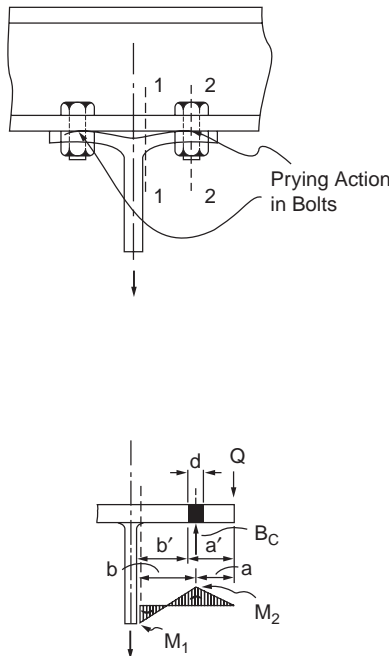


FIGURE 48.15 Prying action in hanger connections.

Two failure modes can be identified for hanger-type connections: formation of plastic hinges in the tee flange or angle leg at cross sections 1 and 2, and tensile failure of the bolts when the tensile force, including prying action  $B_c (=B + Q)$ , exceeds the tensile capacity of bolt  $B$ . Since the determination of the actual prying force is rather complex, the design equation for the required thickness for the tee flange or angle leg is semiempirical in nature. It is given by the following:

If ASD is used:

$$t_{reqd} = \sqrt{\frac{8Tb'}{pF_y(1+\delta\alpha')}} \quad (48.109)$$

where  $T$  is the tensile force per bolt due to the service load, exclusive of initial tightening and prying force (kips).

The other variables are as defined in Eq. (48.110) below, except that  $B$  in the equation for  $\alpha'$  is defined as the allowable tensile force per bolt. A design is considered satisfactory if the thickness of the tee flange or angle leg  $t_f$  exceeds  $t_{reqd}$  and  $B > T$ .

If LRFD is used:

$$t_{reqd} = \sqrt{\frac{4T_u b'}{\phi_b p F_y (1 + \delta \alpha')}} \quad (48.110)$$

where  $\phi_b = 0.90$

$T_u$  = the factored tensile force per bolt, exclusive of initial tightening and prying force (kips)

$p$  = the length of flange tributary to each bolt measured along the longitudinal axis of the tee or double-angle section (in.)

$\delta$  = the ratio of the net area at the bolt line to the gross area at the angle leg or stem face,  $(p - d')/p$

$d'$  = the diameter of bolt hole, bolt diameter + 1/8 in.

$$\alpha' = \frac{(B/T_u - 1)(a'/b')}{\delta[1 - (B/T_u - 1)(a'/b')]} \leq 1; \text{ (if } \alpha' \text{ is less than zero, use } \alpha' = 1)$$

$B$  = the design tensile strength of one bolt,  $\phi F_t A_b$  (kips) ( $\phi F_t$  is given in Table 48.11 and  $A_b$  is the nominal diameter of the bolt)

$$a' = a + d/2$$

$$b' = b - d/2$$

$a$  = the distance from the bolt centerline to the edge of the tee flange or angle leg, but not more than  $1.25b$  (in.)

$b$  = the distance from the bolt centerline to the face of the tee stem or outstanding leg (in.).

A design is considered satisfactory if the thickness of the tee flange or angle leg  $t_f$  exceeds  $t_{reqd}$  and  $B > T_u$ . Note that if  $t_f$  is much larger than  $t_{reqd}$ , the design will be too conservative. In this case,  $\alpha'$  should be recomputed using the equation

$$\alpha' = \frac{1}{\delta} \left[ \frac{4T_u b'}{\phi_b p t_f^2 F_y} - 1 \right] \quad (48.111)$$

As before, the value of  $\alpha'$  should be limited to the range  $0 \leq \alpha' \leq 1$ . This new value of  $\alpha'$  is to be used in Eq. (48.110) to recalculate  $t_{reqd}$ .

## Bolted Bracket-Type Connections

Figure 48.16 shows three commonly used bracket-type connections. The bracing connection shown in Fig. 48.16a should preferably be designed so that the line of action of the force will pass through the centroid of the bolt group. It is apparent that the bolts connecting the bracket to the column flange are subjected to combined tension and shear. As a result, the combined tensile–shear capacities of the bolts should be checked in accordance with Eq. (48.107) in ASD or Eq. (48.108) in LRFD. For simplicity,  $f_v$  and  $f_t$  are to be computed assuming that both the tensile and shear components of the force are distributed evenly to all bolts. In addition to checking for the bolt capacities, the bearing capacities of the column flange and the bracket should also be checked. If the axial component of the force is significant, the effect of prying should also be considered.

In the design of the eccentrically loaded connections shown in Fig. 48.16b, it is assumed that the neutral axis of the connection lies at the center of gravity of the bolt group. As a result, the bolts above the neutral axis will be subjected to combined tension and shear, so Eq. (48.107) or (48.108) needs to be checked. The bolts below the neutral axis are subjected to shear only, so Eq. (48.104) or (48.105) applies. In calculating  $f_v$ , one can assume that all bolts in the bolt group carry an equal share of the shear force. In calculating  $f_t$ , one can assume that the tensile force varies linearly from a value of zero at the neutral axis to a maximum value at the bolt farthest away from the neutral axis. Using this assumption,  $f_t$  can be calculated from the equation  $Pey/I$ , where  $y$  is the distance from the neutral axis to the location of the bolt above the neutral axis and  $I = \Sigma A_b y^2$  is the moment of inertia of the bolt areas, with  $A_b$  being the cross-sectional area of each bolt. The capacity of the connection is determined by the capacities of the bolts and the bearing capacity of the connected parts.

For the eccentrically loaded bracket connection shown in Fig. 48.16c, the bolts are subjected to shear. The shear force in each bolt can be obtained by adding vectorally the shear caused by the applied load  $P$  and the moment  $Px_0$ . The design of this type of connections is facilitated by the use of tables contained in the AISC-ASD and AISC-LRFD manuals [AISC, 1989, 2001].

In addition to checking for bolt shear capacity, one needs to check the bearing and shear rupture capacities of the bracket plate to ensure that failure will not occur in the plate.

## Bolted Shear Connections

Shear connections are connections designed to resist shear force only. They are used in type 2 or type 3 construction in ASD and in type PR construction in LRFD. These connections are not expected to provide appreciable moment restraint to the connection members. Examples of these connections are shown in Fig. 48.17. The framed beam connection shown in Fig. 48.17a consists of two web angles that are often shop-bolted to the beam web and then field-bolted to the column flange. The seated beam connection shown in Fig. 48.17b consists of two flange angles often shop-bolted to the beam flange and field-bolted to the column flange. To enhance the strength and stiffness of the seated beam connection, a stiffened seated beam connection, shown in Fig. 48.17c, is sometimes used to resist large shear force. Shear connections must be designed to sustain appreciable deformation, and yielding of the connections is expected. The need for ductility often limits the thickness of the angles that can be used. Most of these connections are designed with angle thicknesses not exceeding 5/8 in.

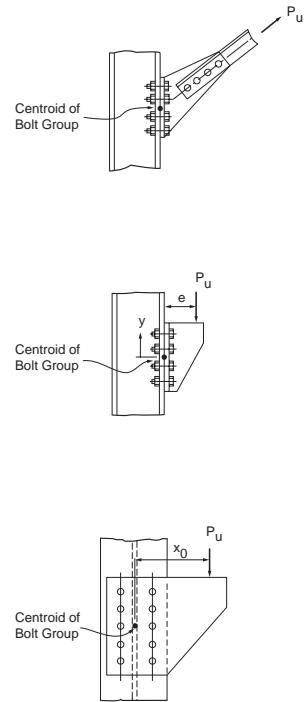
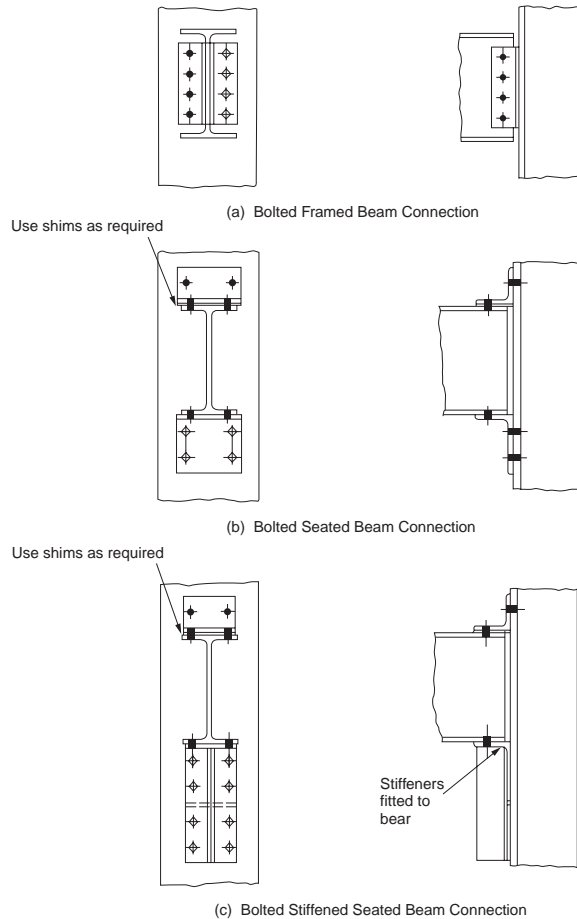


FIGURE 48.16 Bolted bracket-type connections.



**FIGURE 48.17** Bolted shear connections: (a) bolted framed beam connection, (b) bolted seated beam connection, (c) bolted stiffened seated beam connection.

The design of the connections shown in Fig. 48.17 is facilitated by the use of design tables contained in the AISC-ASD and AISC-LRFD manuals. These tables give design loads for the connections with specific dimensions based on the limit states of bolt shear, the bearing strength of the connection, the bolt bearing with different edge distances, and the block shear (for coped beams).

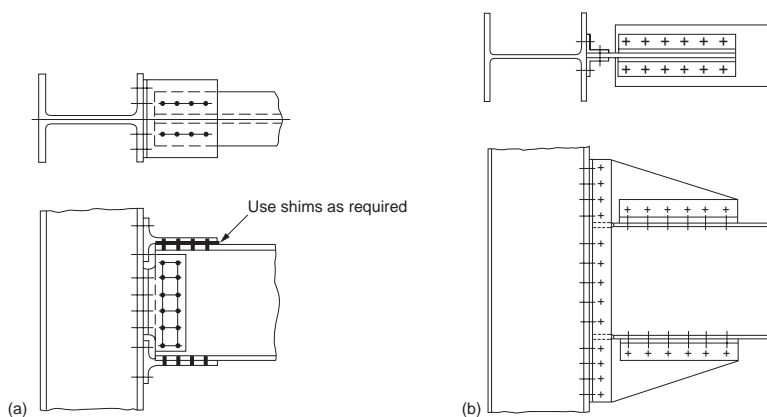
### Bolted Moment-Resisting Connections

Moment-resisting connections are connections designed to resist both moment and shear. They are used in type 1 construction in ASD and in type FR construction in LRFD. These connections are often referred to as rigid or fully restrained connections, as they provide full continuity between the connected members and are designed to carry the full factored moments. Figure 48.18 shows some examples of moment-resisting connections. Additional examples can be found in the AISC-ASD and AISC-LRFD manuals and in Chapter 4 of the AISC manual on connections [AISC, 1992].

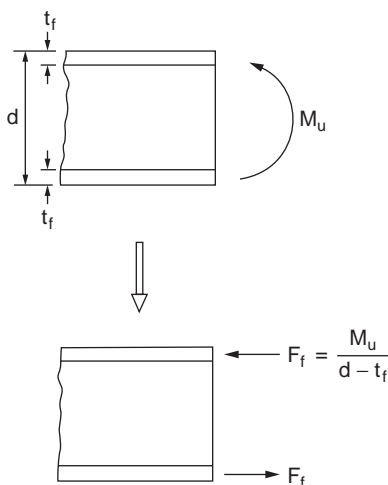
### Design of Moment-Resisting Connections

An assumption used quite often in the design of moment connections is that the moment is carried solely by the flanges of the beam. The moment is converted to a couple  $F_f$  given by  $F_f = M/(d - t_f)$  acting on the beam flanges, as shown in Fig. 48.19.





**FIGURE 48.18** Bolted moment connections.



**FIGURE 48.19** Flange forces in moment connections.

The design of the connection for moment is considered satisfactory if the capacities of the bolts and connecting plates or structural elements are adequate to carry the flange force  $F_f$ . Depending on the geometry of the bolted connection, this may involve checking: (1) the shear and tensile capacities of the bolts, (2) the yield and fracture strengths of the moment plate, (3) the bearing strength of the connected parts, and (4) the bolt spacing and edge distance, as discussed in the foregoing sections.

As for shear, it is common practice to assume that all the shear resistance is provided by the shear plates or angles. The design of the shear plates or angles is governed by the limit states of bolt shear, the bearing of the connected parts, and shear rupture.

If the moment to be resisted is large, the flange force may cause bending of the column flange or local yielding, crippling, or buckling of the column web. To prevent failure due to bending of the column flange or local yielding of the column web (for a tensile  $F_f$ ), as well as local yielding, crippling, or buckling of the column web (for a compressive  $F_f$ ), column stiffeners should be provided if any one of the conditions discussed in Section 48.5 under Criteria for Concentrated Loads is violated.

Following is a set of guidelines for the design of column web stiffeners [AISC, 1989, 2001]:

1. If local web yielding controls, the area of the stiffeners (provided in pairs) shall be determined based on any excess force beyond that which can be resisted by the web alone. The stiffeners need not extend more than one half the depth of the column web if the concentrated beam flange force  $F_f$  is applied at only one column flange.
2. If web crippling or compression buckling of the web controls, the stiffeners shall be designed as axially loaded compression members (see Section 48.4). The stiffeners shall extend the entire depth of the column web.
3. The welds that connect the stiffeners to the column shall be designed to develop the full strength of the stiffeners.

In addition, the following recommendations are given:

1. The width of the stiffener plus one half of the column web thickness should not be less than one half the width of the beam flange or the moment connection plate that applies the force.
2. The stiffener thickness should not be less than one half the thickness of the beam flange.
3. If only one flange of the column is connected by a moment connection, the length of the stiffener plate does not have to exceed one half the column depth.
4. If both flanges of the column are connected by moment connections, the stiffener plate should extend through the depth of the column web, and welds should be used to connect the stiffener plate to the column web with sufficient strength to carry the unbalanced moment on opposite sides of the column.
5. If column stiffeners are required on both the tension and compression sides of the beam, the size of the stiffeners on the tension side of the beam should be equal to that on the compression side for ease of construction.

In lieu of stiffener plates, a stronger column section should be used to preclude failure in the column flange and web.

For a more thorough discussion of bolted connections, see the book by Kulak et al. [1987]. Examples on the design of a variety of bolted connections can be found in the AISC-LRFD manual [AISC, 2001] and in the AISC manual on connections [AISC, 1992].

## Welded Connections

Welded connections are connections whose components are joined together primarily by welds. The four most commonly used welding processes are discussed in Section 48.1 under Structural Fasteners. Welds can be classified according to:

- the types of welds: groove, fillet, plug, and slot
- the positions of the welds: horizontal, vertical, overhead, and flat
- the types of joints: butt, lap, corner, edge, and tee

Although fillet welds are generally weaker than groove welds, they are used more often because they allow for larger tolerances during erection than groove welds. Plug and slot welds are expensive to make and do not provide much reliability in transmitting tensile forces perpendicular to the faying surfaces. Furthermore, quality control of such welds is difficult because inspection of the welds is rather arduous. As a result, plug and slot welds are normally used just for stitching different parts of the members together.

## Welding Symbols

A shorthand notation giving important information on the location, size, length, etc. for various types of welds was developed by the American Welding Society [AWS, 1987] to facilitate the detailing of welds. This system of notation is reproduced in [Fig. 48.20](#).

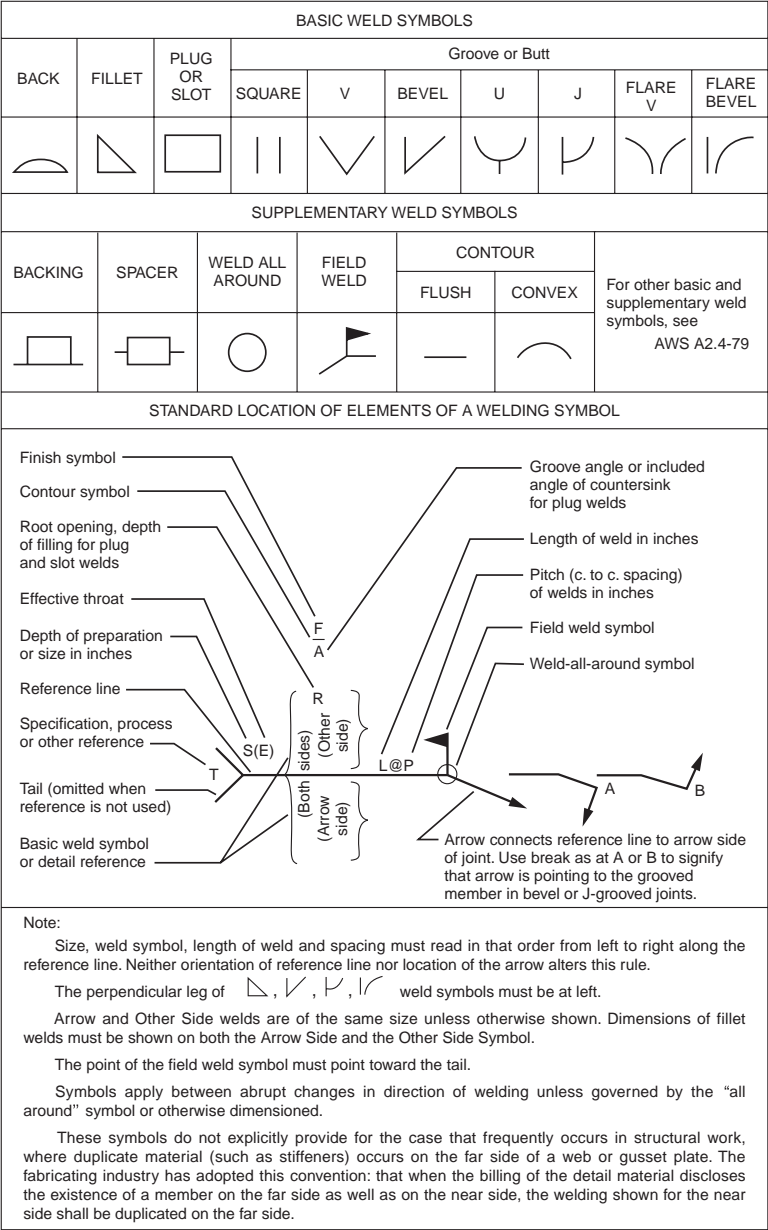


FIGURE 48.20 Basic weld symbols.

Strength of Welds

In ASD, the strength of welds is expressed in terms of allowable stress. In LRFD, the design strength of welds is taken as the smaller of the design strength of the base material  $\phi F_{BM}$  (expressed as a function of the yield stress of the material) and the design strength of the weld electrode  $\phi F_W$  (expressed as a function of the strength of the electrode  $F_{EXX}$ ). These allowable stresses and design strengths are summarized in Table 48.18 [AISC, 1989, 1999]. When using ASD, the computed stress in the weld shall not exceed its allowable value. When using LRFD, the design strength of welds should exceed the required strength, obtained by dividing the load to be transmitted by the effective area of the welds.

**TABLE 48.18** Strength of Welds

| Types of Weld and Stress <sup>a</sup>                        | Material               | ASD Allowable Stress   | LRFD $\phi F_{BM}$ or $\phi F_W$       | Required Weld Strength Level <sup>b,c</sup>   |
|--|------------------------|--|--|---|
| Full-Penetration Groove Weld                                 |                        |  |  |   |
| Tension normal to effective area                             | Base                   | Same as base metal   | $0.90F_y$                              | “Matching” weld must be used  |
| Compression normal to effective area                         | Base                   | Same as base metal   | $0.90F_y$                              | Weld metal with a strength level equal to or less than that of the “matching” weld metal must be used |
| Tension or compression parallel to axis of weld              | Base                   | Same as base metal   | $0.90F_y$                              |   |
| Shear on effective area                                      | Base<br>Weld electrode | $0.30 \times$ nominal tensile strength of weld metal   | $0.90[0.60F_y]$<br>$0.80[0.60F_{EXX}]$ |   |
| Partial-Penetration Groove Welds                             |                        |  |  |   |
| Compression normal to effective area                         | Base                   | Same as base metal   | $0.90F_y$                              | Weld metal with a strength level equal to or less than that of the “matching” weld metal may be used  |
| Tension or compression parallel to axis of weld <sup>d</sup> | Base                   |  |  |   |
| Shear parallel to axis of weld                               | Base<br>Weld electrode | $0.30 \times$ nominal tensile strength of weld metal   | $0.75[0.60F_{EXX}]$                    |   |
| Tension normal to effective area                             | Base<br>Weld electrode | $0.30 \times$ nominal tensile strength of weld metal $\leq 0.18 \times$ yield stress of base metal | $0.90F_y$<br>$0.80[0.60F_{EXX}]$       |   |
| Fillet Welds   |                        |  |  |   |
| Stress on effective area                                     | Base<br>Weld electrode | $0.30 \times$ nominal tensile strength of weld metal   | $0.75[0.60F_{EXX}]$                    | Weld metal with a strength level equal to or less than that of “matching” weld metal may be used      |
| Tension or compression parallel to axis of weld <sup>d</sup> | Base                   | Same as base metal   | $0.90F_y$                              |   |
| Plug or Slot Welds   |                        |  |  |   |
| Shear parallel to faying surfaces (on effective area)        | Base<br>Weld electrode | $0.30 \times$ nominal tensile strength of weld metal   | $0.75[0.60F_{EXX}]$                    | Weld metal with a strength level equal to or less than that of “matching” weld metal may be used      |

<sup>a</sup> See below for effective area.

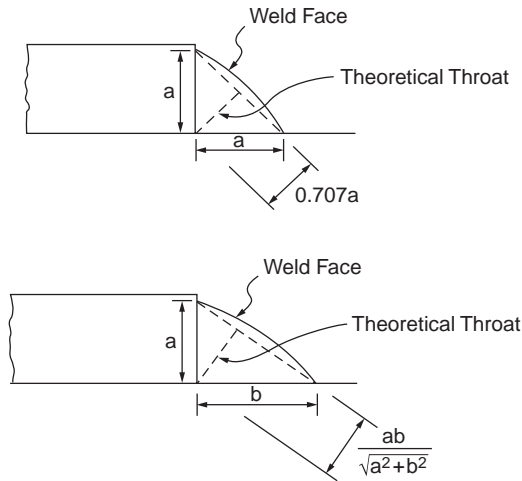
<sup>b</sup> See AWS D1.1 for “matching” weld material.

<sup>c</sup> Weld metal one strength level stronger than “matching” weld metal will be permitted.

<sup>d</sup> Fillet welds and partial-penetration groove welds joining component elements of built-up members, such as flange-to-web connections, may be designed without regard to the tensile or compressive stress in these elements parallel to the axis of the welds.

## Effective Area of Welds

The effective area of groove welds is equal to the product of the width of the part joined and the effective throat thickness. The effective throat thickness of a full-penetration groove weld is taken as the thickness of the thinner part joined. The effective throat thickness of a partial-penetration groove weld is taken as



**FIGURE 48.21** Effective throat of fillet welds.

the depth of the chamfer for J, U, bevel, or V (with bevel  $\geq 60^\circ$ ) joints, and it is taken as the depth of the chamfer minus 1/8 in. (3 mm) for bevel or V joints if the bevel is between  $45^\circ$  and  $60^\circ$ . For flare bevel-groove welds the effective throat thickness is taken as  $5R/16$ , and for flare V-groove welds the effective throat thickness is taken as  $R/2$  (or  $3R/8$  for the GMAW process when  $R \geq 1$  in.).  $R$  is the radius of the bar or bend.

The effective area of fillet welds is equal to the product of the length of the fillets, including returns, and the effective throat thickness. The effective throat thickness of a fillet weld is the shortest distance from the root of the joint to the face of the diagrammatic weld, as shown in Fig. 48.21. Thus, for an equal leg fillet weld, the effective throat is given by 0.707 times the leg dimension. For a fillet weld made by the SAW process, the effective throat thickness is taken as the leg size (for 3/8-in. or 9.5-mm and smaller fillet welds) or as the theoretical throat plus 0.11 in. or 3 mm (for fillet welds over 3/8 in. or 9.5 mm). A larger value for the effective throat thickness is permitted for welds made by the SAW process to account for the inherently superior quality of such welds.

The effective area of plug and slot welds is taken as the nominal cross-sectional area of the hole or slot in the plane of the faying surface.

### Size and Length Limitations of Welds

To ensure effectiveness, certain size and length limitations are imposed for welds. For partial-penetration groove welds, minimum values for the effective throat thickness are given in Table 48.19.

**TABLE 48.19** Minimum Effective Throat Thickness for Partial-Penetration Groove Welds (in.)

| Thickness of the Thicker Part Joined, $t$ | Minimum Effective Throat Thickness |
|---|------------------------------------|
| $t \leq 1/4$                              | 1/8                                |
| $1/4 < t \leq 1/2$                        | 3/16                               |
| $1/2 < t \leq 3/4$                        | 1/4                                |
| $3/4 < t \leq 1\frac{1}{2}$               | 5/16                               |
| $1\frac{1}{2} < t \leq 2\frac{1}{4}$      | 3/8                                |
| $2\frac{1}{4} < t \leq 6$                 | 1/2                                |
| $> 6$                                     | 5/8                                |

Note: 1 in. = 25.4 mm

For plug welds, the hole diameter shall not be less than the thickness of the part that contains the weld plus 5/16 in. (8 mm), rounded to the next larger odd 1/16 in. (or even mm), or greater than the minimum diameter plus 1/8 in. (3 mm) or 2.25 times the thickness of the weld. The center-to-center spacing of plug welds shall not be less than four times the hole diameter. The thickness of a plug weld in material less than 5/8 in. (16 mm) thick shall be equal to the thickness of the material. In material over 5/8 in. (16 mm) thick, the thickness of the weld shall be at least one half the thickness of the material, but not less than 5/8 in. (16 mm).

For slot welds, the slot length shall not exceed ten times the thickness of the weld. The slot width shall not be less than the thickness of the part that contains the weld plus 5/16 in. (8 mm), rounded to the nearest larger odd 1/16 in. (or even mm), or larger than 2.25 times the thickness of the weld. The spacing of lines of slot welds in a direction transverse to their length shall not be less than four times the width of the slot. The center-to-center spacing of two slot welds on any line in the longitudinal direction shall not be less than two times the length of the slot. The thickness of a slot weld in material less than 5/8 in. (16 mm) thick shall be equal to the thickness of the material. In material over 5/8 in. (16 mm) thick, the thickness of the weld shall be at least one half the thickness of the material, but not less than 5/8 in. (16 mm).

For fillet welds, the minimum leg size is given in [Table 48.20](#). The maximum leg size is equal to the thickness of the connected part along the edge of a connected part less than 1/4 in. (6 mm) thick. For thicker parts, the maximum leg size is  $t$  minus 1/16 in. (2 mm), where  $t$  is the thickness of the part. The minimum effective length of a fillet weld is four times its nominal size. If a shorter length is used, the leg size of the weld shall be taken as 1/4 in. (6 mm) its effective length for the purpose of stress computation. The effective length of end-loaded fillet welds with lengths up to 100 times the leg dimension can be set equal to the actual length. If the length exceeds 100 times the weld size, the effective length shall be taken as the actual length multiplied by a reduction factor, given by  $[1.2 - 0.002(L/w)] \leq 1.0$ , where  $L$  is the actual length of the end-loaded fillet weld and  $w$  is the leg size. The length of longitudinal fillet welds used alone for flat-bar tension members shall not be less than the perpendicular distance between the welds. The effective length of any segment of an intermittent fillet weld shall not be less than four times the weld size or 1½ in. (38 mm).

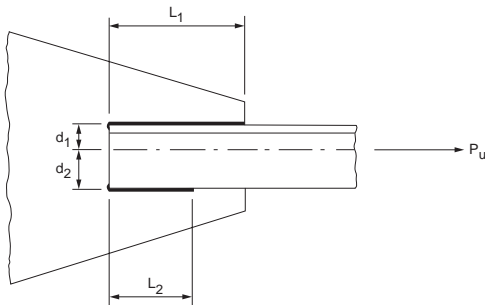
**TABLE 48.20** Minimum Leg Size of Fillet Welds (in.)

| Thickness of Thicker Part Joined, $t$ | Minimum Leg Size |
|---------------------------------------|------------------|
| $t \leq 1/4$                          | 1/8              |
| $1/4 < t \leq 1/2$                    | 3/16             |
| $1/2 < t \leq 3/4$                    | 1/4              |
| $t > 3/4$                             | 5/16             |

*Note:* 1 in. = 25.4 mm

### Welded Connections for Tension Members

[Figure 48.22](#) shows a tension angle member connected to a gusset plate by fillet welds. The applied tensile force  $P$  is assumed to act along the center of gravity of the angle. To avoid eccentricity, the lengths of the two fillet welds must be proportioned so that their resultant will act along the center of gravity of the angle. For example, if LRFD is used, the following equilibrium equations can be written:



**FIGURE 48.22** An eccentrically loaded welded tension connection.

Summing force along the axis of the angle:

$$(\phi F_M) t_{eff} L_1 + (\phi F_M) t_{eff} L_2 = P_u \quad (48.112)$$

Summing moment about the center of gravity of the angle:

$$(\phi F_M) t_{eff} L_1 d_1 = (\phi F_M) t_{eff} L_2 d_2 \quad (48.113)$$

where  $P_u$  = the factored axial force

$\phi F_M$  = the design strength of the welds as given in Table 48.18

$t_{eff}$  = the effective throat thickness

$L_1$  and  $L_2$  = the lengths of the welds

$d_1$  and  $d_2$  = the transverse distances from the center of gravity of the angle to the welds.

The two equations can be used to solve for  $L_1$  and  $L_2$ .

### Welded Bracket-Type Connections

A typical welded bracket connection is shown in Fig. 48.23. Because the load is eccentric with respect to the center of gravity of the weld group, the connection is subjected to both moment and shear. The welds must be designed to resist the combined effect of direct shear for the applied load and any additional shear from the induced moment. The design of the welded bracket connection is facilitated by the use of the design tables in the AISC-ASD and AISC-LRFD manuals. In both ASD and LRFD, the load capacity for the connection is given by

$$P = CC_1 D l \quad (48.114)$$

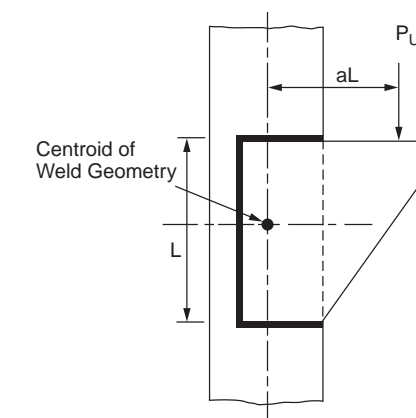


FIGURE 48.23 An eccentrically loaded welded bracket connection.

where  $P$  = the allowable load (in ASD) or factored load,  $P_u$  (in LRFD) (kips)

$l$  = the length of the vertical weld (in.)

$D$  = the number of sixteenths of an inch in the fillet weld size

$C$  = the coefficient tabulated in the AISC-ASD and AISC-LRFD manuals. In the tables, values of  $C$  for a variety of weld geometries and dimensions are given

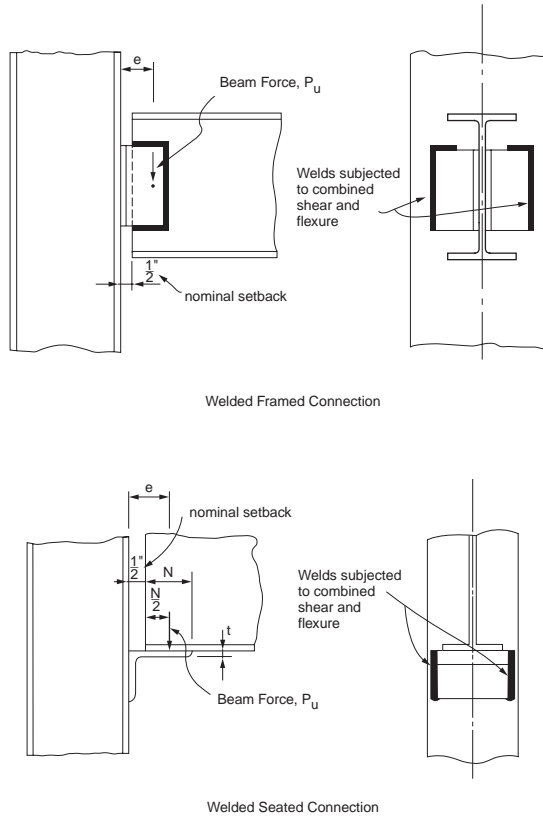
$C_1$  = the coefficient for electrode used (see the following table).

| Electrode |                 | E60   | E70 | E80  | E90  | E100 | E110 |
|-----------|-----------------|-------|-----|------|------|------|------|
| ASD       | $F_v$ (ksi)     | 18    | 21  | 24   | 27   | 30   | 33   |
|           | $C_1$           | 0.857 | 1.0 | 1.14 | 1.29 | 1.43 | 1.57 |
| LRFD      | $F_{EXX}$ (ksi) | 60    | 70  | 80   | 90   | 100  | 110  |
|           | $C_1$           | 0.857 | 1.0 | 1.03 | 1.16 | 1.21 | 1.34 |

### Welded Connections with Welds Subjected to Combined Shear and Flexure

Figure 48.24 shows a welded framed connection and a welded seated connection. The welds for these connections are subjected to combined shear and flexure. For the purpose of design, it is common practice to assume that the shear force per unit length,  $R_s$ , acting on the welds is a constant and is given by

$$R_s = \frac{P}{2l} \quad (48.115)$$



**FIGURE 48.24** Welds subjected to combined shear and flexure.

where  $P$  = the allowable load (in ASD) or factored load,  $P_u$  (in LRFD)  
 $l$  = the length of the vertical weld.

In addition to shear, the welds are subjected to flexure as a result of load eccentricity. There is no general agreement on how the flexure stress should be distributed on the welds. One approach is to assume that the stress distribution is linear, with half the weld subjected to tensile flexure stress and half subjected to compressive flexure stress. Based on this stress distribution and ignoring the returns, the flexure tension force per unit length of weld,  $R_p$  acting at the top of the weld can be written as

$$R_F = \frac{Mc}{I} = \frac{Pe(l/2)}{2l^3/12} = \frac{3Pe}{l^2} \quad (48.116)$$

where  $e$  is the load eccentricity.

The resultant force per unit length acting on the weld,  $R$ , is then

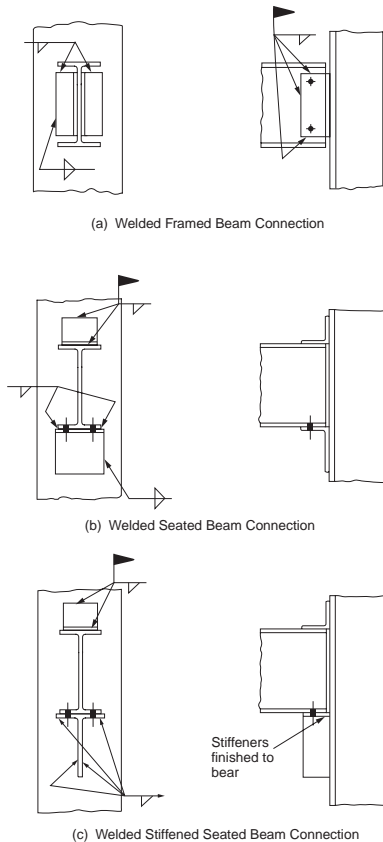
$$R = \sqrt{R_S^2 + R_F^2} \quad (48.117)$$

For a satisfactory design, the value  $R/t_{eff}$ , where  $t_{eff}$  is the effective throat thickness of the weld, should not exceed the allowable values or design strengths given in [Table 48.18](#).

### Welded Shear Connections

[Figure 48.25](#) shows three commonly used welded shear connections: a framed beam connection, a seated beam connection, and a stiffened seated beam connection. These connections can be designed by using





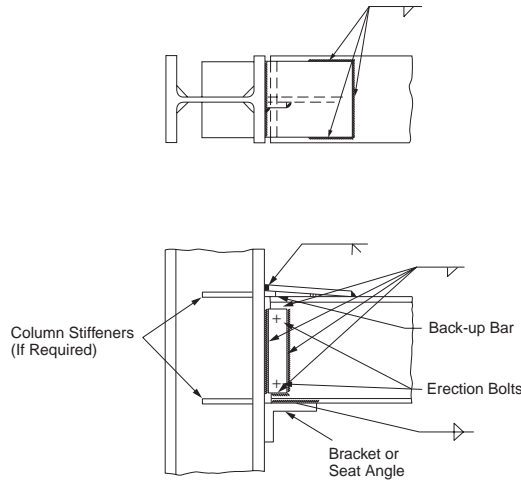
**FIGURE 48.25** Welded shear connections: (a) framed beam connection, (b) seated beam connection, (c) stiffened seated beam connection.

the information presented in the earlier sections on welds subjected to eccentric shear and welds subjected to combined tension and flexure. For example, the welds that connect the angles to the beam web in the framed beam connection can be considered eccentrically loaded welds, and Eq. (48.114) can be used for their design. The welds that connect the angles to the column flange can be considered welds subjected to combined tension and flexure, and Eq. (48.117) can be used for their design. Like bolted shear connections, welded shear connections are expected to exhibit appreciable ductility, so the use of angles with thicknesses in excess of 5/8 in. should be avoided. To prevent shear rupture failure, the shear rupture strength of the critically loaded connected parts should be checked.

To facilitate the design of these connections, the AISC-ASD and AISC-LRFD manuals provide design tables by which the weld capacities and shear rupture strengths for different connection dimensions can be checked readily.

### Welded Moment-Resisting Connections

Welded moment-resisting connections (Fig. 48.26), like bolted moment-resisting connections, must be designed to carry both moment and shear. To simplify the design procedure, it is customary to assume that the moment, to be represented by a couple  $F_f$ , as shown in Fig. 48.19, is to be carried by the beam flanges and that the shear is to be carried by the beam web. The connected parts (e.g., the moment plates, welds, etc.) are then designed to resist the forces  $F_f$  and shear. Depending on the geometry of the welded connection, this may include checking: (1) the yield or fracture strength of the moment plate, (2) the shear or tensile capacity of the welds, and (3) the shear rupture strength of the shear plate.



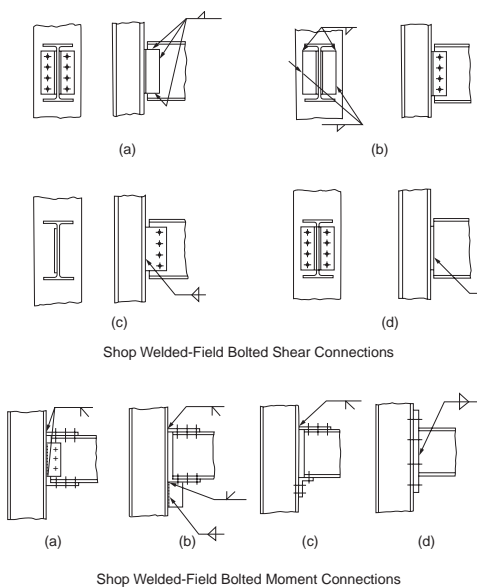
**FIGURE 48.26** Welded moment connections.

If the column to which the connection is attached is weak, the designer should consider the use of column stiffeners to prevent failure of the column flange and web due to bending, yielding, crippling, or buckling (see above, under Design of Moment-Resisting Connections).

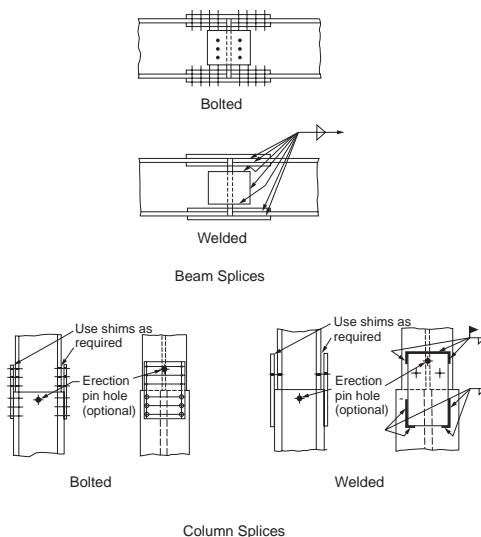
Examples of the design of a variety of welded shear and moment-resisting connections can be found in the AISC manual on connections [AISC, 1992] and in the AISC-LRFD manual [AISC, 2001].

## Shop-Welded and Field-Bolted Connections

A large percentage of connections used for construction are shop-welded and field-bolted types. These connections are usually more cost-effective than fully welded connections, and their strength and ductility characteristics often rival those of fully welded connections. Figure 48.27 shows some of these connections. The design of shop-welded and field-bolted connections is also covered in the AISC manual on connections and in the AISC-LRFD manual. In general, the following should be checked: (1) shear and



**FIGURE 48.27** Shop-welded and field-bolted connections.



**FIGURE 48.28** Bolted and welded beam and column splices.

tensile capacities of the bolts and welds, (2) bearing strength of the connected parts, (3) yield or fracture strength of the moment plate, and (4) shear rupture strength of the shear plate. Also, as for any other types of moment connections, column stiffeners shall be provided if any one of the criteria — column flange bending, local web yielding, crippling, and compression buckling of the column web — is violated.

## Beam and Column Splices

Beam and column splices (Fig. 48.28) are used to connect beam or column sections of different sizes. They are also used to connect beams or columns of the same size if the design calls for an extraordinarily long span. Splices should be designed for both moment and shear, unless the designer intends to utilize the splices as internal hinges. If splices are used for internal hinges, provisions must be made to ensure that the connections possess adequate ductility to allow for large hinge rotation.

Splice plates are designed according to their intended functions. Moment splices should be designed to resist the flange force  $F_f = M/(d - t_f)$  (Fig. 48.19) at the splice location. In particular, the following limit states need to be checked: yielding of the gross area of the plate, fracture of the net area of the plate (for bolted splices), bearing strengths of connected parts (for bolted splices), shear capacity of bolts (for bolted splices), and weld capacity (for welded splices). Shear splices should be designed to resist the shear forces acting at the locations of the splices. The limit states that need to be checked include the shear rupture of the splice plates, the shear capacity of bolts under an eccentric load (for bolted splices), the bearing capacity of the connected parts (for bolted splices), the shear capacity of bolts (for bolted splices), and the weld capacity under an eccentric load (for welded splices). Design examples of beam and column splices can be found in the AISC manual of connections [AISC, 1992] and in the AISC-LRFD manual [AISC, 2001].

## 48.12 Column Base Plates and Beam Bearing Plates (LRFD Approach)

### Column Base Plates

Column base plates are steel plates placed at the bottom of columns whose function is to transmit column loads to the concrete pedestal. The design of a column base plate involves two major steps: (1) determining the size  $N \times B$  of the plate, and (2) determining the thickness  $t_p$  of the plate. Generally, the size of the

plate is determined based on the limit state of bearing on concrete, and the thickness of the plate is determined based on the limit state of plastic bending of critical sections in the plate. Depending on the types of forces (axial force, bending moment, shear force) the plate will be subjected to, the design procedures differ slightly. In all cases, a layer of grout should be placed between the base plate and its support for the purpose of leveling, and anchor bolts should be provided to stabilize the column during erection or to prevent uplift for cases involving a large bending moment.

### Axially Loaded Base Plates

Base plates supporting concentrically loaded columns in frames in which the column bases are assumed pinned are designed with the assumption that the column factored load  $P_u$  is distributed uniformly to the area of concrete under the base plate. The size of the base plate is determined from the limit state of bearing on concrete. The design bearing strength of concrete is given by

$$\phi_c P_p = 0.60 \left[ 0.85 f'_c A_1 \sqrt{\frac{A_2}{A_1}} \right] \quad (48.118)$$

where  $f'_c$  = the compressive strength of concrete

$A_1$  = the area of the base plate

$A_2$  = the area of concrete pedestal that is geometrically similar to and concentric with the loaded area,  $A_1 \leq A_2 \leq 4A_1$

From Eq. (48.118), it can be seen that the bearing capacity increases when the concrete area is greater than the plate area. This accounts for the beneficial effect of confinement. The upper limit of the bearing strength is obtained when  $A_2 = 4A_1$ . Presumably, the concrete area in excess of  $4A_1$  is not effective in resisting the load transferred through the base plate.

Setting the column factored load,  $P_u$ , equal to the bearing capacity of the concrete pedestal,  $\phi_c P_p$ , and solving for  $A_1$  from Eq. (48.118), we have

$$A_1 = \frac{1}{A_2} \left[ \frac{P_u}{0.6(0.85 f'_c)} \right]^2 \quad (48.119)$$

The length,  $N$ , and width,  $B$ , of the plate should be established so that  $N \times B > A_1$ . For an efficient design, the length can be determined from the equation

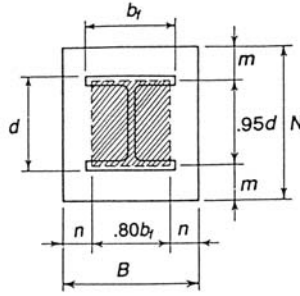
$$N \approx \sqrt{A_1} + 0.50(0.95d - 0.80b_f) \quad (48.120)$$

where  $0.95d$  and  $0.80b_f$  define the so-called effective load-bearing area shown crosshatched in Fig. 48.29a. Once  $N$  is obtained,  $B$  can be solved from the equation

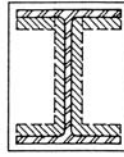
$$B = \frac{A_1}{N} \quad (48.121)$$

Both  $N$  and  $B$  should be rounded up to the nearest full inches.

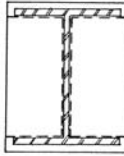
The required plate thickness,  $t_{req'd}$ , is to be determined from the limit state of yield line formation along the most severely stressed sections. A yield line develops when the cross section moment capacity is equal to its plastic moment capacity. Depending on the size of the column relative to the plate and the magnitude of the factored axial load, yield lines can form in various patterns on the plate. Figure 48.29 shows three models of plate failure in axially loaded plates. If the plate is large compared to the column, yield lines are assumed to form around the perimeter of the effective load-bearing area (the crosshatched area), as shown in Fig. 48.29a. If the plate is small and the column factored load is light, yield lines are



(a) Plate with Large  $m, n$



(b) Lightly Loaded Plate with Small  $m, n$



(c) Heavily Loaded Plate with Small  $m, n$

**FIGURE 48.29** Failure models for centrally loaded column base plates.

assumed to form around the inner perimeter of the I-shaped area, as shown in Fig. 48.29b. If the plate is small and the column factored load is heavy, yield lines are assumed to form around the inner edge of the column flanges and both sides of the column web, as shown in Fig. 48.29c. The following equation can be used to calculate the required plate thickness

$$t_{reqd} = l \sqrt{\frac{2 P_u}{0.90 F_y B N}} \quad (48.122a)$$

where  $l$  is the larger of  $m$ ,  $n$ , and  $\lambda n'$  given by

$$m = \frac{(N - 0.95 d)}{2} \quad (48.122b)$$

$$n = \frac{(B - 0.80 b_f)}{2} \quad (48.122c)$$

$$n' = \frac{\sqrt{d b_f}}{4} \quad (48.122d)$$

and

$$\lambda = \frac{2\sqrt{X}}{1 + \sqrt{1-X}} \leq 1 \quad (48.122e)$$

in which

$$X = \left( \frac{4d b_f}{(d + b_f)^2} \right) \frac{P_u}{\phi_c P_p} \quad (48.122f)$$

### Base Plates for Tubular and Pipe Columns

The design concept for base plates discussed above for I-shaped sections can be applied to the design of base plates for rectangular tubes and circular pipes. The critical section used to determine the plate thickness should be based on 0.95 times the outside column dimension for rectangular tubes and 0.80 times the outside dimension for circular pipes [Dewolf and Ricker, 1990].

### Base Plates with Moments

For columns in frames designed to carry moments at the base, base plates must be designed to support both axial forces and bending moments. If the moment is small compared to the axial force, the base plate can be designed without consideration of the tensile force that may develop in the anchor bolts. However, if the moment is large, this effect should be considered. To quantify the relative magnitude of this moment, an eccentricity  $e = M_u/P_u$  is used. The general procedures for the design of base plates for different values of  $e$  are given below [Dewolf and Ricker, 1990].

#### Small Eccentricity

If  $e$  is small ( $e \leq N/6$ ), the bearing stress is assumed to distribute linearly over the entire area of the base plate (Fig. 48.30). The maximum bearing stress is given by

$$f_{\max} = \frac{P_u}{BN} + \frac{M_u c}{I} \quad (48.123)$$

where  $c = N/2$  and  $I = BN^3/12$ .

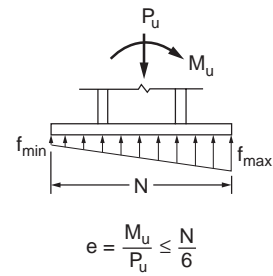
The size of the plate is to be determined by a trial-and-error process. The size of the base plate should be such that the bearing stress calculated using Eq. (48.123) does not exceed  $\phi_c P_p/A_1$ , i.e.,

$$0.60 \left[ 0.85 f'_c \sqrt{\frac{A_2}{A_1}} \right] \leq 0.60 [1.7 f'_c] \quad (48.124)$$

The thickness of the plate is to be determined from

$$t_p = \sqrt{\frac{4M_{plu}}{0.90 F_y}} \quad (48.125)$$

where  $M_{plu}$  is the moment per unit width of critical section in the plate.  $M_{plu}$  is to be determined by assuming that the portion of the plate projecting beyond the critical section acts as an inverted cantilever loaded by the bearing pressure. The moment calculated at the critical section divided by the length of the critical section (i.e.,  $B$ ) gives  $M_{plu}$ .



**FIGURE 48.30** Eccentrically loaded column base plate (small load eccentricity).

### Moderate Eccentricity

For plates subjected to moderate moments ( $N/6 < e \leq N/2$ ), only a portion of the plate will be subjected to bearing stress (Fig. 48.31). Ignoring the tensile force in the anchor bolt in the region of the plate where no bearing occurs and denoting  $A$  as the length of the plate in bearing, the maximum bearing stress can be calculated from force equilibrium consideration as

$$f_{\max} = \frac{2P_u}{AB} \quad (48.126)$$

where  $A = 3(N/2 - e)$  is determined from moment equilibrium. The plate should be proportioned such that  $f_{\max}$  does not exceed the value calculated using Eq. (48.124).  $t_p$  is to be determined from Eq. (48.125).

### Large Eccentricity

For plates subjected to large bending moments so that  $e > N/2$ , one needs to take into consideration the tensile force that develops in the anchor bolts (Fig. 48.32). Denoting  $T$  as the resultant force in the anchor bolts,  $A$  as the depth of the compressive stress block, and  $N'$  as the distance from the line of action of the tensile force to the extreme compression edge of the plate, force equilibrium requires that

$$T + P_u = \frac{f_{\max} AB}{2} \quad (48.127)$$

and moment equilibrium requires that

$$P_u \left( N' - \frac{N}{2} \right) + M = \frac{f_{\max} AB}{2} \left( N' - \frac{A}{3} \right) \quad (48.128)$$

The above equations can be used to solve for  $A$  and  $T$ . The size of the plate is determined using a trial-and-error process. The size should be chosen such that  $f_{\max}$  does not exceed the value calculated using Eq. (48.124),  $A$  should be smaller than  $N'$ , and  $T$  should not exceed the tensile capacity of the bolts.

Once the size of the plate is determined, the plate thickness  $t_p$  is calculated using Eq. (48.125). Note that there are two critical sections on the plate, one on the compression side of the plate and the other on the tension side of the plate. Two values of  $M_{plu}$  are to be calculated, and the larger value should be used to calculate  $t_p$ .

### Base Plates with Shear

Under normal circumstances, the factored column base shear is adequately resisted by the frictional force developed between the plate and its support. Additional shear capacity is also provided by the anchor bolts. For cases in which an exceptionally high shear force is expected, such as in a bracing connection, or in which uplift occurs that reduces the frictional resistance, the use of shear lugs may be necessary. Shear lugs can be designed based on the limit states of bearing on concrete and bending of the lugs. The size of the lug should be proportioned such that the bearing stress on concrete does not exceed  $0.60(0.85f'_c)$ . The thickness of the lug can be determined from Eq. (48.125).  $M_{plu}$  is the moment per unit width at the critical section of the lug. The critical section is taken to be at the junction of the lug and the plate (Fig. 48.33).

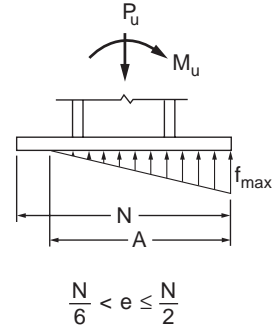


FIGURE 48.31 Eccentrically loaded column base plate (moderate load eccentricity).

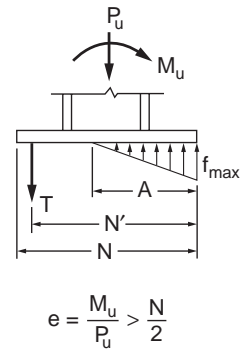
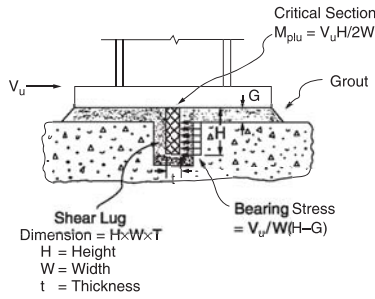
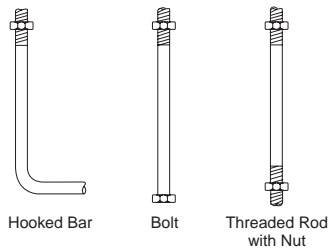


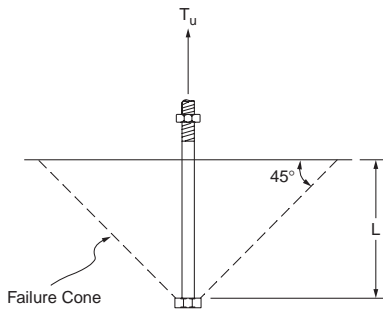
FIGURE 48.32 Eccentrically loaded column base plate (large load eccentricity).



**FIGURE 48.33** Column base plate subjected to shear.



**FIGURE 48.34** Base plate anchors.



**FIGURE 48.35** Cone pullout failure.

## Anchor Bolts

Anchor bolts are provided to stabilize the column during erection and to prevent uplift for cases involving large moments. Anchor bolts can be cast-in-place bolts or drilled-in bolts. The latter are placed after the concrete is set and are not often used. Their design is governed by the manufacturer's specifications. Cast-in-place bolts are hooked bars, bolts, or threaded rods with nuts ([Fig. 48.34](#)) placed before the concrete is set. Anchor rods and threaded rods shall conform to one of the following ASTM specifications: A36/A36M, A193/A193M, A354, A572/A572M, A588/A588M, or F1554.

Of the three types of cast-in-place anchors shown in the figure, the hooked bars are recommended for use only in axially loaded base plates. They are not normally relied upon to carry significant tensile force. Bolts and threaded rods with nuts can be used for both axially loaded base plates or base plates with moments. Threaded rods with nuts are used when the length and size required for the specific design exceed those of standard-size bolts.

Failure of bolts or threaded rods with nuts occurs when the tensile capacities of the bolts are reached. Failure is also considered to occur when a cone of concrete is pulled out from the pedestal. This cone pullout type of failure is depicted schematically in [Fig. 48.35](#). The failure cone is assumed to radiate out from the bolt head or nut at an angle of  $45^\circ$ , with tensile failure occurring along the surface of the cone



at an average stress of  $4\sqrt{f'_c}$ , where  $f'_c$  is the compressive strength of concrete in psi. The load that will cause this cone pullout failure is given by the product of this average stress and the projected area of the cone  $A_p$  [Marsh and Burdette, 1985]. The design of anchor bolts is thus governed by the limit states of tensile fracture of the anchors and cone pullout.

### Limit State of Tensile Fracture

The area of the anchor should be such that

$$A_g \geq \frac{T_u}{\phi_t 0.75 F_u} \quad (48.129)$$

where  $A_g$  = the required gross area of the anchor  
 $F_u$  = the minimum specified tensile strength  
 $\phi_t$  = 0.75 is the resistance factor for tensile fracture

### Limit State of Cone Pullout

From Fig. 48.35, it is clear that the size of the cone is a function of the length of the anchor. Provided that there are sufficient edge distance and spacing between adjacent anchors, the amount of tensile force required to cause cone pullout failure increases with the embedded length of the anchor. This concept can be used to determine the required embedded length of the anchor. Assuming that the failure cone does not intersect with another failure cone or the edge of the pedestal, the required embedded length can be calculated from the equation

$$L \geq \sqrt{\frac{A_p}{\pi}} = \sqrt{\frac{(T_u / \phi_t) 4 \sqrt{f'_c}}{\pi}} \quad (48.130)$$

where  $A_p$  is the projected area of the failure cone,  $T_u$  is the required bolt force in pounds,  $f'_c$  is the compressive strength of concrete in psi, and  $\phi_t$  is the resistance factor (assumed to be equal to 0.75). If failure cones from adjacent anchors overlap one another or intersect with the pedestal edge, the projected area  $A_p$  must be adjusted accordingly (see, for example, Marsh and Burdette [1985]).

The length calculated using the above equation should not be less than the recommended values given by Shipp and Haninger [1983]. These values are reproduced in the following table. Also shown in the table are the recommended minimum edge distances for the anchors.

| Bolt Type<br>(Material) | Minimum<br>Embedded Length | Minimum<br>Edge Distance |
|-------------------------|----------------------------|--------------------------|
| A307 (A36)              | 12d                        | 5d > 4 in.               |
| A325 (A449)             | 17d                        | 7d > 4 in.               |

Note:  $d$  = nominal diameter of the anchor.

## Beam Bearing Plates

Beam bearing plates are provided between main girders and concrete pedestals to distribute the girder reactions to the concrete supports (Fig. 48.36). Beam bearing plates may also be provided between cross beams and girders if the cross beams are designed to sit on the girders.

Beam bearing plates are designed based on the limit states of web yielding, web crippling, bearing on concrete, and plastic bending of the plate. The dimension of the plate along the beam axis, i.e.,  $N$ , is determined from the web yielding or web crippling criterion (see Section 48.5 under Criteria for Concentrated Loads), whichever is more critical. The dimension  $B$  of the plate is determined from Eq. (48.121) with  $A_1$  calculated using Eq. (48.119).  $P_u$  in Eq. (48.119) is to be replaced by  $R_u$ , the factored reaction at the girder support.

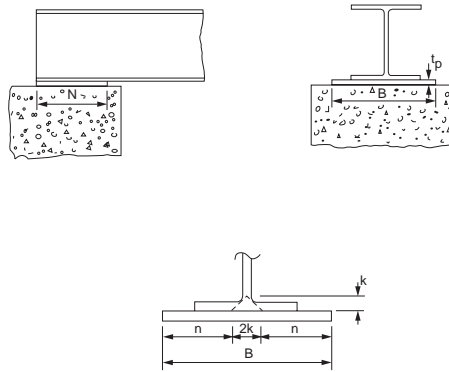


FIGURE 48.36 Beam bearing plate.

Once the size  $B \times N$  is determined, the plate thickness,  $t_p$ , can be calculated using the equation

$$t_p = \sqrt{\frac{2 R_u n^2}{0.90 F_y B N}} \quad (48.131)$$

where  $R_u$  = the factored girder reaction

$F_y$  = the yield stress of the plate

$n = (B - 2k)/2$ , in which  $k$  is the distance from the web toe of the fillet to the outer surface of the flange

The above equation was developed based on the assumption that the critical sections for plastic bending in the plate occur at a distance  $k$  from the centerline of the web.

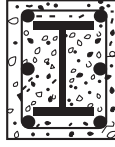
## 48.13 Composite Members (LRFD Approach)

*Composite members* are structural members made from two or more materials. The majority of composite sections used for building constructions are made from steel and concrete, although in recent years the use of fiber-reinforced polymer (FRP) has been on the rise, especially in the area of structural rehabilitation. Composite sections made from steel and concrete utilize the strength provided by steel and the rigidity provided by concrete. The combination of the two materials often results in efficient load-carrying members. Composite members may be concrete encased or concrete filled. For concrete-encased members (Fig. 48.37a), concrete is cast around steel shapes. In addition to enhancing strength and providing rigidity to the steel shapes, the concrete acts as a fireproofing material to the steel shapes. It also serves as a corrosion barrier, shielding the steel from corroding under adverse environmental conditions. For concrete-filled members (Fig. 48.37b), structural steel tubes are filled with concrete. In both concrete-encased and concrete-filled sections, the rigidity of the concrete often eliminates the problem of local buckling experienced by some slender elements of the steel sections.

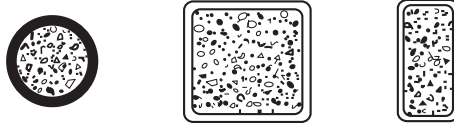
Some disadvantages associated with composite sections are that concrete creeps and shrinks. Furthermore, uncertainties with regard to the mechanical bond developed between the steel shape and the concrete often complicate the design of beam-column joints.

### Composite Columns

According to the LRFD specification [AISC, 1999], a compression member is regarded as a composite column if:



(A) Concrete Encased Composite Section



(B) Concrete Filled Composite Sections

FIGURE 48.37 Composite columns.

1. the cross-sectional area of the steel section is at least 4% of the total composite area. If this condition is not satisfied, the member should be designed as a reinforced concrete column.
2. longitudinal reinforcements and lateral ties are provided for concrete-encased members. The cross-sectional area of the reinforcing bars shall be  $0.007 \text{ in.}^2/\text{in.}$  ( $180 \text{ mm}^2/\text{m}$ ) of bar spacing. To avoid spalling, lateral ties shall be placed at a spacing not greater than two thirds the least dimension of the composite cross section. For fire and corrosion resistance, a minimum clear cover of 1.5 in. (38 mm) shall be provided.
3. the compressive strength of concrete  $f'_c$  used for the composite section falls within the range of 3 ksi (21 MPa) to 8 ksi (55 MPa) for normal weight concrete and not less than 4 ksi (28 MPa) for lightweight concrete. These limits are set because they represent the range of test data available for the development of the design equations.
4. the specified minimum yield stress for the steel sections and reinforcing bars used in calculating the strength of the composite columns does not exceed 60 ksi (415 MPa). This limit is set because this stress corresponds to a strain below which the concrete remains unspalled and stable.
5. the minimum wall thickness of the steel sections for concrete-filled members is equal to  $b\sqrt{(F_y/3E)}$  for rectangular sections of width  $b$  and  $D\sqrt{(F_y/8E)}$  for circular sections of outside diameter  $D$ .

### Design Compressive Strength

The design compressive strength,  $\phi_c P_n$ , shall exceed the factored compressive force,  $P_u$ . The design compressive strength for  $\phi_c \leq 1.5$  is given as

$$\phi_c P_n = \begin{cases} 0.85 \left[ \left( 0.658^{\lambda_c^2} \right) A_s F_{my} \right], & \text{if } \lambda_c \leq 1.5 \\ 0.85 \left[ \left( \frac{0.877}{\lambda_c^2} \right) A_s F_{my} \right], & \text{if } \lambda_c > 1.5 \end{cases} \quad (48.132)$$

where

$$\lambda_c = \frac{KL}{r_m \pi} \sqrt{\frac{F_{my}}{E_m}} \quad (48.133)$$

$$F_{my} = F_y + c_1 F_{yr} \left( \frac{A_r}{A_s} \right) + c_2 f'_c \left( \frac{A_c}{A_s} \right) \quad (48.134)$$

$$E_m = E + c_3 E_c \left( \frac{A_c}{A_s} \right) \quad (48.135)$$

$r_m$  = the radius of gyration of steel section and shall not be less than 0.3 times the overall thickness of the composite cross section in the plane of buckling

$A_c$  = the area of concrete

$A_r$  = the area of longitudinal reinforcing bars

$A_s$  = the area of the steel shape

$E$  = the modulus of elasticity of steel

$E_c$  = the modulus of elasticity of concrete

$F_y$  = the specified minimum yield stress of the steel shape

$F_{yr}$  = the specified minimum yield stress of longitudinal reinforcing bars

$f'_c$  = the specified compressive strength of concrete

$c_1$ ,  $c_2$ , and  $c_3$  = the coefficients given in the table below.

| Type of Composite Section         | $c_1$ | $c_2$ | $c_3$ |
|-----------------------------------|-------|-------|-------|
| Concrete-encased shapes           | 0.7   | 0.6   | 0.2   |
| Concrete-filled pipes and tubings | 1.0   | 0.85  | 0.4   |

In addition to satisfying the condition  $\phi_c P_n \geq P_u$ , shear connectors spaced no more than 16 in. (405 mm) apart on at least two faces of the steel section in a symmetric pattern about the axes of the steel section shall be provided for concrete-encased composite columns to transfer the interface shear force  $V'_u$  between steel and concrete.  $V'_u$  is given by

$$V'_u = \begin{cases} V_u \left( 1 - \frac{A_s F_y}{P_n} \right) & \text{when the force is applied to the steel section} \\ V_u \left( \frac{A_s F_y}{P_n} \right) & \text{when the force is applied to the concrete encasement} \end{cases} \quad (48.136)$$

where  $V_u$  = the axial force in the column

$A_s$  = the area of steel section

$F_y$  = the yield strength of the steel section

$P_n$  = the nominal compressive strength of the composite column without consideration of slenderness effect

If the supporting concrete area in direct bearing is larger than the loaded area, the bearing condition for concrete must also be satisfied. Denoting  $\phi_c P_{nc}$  ( $=\phi_c P_{n, \text{composite section}} - \phi_c P_{n, \text{steel shape alone}}$ ) as the portion of compressive strength resisted by the concrete and  $A_B$  as the loaded area, the condition that needs to be satisfied is

$$\phi_c P_{nc} \leq 0.65 [1.7 f'_c A_B] \quad (48.137)$$

## Composite Beams

Composite beams used in construction can often be found in two forms: steel beams connected to a concrete slab by shear connectors, and concrete-encased steel beams.

### Steel Beams with Shear Connectors

The design flexure strength for steel beams with shear connectors is  $\phi_b M_n$ . The resistance factor  $\phi_b$  and nominal moment  $M_n$  are determined as follows:

| Condition   | $\phi_b$ | $M_n$  |
|---|----------|--|
| Positive moment region and $h/t_w \leq 3.76\sqrt{E/F_{yf}}$ | 0.85     | Determined from plastic stress distribution on the composite section             |
| Positive moment region and $h/t_w > 3.76\sqrt{E/F_{yf}}$    | 0.90     | Determined from elastic stress superposition considering the effects of shoring  |
| Negative moment region                                      | 0.90     | Determined for the steel section alone using equations presented in Section 49.5 |

## Concrete-Encased Steel Beams

For steel beams fully encased in concrete, no additional anchorage for shear transfer is required if (1) at least 1½ in. (38 mm) of concrete cover is provided on top of the beam and at least 2 in. (51 mm) of cover is provided over the sides and at the bottom of the beam, and (2) spalling of concrete is prevented by adequate mesh or other reinforcing steel. The design flexural strength  $\phi_b M_n$  can be computed using either an elastic or plastic analysis.

If an elastic analysis is used,  $\phi_b$  shall be taken as 0.90. A linear strain distribution is assumed for the cross section, with zero strain at the neutral axis and maximum strains at the extreme fibers. The stresses are then computed by multiplying the strains by  $E$  (for steel) or  $E_c$  (for concrete). Maximum stress in steel shall be limited to  $F_y$ , and maximum stress in concrete shall be limited to  $0.85f'_c$ . Tensile strength of concrete shall be neglected.  $M_n$  is to be calculated by integrating the resulting stress block about the neutral axis.

If a plastic analysis is used,  $\phi_b$  shall be taken as 0.90 and  $M_n$  shall be assumed to be equal to  $M_p$ , the plastic moment capacity of the steel section alone.

## Composite Beam-Columns

Composite beam-columns shall be designed to satisfy the interaction of Eq. (48.73) or (48.74), whichever is applicable, with  $\phi_c P_n$  calculated based on Eqs. (48.132) to (48.135),  $P_e$  calculated using  $P_e = A_s F_{my} / \lambda_c^2$ , and  $\phi_b M_n$  calculated using the following equation [Galambos and Chapuis, 1980]:

$$\phi_b M_n = 0.90 \left[ Z F_y + \frac{1}{3} (h_2 - 2c_r) A_r F_{yr} + \left( \frac{h_2}{2} - \frac{A_w F_y}{1.7 f'_c h_1} \right) A_w F_y \right] \quad (48.138)$$

where  $Z$  = the plastic section modulus of the steel section

$c_r$  = the average of the distance measured from the compression face to the longitudinal reinforcement in that face, and the distance measured from the tension face to the longitudinal reinforcement in that face

$h_1$  = the width of the composite section perpendicular to the plane of bending

$h_2$  = the width of the composite section parallel to the plane of bending

$A_r$  = the cross-sectional area of longitudinal reinforcing bars

$A_w$  = the web area of the encased steel shape (=0 for concrete-filled tubes)

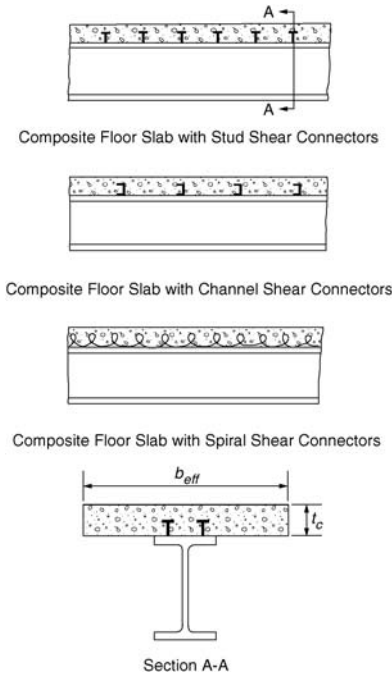
$F_y$  = the yield stress of the steel section

$F_{yr}$  = the yield stress of reinforcing bars

If  $0 < (P_u / \phi_c P_n) \leq 0.3$ , a linear interpolation of  $\phi_b M_n$ , calculated using the above equation and assuming that  $P_u / \phi_c P_n = 0.3$ , and that calculated for beams with  $P_u / \phi_c P_n = 0$  (see above) should be used.

## Composite Floor Slabs

Composite floor slabs (Fig. 48.38) can be designed as shored or unshored. In shored construction, temporary shores are used during construction to support the dead and accidental live loads until the concrete cures. The supporting beams are designed on the basis of their ability to develop composite action to support all factored loads after the concrete cures. In unshored construction, temporary shores



**FIGURE 48.38** Composite floor slabs.

are not used. As a result, the steel beams alone must be designed to support the dead and accidental live loads before the concrete has attained 75% of its specified strength. After the concrete is cured, the composite section should have adequate strength to support all factored loads.

Composite action for the composite floor slabs shown in Fig. 48.38 is developed as a result of the presence of shear connectors. If sufficient shear connectors are provided so that the maximum flexural strength of the composite section can be developed, the section is referred to as fully composite. Otherwise, the section is referred to as partially composite. The flexural strength of a partially composite section is governed by the shear strength of the shear connectors. The horizontal shear force  $V_h$ , which should be designed for at the interface of the steel beam and the concrete slab, is given by:

*In regions of positive moment:*

$$V_h = \min(0.85f'_c A_c, A_s F_y, \sum Q_n) \quad (48.139)$$

*In regions of negative moment:*

$$V_h = \min(A_r F_{yr}, \sum Q_n) \quad (48.140)$$

where  $f'_c$  = the compressive strength of concrete

$A_c$  = the effective area of the concrete slab,  $t_c b_{eff}$

$t_c$  = the thickness of the concrete slab

$b_{eff}$  = the effective width of the concrete slab = smaller of  $(L/4, s)$  for an interior beam, and smaller of  $(L/8 + \text{distance from beam centerline to edge of slab}, s/2 + \text{distance from beam centerline to edge of slab})$  for an exterior beam

$L$  = the beam span measured from center-to-center of the supports

$s$  = the spacing between centerlines of adjacent beams

$A_s$  = the cross-sectional area of the steel beam

$F_y$  = the yield stress of the steel beam  
 $A_r$  = the area of reinforcing steel within the effective area of the concrete slab  
 $F_{yr}$  = the yield stress of the reinforcing steel  
 $\Sigma Q_n$  = the sum of the nominal shear strengths of the shear connectors

The nominal shear strength of a shear connector (used without a formed steel deck) is given by:

For a stud shear connector,

$$Q_n = 0.5A_{sc} \sqrt{f'_c E_c} \leq A_{sc} F_u \quad (48.141)$$

For a channel shear connector,

$$Q_n = 0.3(t_f + 0.5t_w) L_c \sqrt{f'_c E_c} \quad (48.142)$$

where  $A_{sc}$  = the cross-sectional area of the shear stud (in.<sup>2</sup>)  
 $f'_c$  = the compressive strength of concrete (ksi)  
 $E_c$  = the modulus of elasticity of concrete (ksi)  
 $F_u$  = the minimum specified tensile strength of the stud shear connector (ksi)  
 $t_f$  = the flange thickness of the channel shear connector (in.)  
 $t_w$  = the web thickness of the channel shear connector (in.)  
 $L_c$  = the length of the channel shear connector (in.)

If a formed steel deck is used,  $Q_n$  must be reduced by a reduction factor. The reduction factor depends on whether the deck ribs are perpendicular or parallel to the steel beam.

For deck ribs perpendicular to the steel beam,

$$\frac{0.85}{\sqrt{N_r}} \left( \frac{w_r}{h_r} \right) \left[ \left( \frac{H_s}{h_r} \right) - 1.0 \right] \leq 1.0 \quad (48.143)$$

When only a single stud is present in a rib perpendicular to the steel beam, the reduction factor of Eq. (48.143) shall not exceed 0.75.

For deck ribs parallel to steel beam

$$0.6 \left[ \frac{w_r}{h_r} \right] \left[ \left( \frac{H_s}{h_r} \right) - 1.0 \right] \leq 1.0 \quad (48.144)$$

The reduction factor of Eq. (48.144) is applicable only if  $(w_r/h_r) < 1.5$ , where  $N_r$  is the number of stud connectors in one rib at a beam intersection, not to exceed three in computations, regardless of the actual number of studs installed;  $w_r$  is the average width of the concrete rib or haunch;  $h_r$  is the nominal rib height; and  $H_s$  is the length of stud connector after welding, not to exceed the value  $h_r + 3$  in. (75 mm) in computations, regardless of actual length.

For full composite action, the number of connectors required between the maximum moment point and the zero moment point of the beam is given by

$$N = \frac{V_h}{Q_n} \quad (48.145)$$

For partial composite action, the number of connectors required is governed by the condition  $\phi_b M_n \geq M_u$ , where  $\phi_b M_n$  is governed by the shear strength of the connectors.

The placement and spacing of the shear connectors should comply with the following guidelines:

1. The shear connectors shall be uniformly spaced between the points of maximum moment and zero moment. However, the number of shear connectors placed between a concentrated load point and the nearest zero moment point must be sufficient to resist the factored moment  $M_u$ .
2. Except for connectors installed in the ribs of formed steel decks, shear connectors shall have at least 1 in. (25.4 mm) of lateral concrete cover. The slab thickness above a formed steel deck shall not be less than 2 in. (51 mm).
3. Unless located over the web, the diameter of shear studs must not exceed 2.5 times the thickness of the beam flange. For a formed steel deck, the diameter of stud shear connectors shall not exceed 3/4 in. (19 mm) and shall extend not less than 1½ in. (38 mm) above the top of the steel deck.
4. The longitudinal spacing of the studs should fall in the range of six times the stud diameter to eight times the slab thickness if a solid slab is used, or four times the stud diameter to eight times the slab thickness or 36 in. (915 mm), whichever is smaller, if a formed steel deck is used. Also, to resist uplift, the steel deck shall be anchored to all supporting members at a spacing not to exceed 18 in. (460 mm).

The design flexural strength  $\phi_b M_n$  of the composite beam with shear connectors is determined as follows.

In regions of positive moments, for  $h_c/t_w \leq 3.76/\sqrt{E/F_{yf}}$ ,  $\phi_b = 0.85$ ,  $M_n$  is the moment capacity determined using a plastic stress distribution assuming concrete crushes at a stress of  $0.85f'_c$ , and steel yields at a stress of  $F_y$ . If a portion of the concrete slab is in tension, the strength contribution of that portion of concrete is ignored. The determination of  $M_n$  using this method is very similar to the technique used for computing moment capacity of a reinforced concrete beam according to the ultimate strength method.

In regions of positive moments, for  $h_c/t_w > 3.76/\sqrt{E/F_{yf}}$ ,  $\phi_b = 0.90$  and  $M_n$  is the moment capacity determined using the superposition of elastic stress, considering the effect of shoring. The determination of  $M_n$  using this method is quite similar to the technique used for computing the moment capacity of a reinforced concrete beam according to the working stress method.

In regions of negative moment,  $\phi_b M_n$  is to be determined for the steel section alone in accordance with the requirements discussed in Section 48.5. To facilitate design, numerical values of  $\phi_b M_n$  for composite beams with shear studs in solid slabs are given in tabulated form in the AISC-LRFD manual. Values of  $\phi_b M_n$  for composite beams with formed steel decks are given in a publication by the Steel Deck Institute [2001].

## 48.14 Plastic Design

Plastic analysis and design is permitted only for steels with a yield stress not exceeding 65 ksi. The reason for this is that steels with a high yield stress lack the ductility required for inelastic deformation at hinge locations. Without adequate inelastic deformation, moment redistribution, which is an important characteristic for plastic design, cannot take place.

In plastic design, the predominant limit state is the formation of plastic hinges. Failure occurs when enough plastic hinges have formed for a collapse mechanism to develop. To ensure that plastic hinges can form and can undergo large inelastic rotation, the following conditions must be satisfied:

1. Sections must be compact, that is, the width–thickness ratios of flanges in compression and webs must not exceed  $\lambda_p$  in Table 48.8.
2. For columns, the slenderness parameter  $\lambda_c$  (see Section 48.4) shall not exceed  $1.5K$ , where  $K$  is the effective length factor, and  $P_u$  from gravity and horizontal loads shall not exceed  $0.75A_g F_y$ .
3. For beams, the lateral unbraced length  $L_b$  shall not exceed  $L_{pd}$ , where for doubly and singly symmetric I-shaped members loaded in the plane of the web,

$$L_{pd} = \frac{3600 + 2200 \left( M_1 / M_p \right)}{F_y} r_y \quad (48.146)$$



and for solid rectangular bars and symmetric box beams

$$L_{pd} = \frac{5000 + 3000(M_1/M_p)}{F_y} r_y \geq \frac{3000 r_y}{F_y} \quad (48.147)$$

In the above equations,  $M_1$  is the smaller end moment within the unbraced length of the beam,  $M_p$  is the plastic moment ( $=Z_x F_y$ ) of the cross-section,  $r_y$  is the radius of gyration about the minor axis (in.), and  $F_y$  is the specified minimum yield stress (ksi).

$L_{pd}$  is not defined for beams bent about their minor axes or for beams with circular and square cross sections, because these beams do not experience lateral torsional bucking when loaded.

## Plastic Design of Columns and Beams

Provided that the above limitations are satisfied, the design of columns shall meet the condition  $1.7F_a A \geq P_u$ , where  $F_a$  is the allowable compressive stress given in Eq. (48.16),  $A$  is the gross cross-sectional area, and  $P_u$  is the factored axial load.

The design of beams shall satisfy the conditions  $M_p \geq M_u$  and  $0.55F_y t_w d \geq V_u$ , where  $M_u$  and  $V_u$  are the factored moment and shear, respectively.  $M_p$  is the plastic moment capacity,  $F_y$  is the minimum specified yield stress,  $t_w$  is the beam web thickness, and  $d$  is the beam depth. For beams subjected to concentrated loads, all failure modes associated with concentrated loads (see Section 48.5 under Criteria for Concentrated Loads) should also be prevented.

Except at the location where the last hinge forms, a beam bending about its major axis must be braced to resist lateral and torsional displacements at plastic hinge locations. The distance between adjacent braced points should not exceed  $l_{cr}$ , given by

$$l_{cr} = \begin{cases} \left( \frac{1375}{F_y} + 25 \right) r_y, & \text{if } -0.5 < \frac{M}{M_p} < 1.0 \\ \left( \frac{1375}{F_y} \right) r_y, & \text{if } -1.0 < \frac{M}{M_p} \leq -0.5 \end{cases} \quad (48.148)$$

where  $r_y$  = the radius of gyration about the weak axis  
 $M$  = the smaller of the two end moments of the unbraced segment  
 $M_p$  = the plastic moment capacity

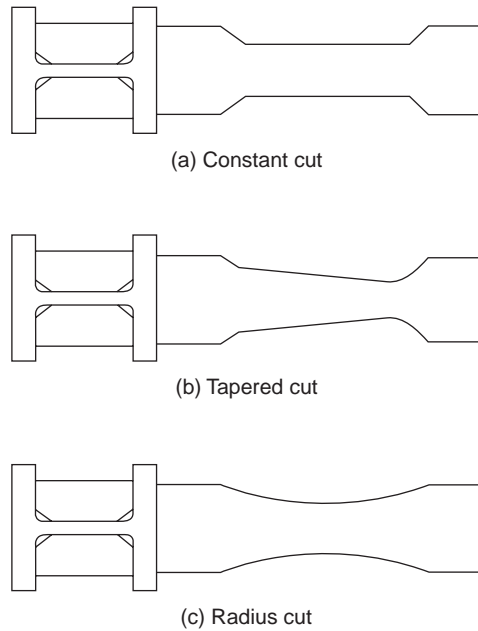
$M/M_p$  is taken as positive if the unbraced segment bends in reverse curvature and as negative if the unbraced segment bends in single curvature.

## Plastic Design of Beam-Columns

Beam-columns designed on the basis of plastic analysis shall satisfy the following interaction equations for stability (Eq. (48.149)) and strength (Eq. (48.150)):

$$\frac{P_u}{P_{cr}} + \frac{C_m M_u}{\left( 1 - \frac{P_u}{P_e} \right) M_m} \leq 1.0 \quad (48.149)$$

$$\frac{P_u}{P_y} + \frac{M_u}{1.18 M_p} \leq 1.0 \quad (48.150)$$



**FIGURE 48.39** Reduced beam section-cut geometries.

where  $P_u$  = the factored axial load

$P_{cr} = 1.7F_a A$ ,  $F_a$  is defined in Eq. (48.16) ( $A$  is the cross-sectional area)

$P_y$  = the yield load,  $AF_y$

$P_e$  = the Euler buckling load,  $\pi^2 EI / (Kl)^2$

$C_m$  = the coefficient defined in Section 48.4

$M_u$  = the factored moment

$M_p$  = the plastic moment,  $ZF_y$

$M_m$  = the maximum moment that can be resisted by the member in the absence of an axial load,  
 $M_{px}$  if the member is braced in the weak direction and  $\{1.07 - [(l/r_y)\sqrt{F_y}]/3160\}M_{px} \leq M_{px}$   
 if the member is unbraced in the weak direction

$l$  = the unbraced length of the member

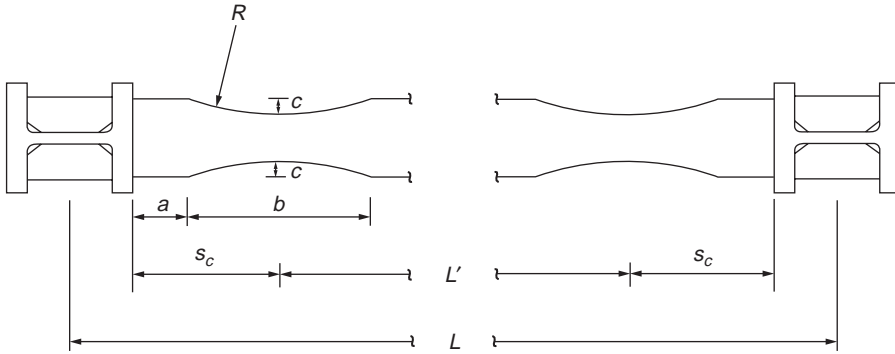
$r_y$  = the radius of gyration about the minor axis

$M_{px}$  = the plastic moment about the major axis,  $Z_x F_y$

$F_y$  = the minimum specified yield stress (ksi)

## Reduced Beam Section

*Reduced beam section* (RBS), or dogbone connection, is a type of connection in welded steel moment frames in which portions of the bottom beam flange or both top and bottom flanges are cut near the beam-to-column connection, thereby reducing the flexural strength of the beam at the RBS region and thus forcing a plastic hinge to form in a region away from the connection [Iwankiw and Carter, 1996; Engelhardt et al., 1996; Plumier, 1997]. The presence of this reduced section in the beam also tends to decrease the force demand on the beam flange welds and so mitigate the distress that may cause fracture in the connection. RBS can be bottom flange cut only or both top and bottom flange cut. Bottom-flange RBS is used if it is difficult or impossible to cut the top flange of an existing beam (e.g., if the beam is attached to a concrete floor slab). Figure 48.39 shows some typical cut geometries for RBS. The constant cut offers the advantage of ease of fabrication. The tapered cut has the advantage of matching the beam's flexural strength to the flexural demand on the beam under a gravity load. The radius cut is relatively easy to fabricate, and because



**FIGURE 48.40** Key dimensions of a radius-cut reduced beam section.

the change in geometry of the cross section is rather gradual, it also has the advantage of minimizing stress concentration. Based on experimental investigations [Engelhardt et al., 1998; Moore et al., 1999], the radius-cut RBS has been shown to be a reliable connection for welded steel moment frames.

The key dimensions of a radius-cut RBS are shown in Fig. 48.40. The distance from the face of the column to the start of the cut is designated as  $a$ ; the length and depth of the cut are denoted as  $b$  and  $c$ , respectively. Values of  $a$ ,  $b$ , and  $c$  are given as follows [Engelhardt et al., 1998; Gross et al., 1999]:

$$a \approx (0.5 \text{ to } 0.75)b_f \quad (48.151)$$

$$b \approx (0.65 \text{ to } 0.85)d \quad (48.152)$$

$$c \approx 0.25b_f \quad \text{for a bottom flange RBS} \quad (48.153)$$

where  $b_f$  is the beam flange width and  $d$  is the beam depth. Using geometry, the cut radius  $R$  can be calculated as

$$R = \frac{4c^2 + b^2}{8c} \quad (48.154)$$

and the distance from the face of the column to the critical plastic section  $s_c$  is given by

$$s_c = a + \frac{b}{2} \quad (48.155)$$

An optimal RBS is one in which the moment at the face of the column will be minimized. To achieve this condition, the following procedure is recommended [Gross et al., 1999]:

- Set  $c = 0.25b_f$ .
- Compute the RBS plastic section modulus using the equation

$$Z_{RBS} = Z_b - \frac{(ct_f)^2}{t_w} - ct_f(d - t_f) \quad (48.156)$$

- where  $Z_b$  is the plastic section modulus of the full-beam cross-section;  $c$  is the depth of cut, as shown in Fig. 48.40; and  $d$ ,  $t_f$ , and  $t_w$  are the beam depth, beam flange thickness, and web thickness, respectively.

- Compute  $\eta$ , the ratio of the moment at the face of the column to the plastic moment of the connecting beam, from the equation

$$\eta = 1.1 \left( 1 + \frac{2s_c}{L'} \right) \frac{Z_{RBS}}{Z_b} + \frac{wL's_c}{2Z_b F_{yf}} \quad (48.157)$$

- where  $s_c$  is given in Eq. (48.155) and shown in Fig. 48.40,  $L'$  is the beam span between critical plastic sections (see Fig. 48.40),  $Z_{RBS}$  and  $Z_b$  are defined in Eq. (48.156),  $w$  is the magnitude of the uniformly distributed load on the beam, and  $F_{yf}$  is the beam flange yield strength.
- If  $\eta < 1.05$ , then the RBS dimensions are satisfactory. Otherwise, use RBS cutouts in both the top and bottom flanges, or consider using other types of moment connections [Gross et al., 1999].

Experimental studies [Uang and Fan, 1999; Yu et al., 2000; Gilton et al., 2000, Engelhardt et al., 2000] of a number of radius-cut RBS, with or without the presence of a concrete slab, have shown that the connections perform satisfactorily and exhibit sufficient ductility under cyclic loading. However, the use of RBS beams in a moment-resistant frame tends to cause an overall reduction in frame stiffness of around 4 to 7% [Grubbs, 1997]. If the increase in frame drift due to this reduction in frame stiffness is appreciable, proper allowances must be made in the analysis and design of the frame.

## Defining Terms

**ASD** — Acronym for allowable stress design.

**Beam-columns** — Structural members whose primary function is to carry loads both along and transverse to their longitudinal axes.

**Biaxial bending** — Simultaneous bending of a member about two orthogonal axes of the cross section.

**Built-up members** — Structural members made of structural elements jointed together by bolts, welds, or rivets.

**Composite members** — Structural members made of both steel and concrete.

**Compression members** — Structural members whose primary function is to carry loads along their longitudinal axes.

**Design strength** — Resistance provided by the structural member obtained by multiplying the nominal strength of the member by a resistance factor.

**Drift** — Lateral deflection of a building.

**Factored load** — The product of the nominal load and a load factor.

**Flexural members** — Structural members whose primary function is to carry loads transverse to their longitudinal axes.

**Limit state** — A condition in which a structure or structural component becomes unsafe (strength limit state) or unfit for its intended function (serviceability limit state).

**Load factor** — A factor to account for the unavoidable deviations of the actual load from its nominal value and uncertainties in structural analysis in transforming the applied load into a load effect (axial force, shear, moment, etc.).

**LRFD** — Acronym for load and resistance factor design.

**PD** — Acronym for plastic design.

**Plastic hinge** — A yielded zone of a structural member in which the internal moment is equal to the plastic moment of the cross section.

**Reduced beam section** — A beam section with portions of flanges cut out to reduce the section moment capacity.

**Resistance factor** — A factor to account for the unavoidable deviations of the actual resistance of a member from its nominal value.

**Service load** — Nominal load expected to be supported by the structure or structural component under normal usage.

**Shear lag** — The phenomenon in which the stiffer (or more rigid) regions of a structure or structural component attract more stresses than the more flexible regions of the structure or structural component. Shear lag causes stresses to be unevenly distributed over the cross section of the structure or structural component.

**Side sway inhibited frames** — Frames in which lateral deflections are prevented by a system of bracing.

**Side sway uninhibited frames** — Frames in which lateral deflections are not prevented by a system of bracing.

**Tension field action** — Postbuckling shear strength developed in the web of a plate girder. Tension field action can develop only if sufficient transverse stiffeners are provided to allow the girder to carry the applied load using truss-type action after the web has buckled.

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## Further Information

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Design Guide 2, *Design of Steel and Composite Beams with Web Openings*, Darwin.

Design Guide 3, *Considerations for Low-Rise Buildings*, Fisher and West.

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