APPENDIX A – MATHEMATICAL FORMULAE

Version 3.1. Jan 99. A.5. corrected equation for trapezium inertia.
Version 3.2. May 2000. A.5. corrected equation for trapezium inertia.

A.1 **Trigonometric functions**

$$\sin x = \frac{e^{ix} - e^{-ix}}{2i}$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \quad \tan A \tan B}$$

$$\sin A + \sin B = 2 \sin\frac{A+B}{2} \cos\frac{A-B}{2}$$

$$\cos A + \cos B = 2 \cos\frac{A+B}{2} \cos\frac{A-B}{2}$$

$$\sin A \ \sin B = \frac{1}{2}\left[\cos(A-B) - \cos(A+B)\right]$$

$$\sin A \ \cos B = \frac{1}{2}\left[\sin(A+B) + \sin(A-B)\right]$$

$$\sin^2 x = \frac{1}{2}\left[1 - \cos 2x\right]$$

$$\sin^3 x = \frac{1}{4}\left[3 \sin x - \sin 3x\right]$$

$$\cos x = \frac{e^{ix} + e^{-ix}}{2}$$

$$\cos(A \pm B) = \cos A \cos B \quad \sin A \sin B$$

$$\sin A - \sin B = 2\cos\frac{A+B}{2} \sin\frac{A-B}{2}$$

$$\cos A - \cos B = -2\sin\frac{A+B}{2} \sin\frac{A-B}{2}$$

$$\cos A \cos B = \frac{1}{2}\left[\cos(A+B) + \cos(A-B)\right]$$

$$\cos^2 x = \frac{1}{2}\left[1 + \cos 2x\right]$$

$$\cos^3 x = \frac{1}{4}\left[3 \cos x + \cos 3x\right]$$

A.2 **Hyperbolic functions**

$$\cosh x = \frac{e^{x} + e^{-x}}{2}$$

$$\cosh ix = \cos x$$

$$\sinh ix = i \sin x$$

$$\cosh^2 x - \sinh^2 x = 1$$

$$\cosh(x \pm y) = \cosh x \cosh y \pm \sinh x \sinh y$$

$$\sinh(x \pm y) = \sinh x \cosh y \pm \cosh x \sinh y$$

$$\cosh(x \pm iy) = \cosh x \cos y \pm i \sinh x \sin y$$

$$\sinh(x \pm iy) = \sinh x \cos y \pm i \cosh x \sin y$$

$$\sinh x = \frac{e^{x} - e^{x}}{2}$$

$$\cos ix = \cosh x$$

$$\sin ix = i \sinh x$$

A.3 **Standard indefinite integral**

Integrand	Integral	Integrand	Integral
$\sin x$	$-\cos x$	$\sinh x$	$\cosh x$
$\cos x$	$\sin x$	$\cosh x$	$\sinh x$
$\tan x$	$-\ln(\cos x)$	$\tanh x$	$\ln(\cosh x)$
$\operatorname{cosec} x$	$\ln(\tan x/2)$	$\operatorname{cosech} x$	$\ln(\tanh x/2)$
$\sec x$	$\ln(\tan x + \sec x)$	$\operatorname{sech} x$	$2\tan^{-1}(e^x)$
$\cot x$	$\ln(\sin x)$	$\coth x$	$\ln(\sinh x)$
$\sec^2 x$	$\tan x$	$\operatorname{sech}^2 x$	$\tanh x$
$\tan x \sec x$	$\sec x$	$\tanh x \operatorname{sech} x$	$-\operatorname{sech} x$
$\cot x \operatorname{cosec} x$	$-\operatorname{cosec} x$	$\coth x \operatorname{cosech} x$	$-\operatorname{cosech} x$
$\frac{1}{\sqrt{a^2 - x^2}}$	$\sin^{-1}\left(\frac{x}{a}\right)$ or $-\cos^{-1}\left(\frac{x}{a}\right)$		
$\frac{1}{\sqrt{x^2 + a^2}}$	$\sinh^{-1}\left(\frac{x}{a}\right)$ or $\ln\left(x + \sqrt{x^2 + a^2}\right)$		
$\frac{1}{\sqrt{x^2 - a^2}}$	$\cosh^{-1}\left(\frac{x}{a}\right)$ or $\ln\left(x + \sqrt{x^2 - a^2}\right)$		
$\frac{1}{x^2 + a^2}$	$\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$		

A.4 **Standard substitutions for integration**

If the integrand is a function of : Substitute:

$(a^2 - x^2)$ or $\sqrt{a^2 - x^2}$	$x = a\sin\theta$ or $x = a\cos\theta$
$(a^2 + x^2)$ or $\sqrt{a^2 + x^2}$	$x = a\tan\theta$ or $x = a\sinh\theta$
$(x^2 - a^2)$ or $\sqrt{x^2 - a^2}$	$x = a\sec\theta$ or $x = a\cosh\theta$

or of the form:

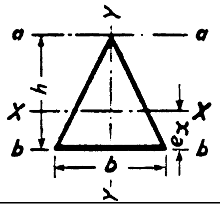
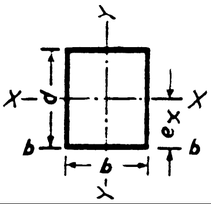
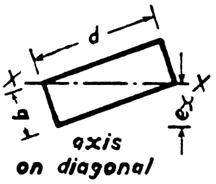
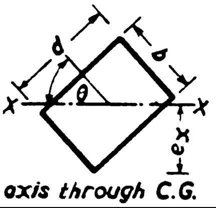
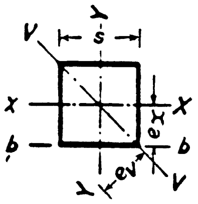
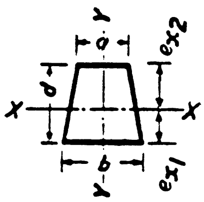
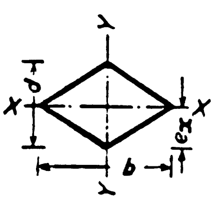
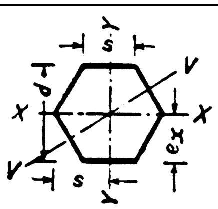
$\left\{(ax+b)\sqrt{px+q}\right\}^{-1}$	$px + q = u^2$
$\left\{(ax+b)\sqrt{px^2+qx+r}\right\}^{-1}$	$ax + b = \frac{1}{u}$

or a rational function of:

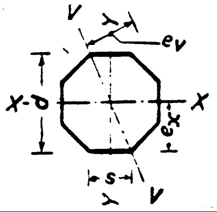
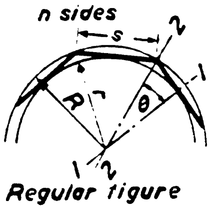
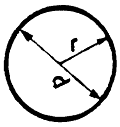
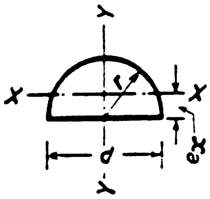
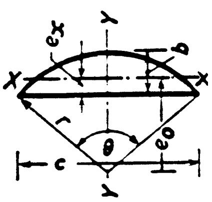
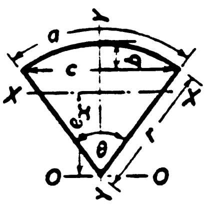
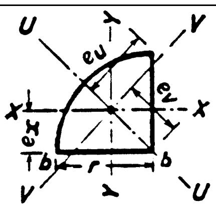
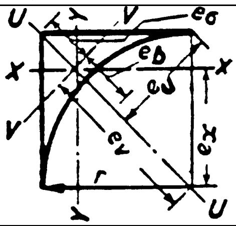
$\sin x$ *and / or* $\cos x$	$t = \tan\frac{x}{2}$

$$\left[\textit{whence} \quad \sin x = \frac{2t}{1+t^2} \qquad \cos x = \frac{1-t^2}{1+t^2} \qquad dx = \frac{2\,dt}{1+t^2}\right]$$

A.5 **Geometric properties of plane sections**

section		Area	Position of centroid	Moments of inertia	Section Moduli
Triangle		$A = \frac{bh}{2}$	$e_x = \frac{h}{3}$	$I_{xx} = bh^3/36$ $I_{yy} = hb^3/48$ $I_{aa} = bh^3/4$ $I_{bb} = bh^3/12$	Z_{xx} $base = bh^2/12$ $apex = bh^2/24$ $Z_{yy} = bh^2/24$
Rectangle		$A = bd$	$e_x = \frac{h}{2}$	$I_{xx} = bd^3/12$ $I_{yy} = db^3/12$ $I_{bb} = bd^3/3$	$Z_{xx} = bd^2/6$ $Z_{yy} = db^2/6$
Rectangle		$A = bd$	$e_x = \frac{bd}{\sqrt{b^2 + d^2}}$	$I_{xx} = \frac{b^3 d^3}{6\left(b^2 + d^2\right)}$	$Z_{xx} = \frac{b^2 d^2}{6\sqrt{b^2 + d^2}}$
Rectangle		$A = bd$	$e_x =$ $\frac{b\sin\theta + d\cos\theta}{2}$	$I_{xx} =$ $\frac{bd\left(b^2\sin^2\theta + d^2\cos^2\theta\right)}{12}$	$Z_{xx} =$ $\frac{bd\left(b^2\sin^2\theta + d^2\cos^2\theta\right)}{6\left(b\sin\theta + d\cos\theta\right)}$
Square		$A = s^2$	$e_x = \frac{s}{2}$ $e_v = \frac{s}{\sqrt{2}}$	$I_{xx} = I_{yy} = s^4/12$ $I_{bb} = s^4/3$ $I_{vv} = s^4/12$	$Z_{xx} = Z_{yy} = \frac{s^3}{6}$ $Z_{vv} = \frac{s^3}{6\sqrt{2}}$
Trapezium		$A = \frac{d(a+b)}{2}$	$e_{x1} = \frac{d(2a+b)}{3(a+b)}$	$I_{xx} = \frac{d^3\left(a^2 + 4ab + b^2\right)}{36(\mathrm{a}+\mathrm{b})}$ $I_{yy} =$ $\frac{d\left(a^3 + a^2 b + ab^2 + b^3\right)}{48}$	$Z_{xx} = \frac{I_{xx}}{d - e_x}$ (*two values*) $Z_{yy} = \frac{2I_{yy}}{b}$
Diamond		$A = \frac{bd}{2}$	$e_x = \frac{d}{2}$	$I_{xx} = \frac{bd^3}{48}$ $I_{yy} = \frac{db^3}{48}$	$Z_{xx} = \frac{bd^2}{24}$ $Z_{yy} = \frac{db^2}{24}$
Hexagon		$A = 0.866d^2$	$e_x = 0.866s$ $= \frac{d}{2}$	$I_{xx} = I_{yy} = I_{vv}$ $= 0.0601d^4$	$Z_{xx} = 0.1203d^3$ $Z_{yy} = Z_{vv}$ $= 0.1042d^3$

Geometric properties of plane sections (cont.)

Section		Area	Position of centroid	Moments of inertia	Section Moduli
Octagon		$A = 0.8284d^2$ $s = 0.4142d$	$e_x = \frac{d}{2}$ $e_v = 0.541d$	$I_{xx} = I_{yy} = I_{vv}$ $= 0.0547d^4$	$Z_{xx} = Z_{yy}$ $= 0.1095d^3$ $Z_{vv} = 0.1011d^3$
Polygon		$A = \frac{n s^2 \cot\theta}{4}$ $A = n r^2 \tan\theta$ $A = \frac{n R^2 \sin 2\theta}{2}$	$e = r$ *or* R *depending on the axis and value of* n	$I_1 = I_2$ $= \frac{A(6R^2 - s^2)}{24}$ $= \frac{A(12r^2 + s^2)}{48}$	$Z = \frac{I}{e}$
Circle		$A = \pi r^2$ $A = 0.7854d^2$	$e = r = \frac{d}{2}$	$I = \frac{\pi d^4}{64}$ $I = 0.7854r^2$	$Z = \frac{\pi d^3}{32}$ $Z = 0.7854r^3$
Semi-Circle		$A = 1.5708r^2$	$e_x = 0.424r$	$I_{xx} = 0.1098r^4$ $I_{yy} = 0.3927r^4$	$Z_{xx} =$ $base = 0.2587r^3$ $crown = 0.1907r^3$ $Z_{yy} = 0.3927r^3$
Segment		$A =$ $\frac{r^2}{2}\left(\frac{\pi\theta°}{180°} - \sin\theta\right)$	$e_0 = \frac{c^3}{12A}$ $e_x = e_0 - r\cos\frac{\theta}{2}$	$I_{xx} = \frac{r^4}{16}\left(\frac{\pi\theta°}{90°} - \sin 2\theta\right)$ $- \frac{20r^4(1-\cos\theta)^3}{\pi\theta° - 180°\sin\theta}$ $I_{yy} = \frac{r^4}{48}\left(\frac{\pi\theta°}{30°} - 8\sin\theta + \sin 2\theta\right)$	Z_{xx} $base = I_{xx} / e_I$ $crown = \frac{I_{xx}}{b - e_I}$ $Z_{vv} = \frac{2I_{yy}}{c}$
Sector		$A = \frac{\theta°}{360°}\pi r^2$	$e_x = \frac{2}{3} r \frac{c}{a}$ $e_x = \frac{r^2 c}{3A}$	$I_{xx} = I_o - \frac{360°}{\theta°\pi}\sin^2\frac{\theta}{2}\frac{4r^4}{9}$ $I_{yy} = \frac{r^4}{8}\left(\frac{\pi\theta°}{180°} - \sin\theta\right)$ $I_0 = \frac{r^4}{8}\left(\frac{\pi\theta°}{180°} + \sin\theta\right)$	Z_{xx} $centre = \frac{I_{xx}}{e_x}$ $crown = \frac{I_{xx}}{r - e_x}$ $Z_{yy} = \frac{2I_{yy}}{c}$
Quadrant		$A = \frac{\pi r^2}{4}$	$e_x = 0.424r$ $e_v = 0.6r$ $e_u = 0.707r$	$I_{xx} = I_{yy} = 0.0549r^4$ $I_{bb} = 0.1963r^4$ $I_{uu} = 0.0714r^4$ $I_{vv} = 0.0384r^4$	*Minimum Values* $Z_{xx} = Z_{yy} = 0.0953r^3$ $Z_{uu} = 0.1009r^3$ $Z_{vv} = 0.064r^3$
Complement		$A = 0.2146r^2$	$e_x = 0.777r$ $e_v = 1.098r$ $e_u = 0.707r$ $e_\sigma = 0.316r$ $e_b = 0.391r$	$I_{xx} = I_{yy} = 0.0076r^4$ $I_{uu} = 0.012r^4$ $I_{vv} = 0.0031r^4$	*Minimum Values* $Z_{xx} = Z_{yy} = 0.0097r^3$ $Z_{uu} = 0.017r^3$ $Z_{vv} = 0.0079r^3$

A.6 **Conversion Factors**
Imperial : Metric conversion factors

Mass

1 kg	= 2.205 lb	1 lb	= 0.4536 kg
1 tonne	= 0.9842 ton	1 ton	= 1.016 tonne

Length

1 mm	= 0.03937 in	1 in	= 25.40 mm
1 m	= 3.281 ft	1 ft	= 0.3048 m
1 m	= 1.094 yd	1 yd	= 0.9144 m

Area

1 mm^2	= 0.00155 in^2	1 in^2	= 645.2 mm^2
1 m^2	= 10.76 ft^2	1 ft^2	= 0.09290 m^2
1 m^2	= 1.196 yd^2	1 yd^2	= 0.8361 m^2

Volume

1 mm^3	= 0.000061 in^3	1 in^3	= 16390 mm^3
1 m^3	= 35.32 ft^3	1 ft^3	= 0.02832 m^3
1 m^3	= 1.308 yd^3	1 yd^3	= 0.7646 m^3

Density

1 kg/m^3	= 0.06242 lb/ft^3	1 lb/ft^3	= 16.02 kg/m^3
1 $tonne/m^3$	= 0.7524 ton/yd^3	1 ton/yd^3	= 1.329 $tonne/m^3$

Force

1 N	= 0.102 kgf	1 kgf	= 9.807 N
1 N	= 0.2248 lbf	1 lbf	= 4.448 N
1 kN	= 0.1004 tonf	1 tonf	= 9.964 kN

Stress

1 N/mm^2	= 145.0 lbf/in^2	1 lbf/in^2	= 0.006895 N/mm^2
1 N/m^2	= 0.102 kgf/m^2	1 kgf/m^2	= 9.807 N/m^2
1 kgf/m^2	= 0.2048 lbf/ft^2	1 lbf/ft^2	= 4.882 kgf/m^2
1 N/m^2	= 0.02089 lbf/ft^2	1 lbf/ft^2	= 47.88 N/m^2
1 kN/m^2	= 0.009324 $tonf/ft^2$	1 $tonf/ft^2$	= 107.3 kN/m^2
1 N/mm^2	= 0.06475 $tonf/in^2$	1 $tonf/in^2$	= 15.44 N/mm^2

Strip Loading

1 N/m	= 0.102 kgf/m	1 kgf/m	= 9.807 N/m
1 kgf/m	= 0.6720 lbf/ft	1 lbf/ft	= 1.488 kgf/m
1 kN/m	= 68.53 lbf/ft	1 lbf/ft	= 0.0146 kN/m
1 kN/m	= 0.03059 tonf/ft	1 tonf/ft	= 32.69 kN/m

Moment

1 Nm	= 0.102 kgf.m	1 kgf.m	= 9.807 Nm
1 kgf.m	= 86.80 lbf.in	1 lbf.in	= 0.01152 kgf.m
1 Nm	= 8.851 lbf.in	1 lbf.in	= 0.1130 Nm
1 Nm	= 0.7376 lbf.ft	1 lbf.in	= 1.356 Nm
1 kNm	= 3.951 tonf.in	1 tonf.in	= 0.2531 kNm

Modulus of Elasticity

1 N/mm^2	= 145.00 lbf/in^2	1 lbf/in^2	= 6.895×10^{-3} N/mm^2

Section Modulus

1 mm^3	= 61.01×10^{-6} in^3	1 in^3	= 16390 mm^3

Second Moment of Area

1 mm^4	= 2.403×10^{-6} in^4	1 in^4	= 416200 mm^4