Structural Engineering Handbook
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Basic Principles of Shock Loading

21.1 Introduction

Shock loading presents an interesting set of problems to the design engineer. In the engineering community, design for static loading traditionally has been the most commonly used design procedure. Designing for shock loading, however, requires a change of thinking in several areas. The objectives of this discussion are to introduce the basic principles of shock loading and to consider their effect upon the integrity of structures.

To understand shock loading, one must first establish the various loading scheme definitions. There are four loading modes that are a function of the strain rate and the number of loading cycles experienced by the member. They are the following:

- static loading
- fatigue loading
- shock loading
- shock/fatigue combination loading

Static loading occurs when a force is slowly applied to a member. This is a slow or constant loading process that is equivalent to loading the member over a period of 1 min. Designing for static loading traditionally has been the approach used for a wide variety of components, such as buildings, water towers, dams, and smoke stacks.

Fatigue loading occurs when the member experiences alternating, repeated, or fluctuating stresses. Fatigue loading generally is classified by the number of loading cycles on the member and the stress level. Low cycle fatigue usually is associated with stress levels above the yield point, and fracture initiates in less than 1000 cycles. High cycle fatigue involves 10,000 cycles or more and overall stress levels that are below the yield point, although stress raisers from notch-like geometries and/or the residual stresses of welding result in localized plastic deformation. High cycle fatigue failures often occur with overall maximum stresses below the yield strength of the material.
In shock loading, an impact-type force is applied over a short instant of time. The yield and ultimate tensile strengths of a given member can be higher for loads with accelerated strain rates when compared to static loading (see Figure 21.1) [4]. The total elongation, however, remains constant at strain rates above approximately $10^{-4}$ in./in./s.

![Figure 21.1: Influence of the strain rate on tensile properties. (From Manjoine, M.J., J. Applied Mech., 66, A211, 1944. With permission.)](image)

Shock/fatigue combination loading is equivalent to a shock load applied many times. It exhibits the problems of both fatigue loading and shock loading. Some examples of this loading mode are found in jack hammers, ore crushers, piledrivers, foundry “shake-outs”, and landing gear on aircraft.

### 21.2 Requirements for Optimum Design

For an optimum design, shock loading requires that the entire volume of the member be stressed to the maximum (i.e., the yield point). This refers to both the entire length of the member and the entire cross-section of that member. In practice, however, most members do not have a geometry that allows for the member to be maximally loaded for its entire length, or stressed to the maximum across the entire cross-section.

### 21.3 Absorbing Kinetic Energy

In Figure 21.2, the member on the left acts as a cantilever beam. Under shock loading, the beam is deflected, which allows that force to be absorbed over a greater distance. The result is a decrease in
the impact intensity applied to the beam. The member on the right, however, is hit straight on as a column, and is restricted to little deflection. Therefore, as the column absorbs the same amount of energy, the resulting force is extremely high.

In shock loading, the energy of the applied force is ultimately absorbed, or transferred, to the structural component designed to resist the force. For example, a vehicle striking a column illustrates this transfer of energy, as shown in Figure 21.3. Prior to impact, the vehicle has a certain value of kinetic energy that is equal to \( \frac{1}{2} m V^2 \). The members that absorb the kinetic energy applied by the vehicle are illustrated in Figure 21.4, where the spring represents the absorbed potential energy, \( E_p \), in “a”. Figure 21.4b illustrates a vertical stop much like Figure 21.2a. The entire kinetic energy must be absorbed by the receiving member (less any insignificant conversion byproducts—heat, noise, etc.) and momentarily stored as elastic potential energy. This elastic potential energy is stored in the member due to its stressed or deflected condition.

In Figure 21.5, the maximum potential energy \( (E_p) \) that can be elastically absorbed by a simply supported beam is derived. The maximum potential energy that may be stored in the loaded (deflected) beam when stressed to yield is

\[
E_p = \frac{\sigma_y^2 IL}{6EC^2}
\]  

(21.1)

where
\[
\begin{align*}
E_p & = \text{maximum elastic capacity (max. absorbed potential energy)} \\
\sigma_y & = \text{yield strength of material} \\
I & = \text{moment of inertia of section} \\
L & = \text{length of member} \\
E & = \text{modulus of elasticity of material} \\
C & = \text{distance to outer fiber from neutral axis of section}
\end{align*}
\]

Figure 21.6 shows various equations that can be used to determine the amount of energy stored in a particular beam, depending upon the specific end conditions and point of load application.
Figure 21.7 addresses the shock loading potential in the case of a simply supported beam with a concentrated force applied in the center. To analyze this problem, two issues must be considered. First, the property of the material is \( \sigma_y^2 / E \), and second, the property of the cross-section is \( I / C^2 \). The objective for optimum performance in shock loading is to ensure maximal energy absorption capability. Both contributing elements, material properties and section properties, will be examined later in this chapter.
FIGURE 21.6: Energy storage capability for various end conditions and loading. (From The Lincoln Electric Co. With permission.)

\[ E_p = \frac{F^2 L^3}{96EI} = \frac{\alpha^2 IL}{6EC^2} = \frac{24A EI}{L^3} \]

\[ E_p = \frac{F^2 a^3 b^3}{6EIL} = \frac{\alpha^2 IL}{6EC^2} = \frac{R_1^2 a^4 L}{6EI} = \frac{R_2^2 b^4 L}{6EI} \]

\[ E_p = \frac{F^2 a^3}{12EI} \left(3L - 4a\right) = \frac{\alpha^2 I(3L - 4a)}{12EC^2} \]

\[ E_p = \frac{F^2 L^3}{384EI} = \frac{\alpha^2 IL}{6EC^2} = \frac{R_1^2 L^3}{96EI} \]

\[ E_p = \frac{F^2 L^3}{6EI} = \frac{\alpha^2 IL}{6EC^2} = \frac{R_2^2 L^3}{6EI} \]

\[ E_p = \frac{F^2 L^3}{240EI} = \frac{\frac{F^2 L^3}{240EI}}{4\alpha^2 IL} \left(\frac{3072A^2 EI}{16EC^2} = \frac{125 L^3}{128A^2 EI} \right) \]

\[ E_p = \frac{F^2 L^3}{60EI} = \frac{\alpha^2 IL}{6EC^2} \]

\[ E_p = \frac{F^2 L^3}{640EI} = \frac{\alpha^2 IL}{10EC^2} = \frac{R_2^2 L^3}{60EI} \]

\[ E_p = \frac{F^2 L^3}{945EI} = \frac{\frac{F^2 L^3}{945EI}}{4\alpha^2 IL} \left(\frac{900A^2 IL}{36EC^2} = \frac{105EI}{128A^2 EI} = \frac{f^2 L^5}{945EI} \right) \]

\[ E_p = \frac{F^2 L^3}{1440EI} = \frac{\frac{F^2 L^3}{1440EI}}{4\alpha^2 IL} \left(\frac{1440EI}{360EI} = \frac{R_2^2 L^3}{360EI} \right) \]

\[ E_p = \frac{F^2 L^3}{126EI} = \frac{\alpha^2 IL}{14EC^2} = \frac{R_2^2 L^3}{126EI} = \frac{f L^6}{604EI} \]

\[ E_p = \frac{M_o^2 L}{6EIa} = \frac{\alpha^2 IL}{6Ea^2 \Delta^2} = \frac{R^2(a^3 + b^3)}{6EI} \]

\[ E_p = \frac{M_o^2 L}{2EI} = \frac{\alpha^2 IL}{2EC^2} = \frac{R^2 L^3}{2EI} \]

\[ E_p = \frac{F^2 L^3}{448EI} = \frac{9\alpha^2 IL}{28EC^2} = \frac{R^2 L^3}{112EI} \]

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FIGURE 21.7: An example of shock loading for a simply supported beam. (From The Lincoln Electric Co. With permission.)

Plotted in Figure 21.8 is the applied strain on the horizontal axis and the resisting stress on the vertical axis. If this member is strained up to the yield point stress, the area under that triangle represents the maximum energy that can be absorbed elastically \( E_p \). When the load is removed, it will return to its original position. The remaining area, which is not cross-hatched, represents the plastic (or inelastic) energy if the member is strained. A tremendous amount of plastic energy can be absorbed, but of course, by that time the member is deformed (permanently set) and may be of no further use.

The stress-strain diagram can help illustrate the total elastic potential capability of a material. The area of the triangular region is \( \sigma_Y \varepsilon_Y / 2 \). Since \( E = \sigma_Y / \varepsilon_Y \) and \( \sigma_Y = E \varepsilon_Y \), then this area is equal to \( \sigma_Y^2 / 2E \). This gives the elastic energy potential per unit volume of material. As shown in Figure 21.7, the property of the material was \( \sigma_Y^2 / E \) (as defined by Equation 21.1), which is directly proportional to the triangular area in Figure 21.8.

The stress-strain curve for two members (one smooth and one notched) is illustrated in Figure 21.9. At the top of the illustration, the stress-strain curve is shown. Extending from the stress-strain curve is the length of the member. If the member is smooth, as on the lower left, it can be stressed up to the yield point, resulting in the highest strain that can be attained and the maximum elastic energy per unit volume of material. Applying this along the length of the member, the solid prism represents the total energy absorbed by that member.

On the right side of the illustration, note that the notch shown gives a stress raiser. If the stress raiser is assigned an arbitrary value of three-to-one, the area under the notch will display stresses three times greater than the average stress. Therefore, to keep the stress in the notched region below the yield stress, the applied stress must be reduced to 1/3 of the original amount. This will reduce the
strain to 1/3, which corresponds to only 1/9 of the energy that will be absorbed. In other words, 1/9 of the elastic energy can be absorbed if the stress at the notch is kept to the yield stress. Notches and cracks in members subject to shock loading can become very dangerous, especially if the stress flow is normal to the notch. Therefore, structural members in shock loading require good workmanship and detailing.

21.4 Material Properties for Optimum Design

As shown previously, in Figure 21.8, the elastic energy absorbed during a shock load is equal to the triangular-shaped area under the stress-strain curve up to the yield point. This energy is equal to $\sigma_y^2/2E$, which is directly a function of the material. In order to maximize the amount of energy that can be absorbed ($E_p$), with regard to material properties, one must maximize this triangular-shaped area under the stress-strain curve.

In order to compare the energy absorption capability of various materials in shock loading, a list of materials and their corresponding mechanical properties is compiled in Table 21.1. For each

<table>
<thead>
<tr>
<th>Material</th>
<th>Yield strength, $\sigma_y$ (ksi)</th>
<th>Ultimate tensile strength, $\sigma_u$ (ksi)</th>
<th>Modulus of elasticity, $E$ (psi)</th>
<th>Percent elongation (%)</th>
<th>Absorbed elastic energy, $\sigma_y^2/2E$ (psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mild steel</td>
<td>35</td>
<td>60</td>
<td>$30 \times 10^6$</td>
<td>35</td>
<td>20.4</td>
</tr>
<tr>
<td>Medium carbon steel</td>
<td>45</td>
<td>85</td>
<td>$30 \times 10^6$</td>
<td>25</td>
<td>33.7</td>
</tr>
<tr>
<td>High carbon steel</td>
<td>75</td>
<td>120</td>
<td>$30 \times 10^6$</td>
<td>8</td>
<td>94.0</td>
</tr>
<tr>
<td>A514 Steel</td>
<td>100</td>
<td>115–135</td>
<td>$30 \times 10^6$</td>
<td>18</td>
<td>166.7</td>
</tr>
<tr>
<td>Gray cast iron</td>
<td>6</td>
<td>20</td>
<td>$15 \times 10^6$</td>
<td>5</td>
<td>1.2</td>
</tr>
<tr>
<td>Malleable cast iron</td>
<td>20</td>
<td>50</td>
<td>$23 \times 10^6$</td>
<td>10</td>
<td>8.7</td>
</tr>
<tr>
<td>5056-H 18 Aluminum alloy</td>
<td>59</td>
<td>63</td>
<td>$10 \times 10^6$</td>
<td>10</td>
<td>174.1</td>
</tr>
</tbody>
</table>

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material listed in column 1, typical yield and ultimate tensile strengths are tabulated in columns 2 and 3. The modulus of elasticity is recorded in column 4, and column 5 lists the typical percent elongation. Column 6 details the calculated value of $\sigma^2/2E$ for each material.

Higher strength steel will provide a maximum value of $\sigma^2/2E$, since the modulus of elasticity is approximately constant for all steels. However, if aluminum is an option, 5xxx series alloys will provide moderate-to-high strengths with high values of $\sigma^2/2E$. In addition, the 5xxx series alloys usually have the highest welded strengths among aluminum alloys with good corrosion resistance [3]. Some materials, such as cast iron, should not be used for shock loading. Cast iron has a very low value of $\sigma^2/2E$ and will absorb only a small amount of elastic energy. It is ironic that some engineers believe that cast iron is a better material for shock loading than steel or aluminum alloys; this is not the case.

When choosing a material for shock loading applications, the designer should also remember to consider other characteristics of the material such as machineability, weldability, corrosion resistance, etc. For example, very high strength steels are appropriate for shock loading applications, however, they are not very weldable.

### 21.5 Section Properties for Optimum Design

It was indicated earlier that most engineers are well versed in the principles of steady loading. To demonstrate the differences between steady and shock loading, an example is shown in Figure 21.10. On the left, a steady load is applied to a simply supported beam, and the maximum force that the member can carry for a given stress is $F = 4\sigma I/LC$. From this equation, it is important to realize that if the length of the beam (L) is doubled, then the beam would be able to carry only half of the load at the same stress.

![FIGURE 21.10: Steady load vs. shock load. (From The Lincoln Electric Co. With permission.)](image)

With the shock loading equation, $E_p = \sigma^2IL/6EC^2$, notice that the length is in the numerator. Therefore, under shock loading, doubling the span of the beam will also double the amount of energy that it can absorb for the same stress level. This is just the opposite of what happens under static loading. Under static loading conditions with a fixed load, doubling the length of the beam also doubles the stress level, but with shock loading, doubling the length of the beam reduces the stress level to only 70% of what it was before.

In another example, consider the case of a variable-depth girder shown in the lower portion of Figure 21.11. On the top left there is a prismatic beam (a beam of the same cross-section throughout) under steady load. If, for instance, this is a crane beam in the shop, and there is concern about the weight of the beam, the beam depth can be reduced at the ends. This reduces the dead weight, but leaves the strength capacity of the beam unchanged under a static loading condition. The right-hand side of Figure 21.11 illustrates what happens under shock loading. The equations show that by
reducing the beam depth, thus increasing the amount of bending stresses away from the center of the beam to a maximum, \( E_p \) is actually doubled.

**FIGURE 21.11: Steady load vs. shock load for a variable-depth beam. (From The Lincoln Electric Co. With permission.)**

In Table 21.2, the properties of two beams are analyzed. Beam A, a 12-in. deep, 65-lb beam, was first suggested. It has a moment of inertia of 533.4 in.\(^4\). The steady load section modulus will equal \( S = I/C \), which results in a value of 88 in.\(^3\). Taking a more conventional (i.e., static loading) approach, beam B represents an attempt to create a better beam for shock loading. In this case, the beam size is increased to a depth of 24 in. and a weight of 76 lb. Its moment of inertia is now 2000 in.\(^4\), or four times higher than that of beam A. However, in terms of shock loading alone, beam B has virtually no advantage over beam A because the value of \( I/C^2 \) has not changed. From Equation 21.1, \( I/C^2 \) is directly proportional to the amount of potential energy that can be absorbed, therefore, since \( I/C^2 \) has not changed, no additional energy can be absorbed by beam B as compared to beam A.

**TABLE 21.2 Properties of Beam A and Beam B**

<table>
<thead>
<tr>
<th>Section property</th>
<th>Beam A</th>
<th>Beam B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Moment of inertia, I/C</td>
<td>533.4 in.(^4)</td>
<td>2096.4 in.(^4)</td>
</tr>
<tr>
<td>Steady load strength, ( S = \frac{F}{4aL} )</td>
<td>6.06 in.</td>
<td>11.96 in.</td>
</tr>
<tr>
<td>Shock load strength, ( \frac{I}{C^2} )</td>
<td>533.4/16067 = 88.2 in.(^3)</td>
<td>2096.4/161130 = 125 in.(^3)</td>
</tr>
<tr>
<td>( \frac{F}{C^2} )</td>
<td>14.5 in.(^2)</td>
<td>14.6 in.(^2)</td>
</tr>
</tbody>
</table>

To determine the relative efficiency or economy of the configuration, it is possible to divide \( I/C^2 \) by the cross-sectional area \( A \). Since the material costs are directly related to the material weight (weight is equal to the cross-sectional area multiplied by the density and the length), \( I/C^2 A \) is a good measure of the configuration's efficiency. Figure 21.12 shows that, for the same flange dimension and web thickness, changing the overall depth of section has little effect upon the shock load strength, \( I/C^2 A \). For this reason, when a beam is made deeper, it also becomes more rigid. When a shock load strikes it, the resultant force will be extremely high due to the small deflection. This example

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demonstrates the importance of discarding the preconceived ideas that come from experience with static loads, and embracing a new way of thinking that is appropriate for shock loading situations.

**FIGURE 21.12:** Effect of depth on shock loading capacity. (From The Lincoln Electric Co. With permission.)

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>15 in$^2$</td>
<td>12.5 in$^2$</td>
</tr>
<tr>
<td>I</td>
<td>1270 in$^4$</td>
<td>324.17 in$^4$</td>
</tr>
<tr>
<td>C</td>
<td>11 in</td>
<td>6 in</td>
</tr>
</tbody>
</table>

$\frac{1}{C^2} = 10.496$ in$^2$, $9.005$ in$^2$, $7.628$ in$^2$  

$\frac{1}{C^2A} = 0.6997$, 0.7204, 0.6780

**FIGURE 21.13:** Effect of web thickness on shock loading capacity. (From The Lincoln Electric Co. With permission.)

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>20 in$^2$</td>
<td>15 in$^2$</td>
</tr>
<tr>
<td>I</td>
<td>386.67 in$^4$</td>
<td>345.0 in$^4$</td>
</tr>
<tr>
<td>C</td>
<td>6 in</td>
<td>6 in</td>
</tr>
</tbody>
</table>

$\frac{1}{C^2} = 10.741$ in$^2$, $9.583$ in$^2$, $9.005$ in$^2$  

$\frac{1}{C^2A} = 0.5370$, 0.6389, 0.7204

The effect of the beam web thickness is examined in Figure 21.13. For the same overall depth of section and flange size, decreasing the web thickness increases the shock load strength, $I/C^2A$. This
example makes it clear that decreasing the web thickness actually increases the efficiency of shock load absorption for the same cross-sectional area.

Figure 21.14, on the other hand, demonstrates what happens when the flange width is considered a variable. For the same overall depth of section, increasing the flange width increases the shock load strength, \( I/C^2 A \). This puts the steel on the outer fibers, which is an improvement for shock loading. Two different section geometries are considered in Figure 21.15. The box-shaped section has a much higher moment of inertia than the H-shaped section; however, the distance to the outer fiber in both cases is equal. By implementing the box-shaped design, there is an increased capability to absorb a shock load over the H section. In the examples previously presented, it is important to realize that improvements were made by changing the section, not the material.

\[
\begin{array}{ccc}
I & 22.5 \text{ in}^2 & 12.5 \text{ in}^2 & 7.5 \text{ in}^2 \\
C & 6 \text{ in} & 6 \text{ in} & 6 \text{ in} \\
\frac{1}{C^2} & 17.431 \text{ in}^2 & 9.005 \text{ in}^2 & 4.792 \text{ in}^2 \\
\frac{1}{C^2 A} & 0.7747 & 0.7204 & 0.6389 \\
\end{array}
\]

FIGURE 21.14: Effect of flange width on shock loading capacity. (From The Lincoln Electric Co. With permission.)

\[
\begin{array}{ccc}
I & 16.39 \text{ in}^2 & 16.39 \text{ in}^2 \\
C & 3.5 \text{ in} & 3.5 \text{ in} \\
\frac{1}{C^2} & 2.638 \text{ in}^2 & 9.963 \text{ in}^3 \\
\frac{1}{C^2 A} & 0.1610 & 0.6078 \\
\end{array}
\]

FIGURE 21.15: Effect of section geometry on shock loading capacity. (From The Lincoln Electric Co. With permission.)
21.6 Detailing and Workmanship for Shock Loading

The fabrication of structures subjected to shock loading is generally no different from that required for slowly loaded structures, except that good workmanship is even more important. Wherever shock loading is a factor, the smallest fabrication problem can have negative, sometimes disastrous, consequences. For welded structures, postweld nondestructive inspection of these assemblies is recommended to verify the integrity of the welds.

21.7 An Example of Shock Loading

Figure 21.16 illustrates a real-world example of shock loading. The drawing shows ore falling into a primary gyratory crushe. When the ore comes down the chute, it strikes a bumper plate rather than falling with great impact directly into the crushe. This transforms some of the kinetic energy into the structure that supports the bumper. If the bumper plate is considered as a design problem, the falling ore has a normal velocity component to the surface of the bumper plate. Prior to impact, the normal velocity of the ore could equal 213 in./s. When the ore strikes the bumper plate, this normal velocity becomes zero.

If the force caused by the moving ore is found in terms of an incremental time \((\Delta t)\), the “impulse” (force multiplied by incremental time) is equal to the change in momentum of the falling ore. After determining the force applied to the bumper plate, the plate size can be calculated. Equations 21.2 through 21.7 below outline the calculation procedure for this problem.

Mass of ore:

\[
m = \frac{W}{g} = \frac{AV_1(\Delta t)\delta}{g}
\]  

Impulse:

\[
Imp = F(\Delta t)
\]
Change in momentum:

\[ \Delta M = mV_n - mV_0 \quad (21.4) \]

Impulse equals change in momentum:

\[ \text{Imp} = \Delta M \quad (21.5) \]

\[ F(\Delta t) = \Delta M = mV_n - mV_0 = \left[ \frac{AV_1(\Delta t)\delta}{g} \right] V_n - mV_0 \quad (21.6) \]

\[ F = \frac{AV_1\delta V_n}{g} \quad (21.7) \]

where

\[ m = \text{mass of ore} \]
\[ W = \text{weight of ore} \]
\[ t = \text{time ore falls} \]
\[ A = \text{cross-sectional area of ore stream} \]
\[ \delta = \text{density of ore} \]
\[ g = \text{acceleration of gravity} \]
\[ V_1 = \text{ore velocity prior to impact} \]
\[ V_0 = \text{ore velocity after impact (assume } V_0 = 0) \]
\[ V_n = \text{normal velocity of ore at impact} \]
\[ F = \text{force of impact} \]

This force, \( F \), can then be used to size the bumper plate and the supporting structure of the crusher.

### 21.8 Conclusions

In structural design, historically, the first tool an engineer utilizes is one applicable to static loading. However, in shock loading applications, new techniques are required to prevent failure and to utilize energy absorption. For example, optimum shock loading performance occurs when a member's complete volume is stressed to a maximum.

When sizing a structural member, remember to consider both the material properties and the section properties. The amount of absorbed kinetic energy is directly related to both of these properties. Detailing and workmanship are also crucial because flaws and cracks act as stress raisers that can reduce the member's ability to withstand shock loads.

### 21.9 Defining Terms

Elastic capacity: The maximum amount of potential energy that can be stored in a member.

Fatigue loading: Alternating, repeated, or fluctuating stresses that a member experiences over its operating life.

Impulse: The product of the force on a body and the time during which that force acts; equal to the change in momentum.

Kinetic energy: Energy associated with motion.

Moment of inertia: A measure of the resistance of a body to angular acceleration about a given axis that is equal to the sum of the products of each element of mass in the body and the square of the element's distance from the axis.

Momentum: A property of a moving body that the body has by virtue of its mass and motion and that is equal to the product of the body's mass and velocity.
Potential energy: Energy that a piece of matter has because of its position.
Shock/fatigue combination loading: The application of a shock load repeated many times.
Shock loading: The application of an extremely high force over a very short duration of time.
Static loading: Constant loading of a member.

References


Further Reading