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Plate and Box Girders

Mohamed Elgaaly
*Department of Civil &
Architectural Engineering,
Drexel University,
Philadelphia, PA*

19.1 Introduction

Plate and box girders are used mostly in bridges and industrial buildings, where large loads and/or long spans are frequently encountered. The high torsional strength of box girders makes them ideal for girders curved in plan. Recently, thin steel plate shear walls have been effectively used in buildings. Such walls behave as vertical plate girders with the building columns as flanges and the floor beams as intermediate stiffeners. Although traditionally simply supported plate and box girders are built up to 150 ft span, several three-span continuous girder bridges have been built in the U.S. with center spans exceeding 400 ft.

In its simplest form a plate girder is made of two flange plates welded to a web plate to form an I section, and a box girder has two flanges and two webs for a single-cell box and more than two webs in multi-cell box girders (Figure 19.1). The designer has the freedom in proportioning the cross-section of the girder to achieve the most economical design and taking advantage of available high-strength steels. The larger dimensions of plate and box girders result in the use of slender webs and flanges, making buckling problems more relevant in design. Buckling of plates that are adequately supported along their boundaries is not synonymous with failure, and these plates exhibit post-buckling strength that can be several times their buckling strength, depending on the plate slenderness. Although plate
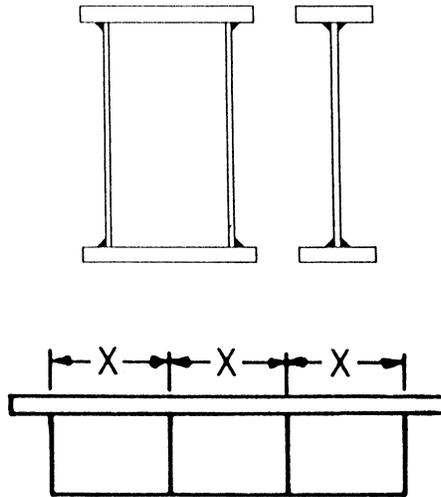
FIGURE 19.1: Plate and box girders.

buckling has not been the basis for design since the early 1960s, buckling strength is often required to calculate the post-buckling strength.

The trend toward limit state format codes placed the emphasis on the development of new design approaches based on the ultimate strength of plate and box girders and their components. The post-buckling strength of plates subjected to shear is due to the diagonal tension field action. The post-buckling strength of plates subjected to uniaxial compression is due to the change in the stress distribution after buckling, higher near the supported edges. An effective width with a uniform stress, equal to the yield stress of the plate material, is used to calculate the post-buckling strength [40].

The flange in a box girder and the web in plate and box girders are often reinforced with stiffeners to allow for the use of thin plates. The designer has to find a combination of plate thickness and stiffener spacing that will optimize the weight and reduce the fabrication cost. The stiffeners in most cases are designed to divide the plate panel into subpanels, which are assumed to be supported along the stiffener lines. Recently, the use of corrugated webs resulted in employing thin webs without the need for stiffeners, thus reducing the fabrication cost and also improving the fatigue life of the girders.

The web of a girder and load-bearing diaphragms can be subjected to in-plane compressive patch loading. The ultimate capacity under this loading condition is controlled by web crippling, which can occur prior to or after local yielding. The presence of openings in plates subjected to in-plane loads is unavoidable in some cases, and the presence of openings affects the stability and ultimate strength of plates.

19.2 Stability of the Compression Flange

The compression flange of a plate girder subjected to bending usually fails in lateral buckling, local torsional buckling, or yielding; if the web is slender the compression flange can fail by vertical buckling into the web (Figure 19.2).
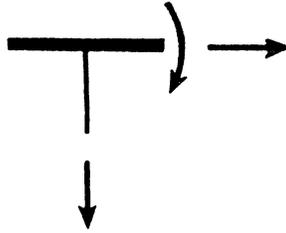
FIGURE 19.2: Compression flange modes of failure.

19.2.1 Vertical Buckling

The following limiting value for the web slenderness ratio to preclude this mode of failure [4] can be used,

$$h/t_w \leq \left[0.68 E / \sqrt{F_{yf} \left(F_{yf} + F_r \right)} \right] \sqrt{A_w / A_f} \tag{19.1}$$

where h and t_w are the web height and thickness, respectively; A_w is the area of the web; A_f is the area of the flange; E is Young's modulus of elasticity; F_{yf} is the yield stress of the flange material; and F_r is the residual tension that must be overcome to achieve uniform yielding in compression. This limiting value may be too conservative since vertical buckling of the compression flange into the web occurs only after general yielding of the flange. This limiting value, however, can be helpful to avoid fatigue cracking under repeated loading due to out-of-plane flexing, and it also facilitates fabrication.

The American Institute of Steel Construction (AISC) specification [32] uses Equation 19.1 when the spacing between the vertical stiffeners, a, is more than 1.5 times the web depth, h ($a/h > 1.5$). In such a case the specification recommends that

$$h/t_w \leq 14{,}000 / \sqrt{F_{yf}(F_{yf} + 16.5)} \tag{19.2}$$

where a minimum value of $A_w/A_f = 0.5$ was assumed and the residual tension was taken to be 16.5 ksi. Furthermore, when a/h is less than or equal to 1.5, higher web slenderness is permitted, namely

$$h/t_w \leq 2000 / \sqrt{F_{yf}} \tag{19.3}$$

19.2.2 Lateral Buckling

When a flange is not adequately supported in the lateral direction, elastic lateral buckling can occur. The compression flange, together with an effective area of the web equal to $A_w/6$, can be treated as a column and the buckling stress can be calculated from the Euler equation [2]:

$$F_{cr} = \pi^2 E / (\lambda)^2 \tag{19.4}$$

where λ is the slenderness ratio, which is equal to L_b/r_T; L_b is the length of the unbraced segment of the beam; and r_T is the radius of gyration of the compression flange plus one-third of the compression portion of the web.

The AISC specification adopted Equation 19.4, rounding $\pi^2 E$ to 286,000 and assuming that elastic buckling will occur when the slenderness ratio, λ, is greater than $\lambda_r (= 756/\sqrt{F_{yf}})$.

Furthermore, Equation 19.4 is based on uniform compression; in most cases the bending is not uniform within the length of the unbraced segment of the beam. To account for nonuniform bending,

Equation 19.4 should be multiplied by a coefficient, C_b [25], where

$$C_b = 12.5 M_{\max}/(2.5 M_{\max} + 3 M_A + 4 M_B + 3 M_C) \tag{19.5}$$

where

$M_{\max}$ = absolute value of maximum moment in the unbraced beam segment
M_A = absolute value of moment at quarter point of the unbraced beam segment
M_B = absolute value of moment at centerline of the unbraced beam segment
M_C = absolute value of moment at three-quarter point of the unbraced beam segment

When the slenderness ratio, λ, is less than or equal to $\lambda_p (= 300/\sqrt{F_{yf}})$, the flange will yield before it buckles, and $F_{cr} = F_{yf}$. When the flange slenderness ratio, λ, is greater than λ_p and smaller than or equal to λ_r, inelastic buckling will occur and a straight line equation must be adopted between yielding ($\lambda \leq \lambda_p$) and elastic buckling ($\lambda > \lambda_r$) to calculate the inelastic buckling stress, namely

$$F_{cr} = C_b F_{yf} \left[1 - 0.5(\lambda - \lambda_p)/(\lambda_r - \lambda_p) \right] \leq F_{yf} \tag{19.6}$$

19.2.3 Torsional Buckling

If the outstanding width-to-thickness ratio of the flange is high, torsional buckling may occur. If one neglects any restraint provided by the web to the flange rotation, then the flange can be treated as a long plate, which is simply supported (hinged) at one edge and free at the other, subjected to uniaxial compression in the longitudinal direction. The elastic buckling stress under these conditions can be calculated from

$$F_{cr} = k_c \pi^2 E/12(1 - \mu^2)\lambda^2 \tag{19.7}$$

where k_c is a buckling coefficient equal to 0.425 for a long plate simply supported and free at its longitudinal edges; λ is equal to $b_f/2t_f$; b_f and t_f are the flange width and thickness, respectively; and E and μ are Young's modulus of elasticity and the Poisson ratio, respectively.

The AISC specification adopted Equation 19.7, rounding $\pi^2 E/12(1 - \mu^2)$ to 26,200 and assuming $k_c = 4\sqrt{h/t_w}$, where $0.35 \leq k_c \geq 0.763$. Furthermore, to allow for nonuniform bending, the buckling stress has to be multiplied by C_b, given by Equation 19.5. Elastic torsional buckling of the compression flange will occur if λ is greater than $\lambda_r (= 230/\sqrt{F_{yf}/k_c})$. When λ is less than or equal to $\lambda_p (= 65/\sqrt{F_{yf}})$, the flange will yield before it buckles, and $F_{cr} = F_{yf}$. When $\lambda_p < \lambda \leq \lambda_r$, inelastic buckling will occur and Equation 19.6 shall be used.

19.2.4 Compression Flange of a Box Girder

Lateral-torsional buckling does not govern the design of the compression flange in a box girder. Unstiffened flanges and flanges stiffened with longitudinal stiffeners can be treated as long plates supported along their longitudinal edges and subjected to uniaxial compression. In the AASHTO (American Association of State Highway and Transportation Officials) specification [1], the nominal flexural stress, F_n, for the compression flange is calculated as follows:
If $w/t \leq 0.57\sqrt{kE/F_{yf}}$, then the flange will yield before it buckles, and

$$F_n = F_{yf} \tag{19.8}$$

If $w/t > 1.23\sqrt{kE/F_{yf}}$, then the flange will elastically buckle, and

$$F_n = k\pi^2 E/12(1 - \mu^2)(w/t)^2$$

or

$$F_n = 26,200 \, k(t/w)^2 \tag{19.9}$$

If $0.57\sqrt{kE/F_{yf}} < w/t \le 1.23\sqrt{kE/F_{yf}}$, then the flange buckles inelastically, and

$$F_n = 0.592F_{yf}[1 + 0.687\sin(c\pi/2)] \tag{19.10}$$

In Equations 19.8 to 19.10,

w = the spacing between the longitudinal stiffeners, or the flange width for unstiffened flanges
c = $\left[1.23 - (w/t)\sqrt{F_{yf}/kE}\right]/0.66$
k = $\left(8I_s/wt^3\right)^{1/3} \le 4.0$, for $n = 1$
k = $\left(14.3I_s/wt^3n^4\right)^{1/3} \le 4.0$, for $n = 2, 3, 4$, or 5
n = number of equally spaced longitudinal stiffeners
I_s = the moment of inertia of the longitudinal stiffener about an axis parallel to the flange and taken at the base of the stiffener

The nominal stress, F_n, shall be reduced for hybrid girders to account for the nonlinear variation of stresses caused by yielding of the lower strength steel in the web of a hybrid girder. Furthermore, another reduction is made for slender webs to account for the nonlinear variation of stresses caused by local bend buckling of the web. The reduction factors for hybrid girders and slender webs will be given in Section 19.3. The longitudinal stiffeners shall be equally spaced across the compression flange width and shall satisfy the following requirements [1].

The projecting width, b_s, of the stiffener shall satisfy:

$$b_s \le 0.48t_s\sqrt{E/F_{yc}} \tag{19.11}$$

where

t_s = thickness of the stiffener
F_{yc} = specified minimum yield strength of the compression flange

The moment of inertia, I_s, of each stiffener about an axis parallel to the flange and taken at the base of the stiffener shall satisfy:

$$I_s \ge \Psi wt^3 \tag{19.12}$$

where

Ψ = $0.125k^3$ for $n = 1$
    = $0.07k^3n^4$ for $n = 2, 3, 4$, or 5
n = number of equally spaced longitudinal compression flange stiffeners
w = larger of the width of compression flange between longitudinal stiffeners or the distance from a web to the nearest longitudinal stiffener
t = compression flange thickness
k = buckling coefficient as defined in connection with Equations 19.8 to 19.10

The presence of the in-plane compression in the flange magnifies the deflection and stresses in the flange from local bending due to traffic loading. The amplification factor, $1/(1 - \sigma_a/\sigma_{cr})$, can be used to increase the deflections and stresses due to local bending; where σ_a and σ_{cr} are the in-plane compressive and buckling stresses, respectively.

19.3 Web Buckling Due to In-Plane Bending

Buckling of the web due to in-plane bending does not exhaust its capacity; however, the distribution of the compressive bending stress changes in the post-buckling range and the web becomes less efficient. Only part of the compression portion of the web can be assumed effective after buckling. A reduction in the girder moment capacity to account for the web bend buckling can be used, and the following reduction factor [4] has been suggested:

$$R = 1 - 0.0005(A_w/A_f)(h/t - 5.7\sqrt{E/F_{yw}}) \tag{19.13}$$

It must be noted that when $h/t = 5.7\sqrt{E/F_{yw}}$, the web will yield before it buckles and there is no reduction in the moment capacity. This can be determined by equating the bend buckling stress to the web yield stress, i.e.,

$$k\pi^2 E / \left[12(1 - \mu^2)(h/t)^2 \right] = F_{yw} \tag{19.14}$$

where k is the web bend buckling coefficient, which is equal to 23.9 if the flange simply supports the web and 39.6 if one assumes that the flange provides full fixity; the 5.7 in Equation 19.13 is based on a k value of 36.

The AISC specification replaces the reduction factor given in Equation 19.13 by

$$R_{PG} = 1 - [a_r/(1,200 + 300a_r)] (h/t - 970/\sqrt{F_{cr}}) \tag{19.15}$$

where a_r is equal to A_w/A_f and 970 is equal to $5.7\sqrt{29000}$; it must be noted that the yield stress in Equation 19.13 was replaced by the flange critical buckling stress, which can be equal to or less than the yield stress as discussed earlier. It must also be noted that in homogeneous girders the yield stresses of the web and flange materials are equal; in hybrid girders another reduction factor, R_e, [39] shall be used:

$$R_e = \left[12 + a_r(3m - m^3) \right] / (12 + 2a_r) \tag{19.16}$$

where a_r is equal to the ratio of the web area to the compression flange area (≤ 10) and m is the ratio of the web yield stress to the flange yield or buckling stress.

19.4 Nominal Moment Strength

The nominal moment strength can be calculated as follows.
Based on tension flange yielding:

$$M_n = S_{xt} R_e F_{yt} \tag{19.17a}$$

or
Based on compression flange buckling:

$$M_n = S_{xc} R_{PG} R_e F_{cr} \tag{19.17b}$$

where S_{xc} and S_{xt} are the section moduli referred to the compression and tension flanges, respectively; F_{yt} is the tension flange yield stress; F_{cr} is the compression flange buckling stress calculated according to Section 19.2; R_{PG} is the reduction factor calculated using Equation 19.15; and R_e is a reduction factor to be used in the case of hybrid girders and can be calculated using Equation 19.16.

19.5 Web Longitudinal Stiffeners for Bending Design

Longitudinal stiffeners can increase the bending strength of plate girders. This increase is due to the control of the web lateral deflection, which increases its flexural stress capacity. The presence of the stiffener also improves the bending resistance of the flange due to a greater web restraint. If one longitudinal stiffener is used, its optimum location is 0.20 times the web depth from the compression flange. In this case the web plate elastic bend buckling stress increases more than five times that without the stiffener. Tests [8] showed that an adequately proportioned longitudinal stiffener at 0.2h from the compression flange eliminates bend buckling in girders with web slenderness, h/t, as large as 450. Girders with larger slenderness will require two or more longitudinal stiffeners to eliminate

web bend buckling. It must be noted that the increase in the bending strength of a longitudinally stiffened thin-web girder is usually small because the web contribution to the bending strength is small. However, longitudinal stiffeners can be important in a girder subjected to repeated loads because they reduce or eliminate the out-of-plane bending of the web, which increases resistance to fatigue cracking at the web-to-flange juncture and allows more slender webs to be used [42].

The AISC specification does not address longitudinal stiffeners; on the other hand, the AASHTO specification states that longitudinal stiffeners should consist of either a plate welded longitudinally to one side of the web or a bolted angle, and shall be located at a distance of 0.4 D_c from the inner surface of the compression flange, where D_c is the depth of the web in compression at the section with the maximum compressive flexural stress. Continuous longitudinal stiffeners placed on the opposite side of the web from the transverse intermediate stiffeners, as shown in Figure 19.3, are preferred. If longitudinal and transverse stiffeners must be placed on the same side of the web, it is preferable
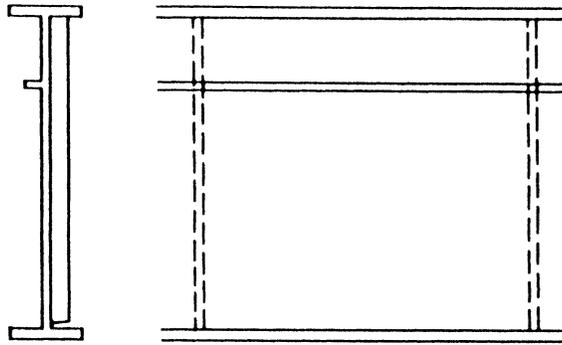
FIGURE 19.3: Longitudinal stiffener for flexure.

that the longitudinal stiffener not be interrupted for the transverse stiffener. Where the transverse stiffeners are interrupted, the interruptions must be carefully detailed with respect to fatigue.

To prevent local buckling, the projecting width, b_s of the stiffener shall satisfy the requirements of Equation 19.11. The section properties of the stiffener shall be based on an effective section consisting of the stiffener and a centrally located strip of the web not exceeding 18 times the web thickness. The moment of inertia of the longitudinal stiffener and the effective web strip about the edge in contact with the web, I_s, and the corresponding radius of gyration, r_s, shall satisfy the following requirements:

$$I_s \geq ht_w^3 \left[2.4(a/h)^2 - 0.13 \right] \tag{19.18}$$

and

$$r_s \geq 0.234a\sqrt{F_{yc}/E} \tag{19.19}$$

where
a = spacing between transverse stiffeners

19.6 Ultimate Shear Capacity of the Web

As stated earlier, in most design codes buckling is not used as a basis for design. Minimum slenderness ratios, however, are specified to control out-of-plane deflection of the web. These ratios are derived to give a small factor of safety against buckling, which is conservative and in some cases extravagant.

Before the web reaches its theoretical buckling load the shear is taken by beam action and the shear stress can be resolved into diagonal tension and compression. After buckling, the diagonal compression ceases to increase and any additional loads will be carried by the diagonal tension. In very thin webs with stiff boundaries, the plate buckling load is very small and can be ignored and the shear is carried by a complete diagonal tension field action [41]. In welded plate and box girders the web is not very slender and the flanges are not very stiff; in such a case the shear is carried by beam action as well as incomplete tension field action.

Based on test results, the analytical model shown in Figure 19.4 can be used to calculate the ultimate shear capacity of the web of a welded plate girder [5]. The flanges are assumed to be too flexible to
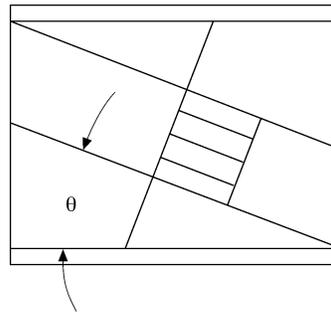
FIGURE 19.4: Tension field model by Basler.

support the vertical component from the tension field. The inclination and width of the tension field were defined by the angle Θ, which is chosen to maximize the shear strength. The ultimate shear capacity of the web, V_u, can be calculated from

$$V_u = \left[\tau_{cr} + 0.5\sigma_{yw}(1 - \tau_{cr}/\tau_{yw})\sin\Theta_d\right]A_w \qquad (19.20)$$

where

τ_{cr} = critical buckling stress in shear
τ_{yw} = yield stress in shear
σ_{yw} = web yield stress
Θ_d = angle of panel diagonal with flange
A_w = area of the web

In Equation 19.20, if $\tau_{cr} \geq 0.8\tau_{yw}$, the buckling will be inelastic and

$$\tau_{cr} = \tau_{cri} = \sqrt{0.8\tau_{cr}\tau_{yw}} \qquad (19.21)$$

It was shown later [23] that Equation 19.20 gives the shear strength for a complete tension field instead of the limited band shown in Figure 19.4. The results obtained from the formula, however, were in good agreement with the test results, and the formula was adopted in the AISC specification.

Many variations of this incomplete tension field model have been developed; are view can be found in the SSRC *Guide to Stability Design Criteria for Metal Structures* [22]. The model shown in Figure 19.5 [36, 38] gives better results and has been adopted in codes in Europe. In the model shown in Figure 19.5, near failure the tensile membrane stress, together with the buckling stress, causes yielding, and failure occurs when hinges form in the flanges to produce a combined mechanism that includes the yield zone ABCD. The vertical component of the tension field is added to the shear at buckling and combined with the frame action shear to calculate the ultimate shear strength. The
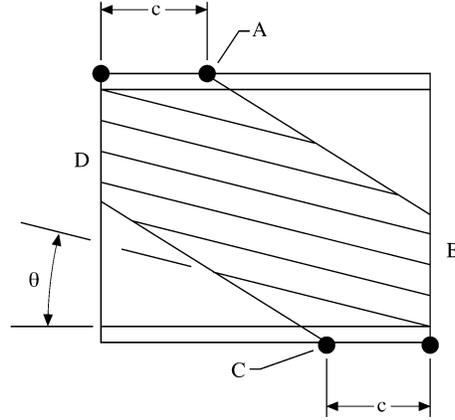
FIGURE 19.5: Tension field model by Rockey et al.

ultimate shear strength is determined by adding the shear at buckling, the vertical component of the tension field, and the frame action shear, and is given by

$$V_u = \tau_{cr} A_w + \sigma_t A_w \left[(2c/h) + \cot \Theta - \cot \Theta_d \right] \sin^2 \Theta + 4M_p/c \qquad (19.22)$$

where

$$\sigma_t = -1.5\tau_{cr} \sin 2\Theta + \sqrt{\sigma_{yw}^2 + (2.25 \sin^2 2\Theta - 3)\tau_{cr}^2}$$

$$c = (2/\sin \Theta)\sqrt{Mp/(\sigma_t t_w)} \quad 0 \le c \le a$$

M_p = plastic moment capacity of the flange with an effective depth of the web, b_e, given by

$$b_e = 30t_w[1 - 2(\tau_{cr}/\tau_{yw})]$$

where $(\tau_{cr}/\tau_{yw}) \le 0.5$; reduction in M_p due to the effect of the flange axial compression shall be considered and when $\tau_{cr} > 0.8\tau_{yw}$,

$$\tau_{cr} = \tau_{cri} = \tau_{yw}[1 - 0.16(\tau_{yw}/\tau_{cr})]$$

The maximum value of V_u must be found by trial; Θ is the only independent variable in Equation 19.22, and the optimum is not difficult to determine by trial since it is between $\Theta_d/2$ and 45 degrees, and V_u is not sensitive to small changes from the optimum Θ.

Recently [2, 33], it has been argued that the post-buckling strength arises not due to a diagonal tension field action, but by redistribution of shear stresses and local yielding in shear along the boundaries. A case in between is to model the web panel as a diagonal tension strip anchored by corner zones carrying shear stresses and act as gussets connecting the diagonal tension strip to the vertical stiffeners which are in compression [9]. On the basis of test results, it can be concluded that unstiffened webs possess a considerable reserve of post-buckling strength [16, 24]. The incomplete diagonal tension field approach, however, is only reasonably accurate up to a maximum aspect ratio (stiffeners spacing: web depth) equal to 6. Research is required to develop an appropriate method of predicting the post-buckling strength of unstiffened girders.

In the AISC specification, the shear capacity of a plate girder web can be calculated, using the model shown in Figure 19.4, as follows:

For $h/t_w \le 187\sqrt{k_v/F_{yw}}$, the web yields before buckling, and

$$V_n = 0.6A_w F_{yw} \qquad (19.23)$$

For $h/t_w > 187\sqrt{k_v/F_{yw}}$, the web will buckle and a tension field will develop, and

$$V_n = 0.6A_wF_{yw}\left[C_v + (1 - C_v)/1.15\sqrt{1 + (a/h)^2}\right] \qquad (19.24)$$

where

k_v = buckling coefficient $= 5 + 5/(a/h)^2$
 = 5, if $(a/h) > 3$ or $[260/(h/t_w)]^2$
C_v = ratio of the web buckling stress to the shear yield stress of the web
 = $187\sqrt{k_v/F_{yw}}/(h/t_w)$, for $187\sqrt{k_v/F_{yw}} \le h/tw \le 234\sqrt{k_v/F_{yw}}$
 = $44{,}000\,k_v/[(h/t_w)^2F_{yw}]$, for $h/t_w > 234\sqrt{k_v/F_{yw}}$

It must be noted that, in the above, the web buckling is elastic when $h/t_w > 234\sqrt{k_v/F_{yw}}$.

The AISC specification does not permit the consideration of tension field action in end panels, hybrid and web-tapered plate girders, and when a/h exceeds 3.0 or $[260/(h/tw)]^2$. This is contrary to the fact that a tension field can develop in all these cases; however, little or no research was conducted. Furthermore, tension field can be considered for end panels if the end stiffener is designed for this purpose. When neglecting the tension field action, the nominal shear capacity can be calculated from

$$V_n = 0.6A_wF_{yw}C_v \qquad (19.25)$$

Care must be exercised in applying the tension field models developed primarily for welded plate girders to the webs of a box girder. The thin flange of a box girder can provide very little or no resistance against movements in the plane of the web. If the web of a box girder is transversely stiffened and if the model shown in Figure 19.4 is used, it may overpredict the web strength. Hence, it is advisable to use the model shown in Figure 19.5, assuming the plastic moment capacity of the flange to be negligible.

19.7 Web Stiffeners for Shear Design

Transverse stiffeners must be stiff enough to prevent out-of-plane displacement along the panel boundaries in computing shear buckling of plate girder webs. To provide the out-of-plane support an equation, developed for an infinitely long web with simply supported edges and equally spaced stiffeners, to calculate the required moment of inertia of the stiffeners, I_s, namely for $a \le h$,

$$I_s = 2.5ht_w^3[(h/a) - 0.7(a/h)] \qquad (19.26)$$

The AASHTO formula for load-factor design is

$$I_s = Jat_w^3 \qquad (19.27)$$

where

J = $[2.5(h/a)^2 - 2] \ge 0.5$

Equation 19.27 is the same as Equation 19.26 except that the coefficient of (a/h) in the second term between brackets is 0.8 instead of 0.7. Equation 19.27 was adopted by the AISC specification as well. The moment of inertia of the transverse stiffener shall be taken about the edge in contact with the web for single-sided stiffeners and about the mid-thickness of the web for double-sided stiffeners.

To prevent local buckling of transverse stiffeners, the width, b_s, of each projecting stiffener element shall satisfy the requirements of Equation 19.11 using the yield stress of the stiffener material rather than that of the flange, as in Equation 19.11. Furthermore, b_s shall also satisfy the following requirements:

$$16.0t_s \ge b_s \le 0.25b_f \qquad (19.28)$$

where

b_f =   the full width of the flange

Transverse stiffeners shall consist of plates or angles welded or bolted to either one or both sides of the web. Stiffeners that are not used as connection plates shall be a tight fit at the compression flange, but need not be in bearing with the tension flange. The distance between the end of the web-to-stiffener weld and the near edge of the web-to-flange fillet weld shall not be less than $4t_w$ or more than $6t_w$. Stiffeners used as connecting plates for diaphragms or cross-frames shall be connected by welding or bolting to both flanges.

In girders with longitudinal stiffeners the transverse stiffener must also support the longitudinal stiffener as it forces a horizontal node in the bend buckling configuration of the web. In such a case it is recommended that the transverse stiffener section modulus, S_T, be equal to $S_L(h/a)$, where S_L is the section modulus of the longitudinal stiffener and h and a are the web depth and the spacing between the transverse stiffeners, respectively. In the AASHTO specification, the moment of inertia of transverse stiffeners used in conjunction with longitudinal stiffeners shall also satisfy

$$I_t \geq (b_t/b_l)(h/3a)I_l \tag{19.29}$$

where b_t and b_l = projecting width of transverse and longitudinal stiffeners, respectively, and I_t and I_l = moment of inertia of transverse and longitudinal stiffeners, respectively.

Transverse stiffeners in girders that rely on a tension field must also be designed for their role in the development of the diagonal tension. In this situation they are compression members, and so must be checked for local buckling. Furthermore, they must have cross-sectional area adequate for the axial force that develops. The axial force, F_s, can be calculated based on the analytical model [5] shown in Figure 19.4, and is given by

$$F_s = 0.5F_{yw}at_w \left[1 - \left(\tau_{cr}/\tau_{yw}\right)\right](1 - \cos \Theta_d) \tag{19.30}$$

The AISC and AASHTO specifications assume that a width of the web equal to $18t_w$ acts with the stiffener and give the following formula for the cross-sectional area, A_s, of the stiffeners:

$$A_s \geq \left[0.15Bht_w(1 - C_v)V_u/0.9V_n - 18t_w^2\right](F_{yw}/F_{ys}) \tag{19.31}$$

where the new notations are

$0.9V_u$ =   shear due to factored loads
B =   1.0 for double-sided stiffeners
      =   1.8 for single-sided angle stiffeners
      =   2.4 for single-sided plate stiffeners

If longitudinally stiffened girders are used, h in Equation 19.31 shall be taken as the depth of the web, since the tension field will occur between the flanges and the transverse stiffeners.

The optimum location of a longitudinal stiffener that is used to increase resistance to shear buckling is at the web mid-depth. In this case the two subpanels buckle simultaneously and the increase in the critical stress is substantial. To obtain the tension field shear resistance one can assume that only one tension field is developed between the flanges and transverse stiffeners even if longitudinal stiffeners are used.

19.8   Flexure-Shear Interaction

The shear capacity of a girder is independent of bending as long as the applied moment is less than the moment that can be taken by the flanges alone, $M_f = \sigma_{yf}A_f h$; any larger moment must be resisted in part by the web, which reduces the shear capacity of the girder. When the girder is subjected to

pure bending with no shear, the maximum moment capacity is equal to the plastic moment capacity, M_p, due to yielding of the girder's entire cross-section. In view of the tension field action and based on test results [3], a simple conservative interaction equation is given by

$$(V/V_u)^2 + (M - M_f)/(M_p - M_f) = 1 \tag{19.32}$$

where M and V are the applied moment and shear, respectively.

In the AISC specification, plate girders with webs designed for tension field action and when the ultimate shear, V_u, is between 0.54 and $0.9V_n$, and the ultimate moment, M_u, is between 0.675 and $0.9M_n$, where V_n and M_n are the nominal shear and moment capacities in absence of one another; the following interaction equation must be satisfied,

$$M_u/0.9M_n + 0.625(V_u/0.9V_n) \leq 1.375 \tag{19.33}$$

19.9 Steel Plate Shear Walls

Although the post-buckling behavior of plates under monotonic loads has been under investigation for more than half a century, post-buckling strength of plates under cyclic loading has not been investigated until recently [7]. The results of this investigation indicate that plates can be subjected to few reversed cycles of loading in the post-buckling domain, without damage. In steel plate shear walls, the boundary members are stiff and the plate is relatively thin; in such cases a complete tension field can be developed. The plate can be modeled as a series of tension bars inclined at an angle, ϕ [27]. The angle of inclination, ϕ, is a function of the panel length and height, the plate thickness, the cross-sectional areas of the surrounding beams and columns, and the moment of inertia of the columns. It can be determined by applying the principle of least work and is given by

$$\tan^4 \alpha = \left[(2/t_w L) + (1/A_c) \right] / \left[(2/t_w L) + (2h/A_b L) + (h^4/180 I_c L^2) \right] \tag{19.34}$$

where

α = angle of inclination of tension field with the vertical axis
L = panel length
h = panel height
t_w = wall thickness
A_b = cross-sectional area of beam
A_c = cross-sectional area of column
I_c = moment of inertia of column about axis perpendicular to the plane of the wall

Although this model can predict the ultimate capacity to a reasonable degree of accuracy it cannot depict the load-deflection characteristics to the same degree of accuracy. Based on test results and finite element analysis [17, 18], the stresses in the inclined tensile plate strips are not uniform but are higher near the supporting boundaries than the center of the plate, and yielding of these strips starts near their ends and propagates toward the midlength. The following method can be used to calculate the ultimate capacity and determine the load-deflection characteristics of a thin-steel-plate shear wall.

The plate in the shear wall is replaced by a series of truss elements in the diagonal tension direction, as shown in Figure 19.6. A minimum of four truss members shall be used to replace the plate panel in order to depict the panel behavior to a reasonable degree of accuracy; however, six members are recommended. The stress-strain relationship for the truss elements shall be assumed to be bilinearly elastic perfectly plastic, as shown in Figure 19.7, where E is Young's modulus of elasticity and σ_y is the tensile yield stress of the plate material. In Figure 19.7 the first slope represents the elastic
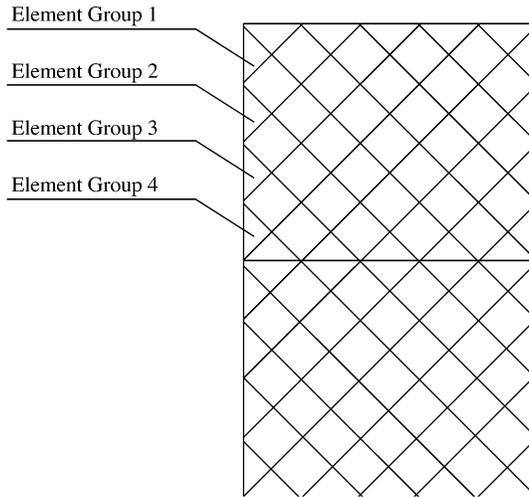
FIGURE 19.6: Steel plate shear wall analytical model.
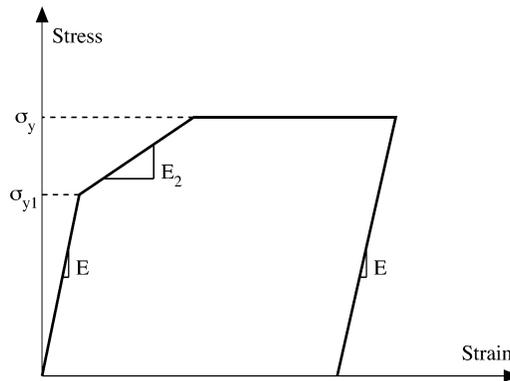
FIGURE 19.7: Stress-strain relationship for truss element.

response and the second represents the reduced stiffness caused by partial yielding; σ_{y1} and E_2 can be determined using a semi-empirical approach for welded as well as bolted shear walls, and can be calculated using the following equations:

$$\sigma_{y1} = (0.423 + 0.816 b_e/L)\sigma_y \tag{19.35}$$

where

$$b_e = \left[14.6\pi^2 E/12(1 - v^2)\tau_y \right]^{0.5} t \tag{19.36}$$

L = length of the strip
t = thickness of the plate
and

$$E_2 = \left[(\sigma_{y2} - \sigma_{y1})(3 - \sigma_{y1})/\sigma_{y2}(2 + \alpha) \right] E \tag{19.37}$$

where α is the ratio between the plastic strain, ε_p, and the strain at the initiation of yielding, ε_y. This ratio is in the range of 5 to 20, depending on the stiffness of the columns relative to the thickness of the plate; a value of 10 can be used in design. In the derivation of Equations 19.35 and 19.36 the inclination angle of the equivalent truss elements was assumed to be 45 degrees. The angle of inclination is usually in the range of 38 to 43 degrees and the effect of this assumption on the overall behavior of the wall is negligible.

In order to define the load displacement relationship of the truss elements in a bolted shear wall, the parameters P_{y1}, K_1, P_y, and K_2 shown in Figure 19.8 need to be calculated. For a bolted plate,
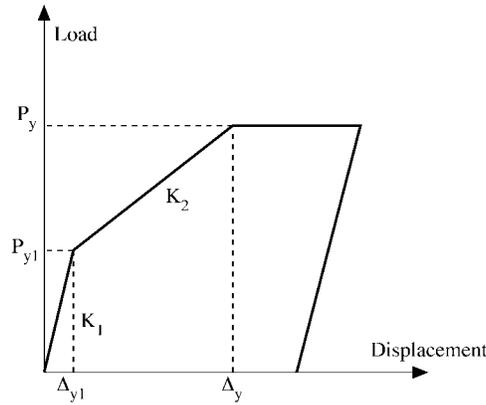
FIGURE 19.8: Load-displacement relationship for truss element.

the initial yielding can occur when the plate at the bolted connection starts slipping or when it locally yields near the boundaries as in the welded plate. The load due to slippage is controlled by the friction coefficient between the connected surfaces and the normal force applied by the bolts. In case the bolts were pre-tensioned to 70% of their ultimate tensile strength, slip will occur at a load equal to

$$P_{y1} = n(0.7\mu F_u^b A^b) \tag{19.38}$$

where n is the number of bolts at one end of the truss element; F_u^b and A^b are the ultimate tensile strength and cross-section area of the bolt, respectively; and μ is the friction coefficient between the connected surfaces. The load that causes initial yielding at the ends of the strip is the same as for the welded plate, and can be obtained from Equation 19.35 by multiplying the stress, σ_{y1}, by the strip cross-sectional area, A^p.

The usable load is the smaller of the loads that causes slippage or initial yielding at the ends of the strip. The initial stiffness of the equivalent truss element can be calculated as follows:

$$K_1 = EA^p/L \tag{19.39}$$

The ultimate load is controlled by the total yielding of the plate strip, tearing at the bolt holes, or shearing of the bolts. The smallest failure load is the controlling ultimate capacity of the truss element. The ultimate load due to the strip yielding along its entire length,

$$P_y' = \sigma_y A^p \tag{19.40}$$

The bolted shear wall should be designed such that plate yielding controls.

Tearing at the bolt holes will occur when the edge distance is small or the bolt spacing is large, which should be avoided in design because it is a brittle failure. The tearing load, denoted as P_y'', can be calculated using the following formula:

$$P_y'' = 1.4n\sigma_u(L_e - D/2)t \tag{19.41}$$

where n is the number of bolts at the end of the truss element, σ_u is the ultimate tensile strength of the plate material, L_e is the distance from the edge of the plate to the centroid of the bolt hole, D is the diameter of the bolt, and t is the thickness of the plate. Note that in the above formula the ultimate strength of the plate material in shear was assumed to be 0.7, its ultimate strength in tension.

The shear failure of the bolts can occur if the shear strength of the bolts is small or the spacing between them is large. In such case the ultimate capacity of the truss element, which is denoted as P_y''', can be calculated from one of the following formulas:

$$P_y''' = 0.45nF_u^b A^b \tag{19.42a}$$

$$P_y''' = 0.60nF_u^b A^b \tag{19.42b}$$

where n is the number of bolts and F_u^b and A^b are the ultimate strength and the gross cross-section area of the bolts, respectively. Equation 19.42a is used if the shear plane is within the threaded part of the bolt and Equation 19.42b is used if the shear plane is not within the threaded part of the bolt.

The ultimate load of the truss element is the smallest value of the plate total yielding capacity, the tearing capacity, and the bolt shearing capacity, i.e.,

$$P_y = \min(P_y', P_y'', P_y''') \tag{19.43}$$

As stated earlier, the plate yielding shall control and the designer must ensure that P_y' is the smallest of the three values.

As can be seen in Figure 19.8, in order to define the stiffness, K_2, the displacement of the truss element when the load reaches the ultimate capacity, P_y, needs to be determined. This ultimate displacement includes the stretching of the element as well as the slippage and the bearing deformation of the plate and the bolts at the connections. The plate strip represented by the truss element is stretched under load; the elongation includes both elastic and plastic deformations. As discussed earlier, due to the nonuniform strain distribution along the length of the strip, the plastic deformation will occur mostly near the ends. If one assumes that the strain distribution along the length of the strip is a second-degree parabola, and using a plastic deformation factor a, the elongation of the truss element due to the plate elastic and plastic deformations can be calculated using the following equation:

$$\Delta_{\text{def}} = (\sigma_y/3E)(2 + \alpha)L \tag{19.44}$$

The elongation of the truss element due to slippage can be approximated by two times the hole clearance, taking into consideration the slippage at both ends of the element [26]. The local deformation at the bolt holes includes the effects of shearing, bending, and bearing deformations of the fastener as well as local deformation of the connected plates, and can be taken as 0.2 times the bolt diameter [21]. Having defined the ultimate elongation of the truss element, its reduced stiffness after slippage and/or initial yielding can be obtained using the following equation:

$$K_2 = (P_y - P_{y1})/\left[0.125 + 0.2D + \sigma_y(\alpha - 1)L/(3E)\right] \tag{19.45}$$

19.10   In-Plane Compressive Edge Loading

Webs of plate and box girders and load-bearing diaphragms in box girders can be subjected to local in-plane compressive loads. Vertical (transverse) stiffeners can be provided at the location of the load to prevent web crippling; however, this is not always possible, such as in the case of a moving load, and it involves higher cost. Failure of the web under this loading is always due to crippling [10], as shown in Figure 19.9; in thin webs crippling occurs before yielding of the web and in stocky webs after yielding. The formula in the AISC specification predicts the crippling load, P_{cr}, to a reasonable
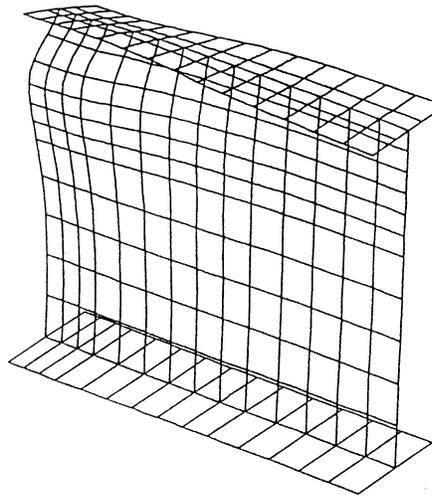
FIGURE 19.9: Deformed shape under in-plane edge loading (only half of the beam is shown).

degree of accuracy [37]; this formula is

$$P_{cr} = 135 t_w^2 \left[1 + 3(N/d) \left(t_w/t_f \right)^{1.5} \right] \left(F_{yw} t_f / t_w \right)^{0.5} \tag{19.46}$$

The formula given by Equation 19.46 is applicable if the load is applied at a distance not less than half the member's depth from its end; if the load is at a distance less than half the member's depth, the following formulae shall be used [14]:

$$\text{For } N/d \leq 0.2, \;\; P_{cr} = 68 t_w^2 \left[1 + 3(N/d) \left(t_w/t_f \right)^{1.5} \right] \left(F_{yw} t_f / t_w \right)^{0.5} \tag{19.47a}$$

$$\text{For } N/d > 0.2, \;\; P_{cr} = 68 t_w^2 \left[1 + (4N/d - 0.2)(t_w/t_f)^{1.5} \right] \left(F_{yw} t_f / t_w \right)^{0.5} \tag{19.47b}$$

In addition to web crippling, the AISC specification requires a web yielding check; furthermore, when the relative lateral movement between the loaded compression flange and the tension flange is not restrained at the point of load application, sidesway buckling must be checked.

19.11 Eccentric Edge Loading

Eccentricities in loading with respect to the plane of the web are unavoidable, and it was found that there is a reduction in the web capacity due to the presence of an eccentricity [13, 14]; for example, in one case, an eccentricity of 0.5 in. reduced the web ultimate capacity to about half its capacity under in-plane load. Furthermore, it was found that the effect of the load eccentricity in reducing the ultimate capacity decreases as the ratio of the flange-to-web thickness increases. A deformed beam subjected to eccentric load near failure is shown in Figure 19.10. Web strength reduction factors for various eccentricities as a function of the flange width and for various flange-to-web thickness ratios are given in Figure 19.11.
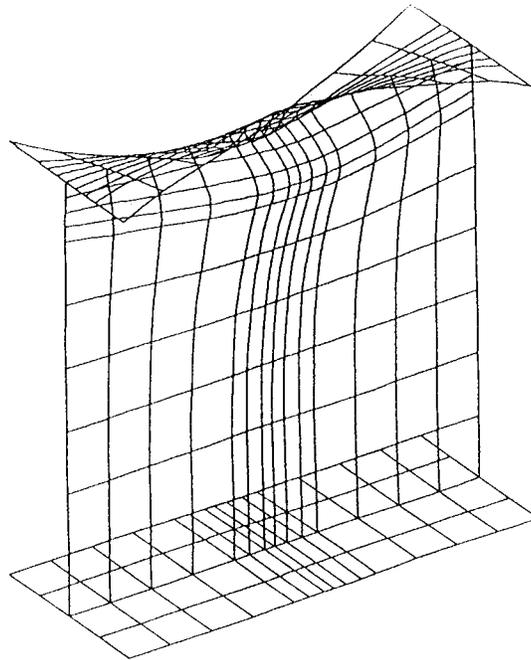
FIGURE 19.10: Deformed shape under eccentric edge loading.

The failure mechanism in the case of eccentric loading is different from that for in-plane loading. The flange twisting moment acting at the web-flange intersection can cause failure due to bending rather than crippling of the web, if the eccentricity is large enough. In most cases, however, the failure mode is due to a combination of web bending and crippling. Failure mechanisms were developed, and formulas to calculate the ultimate capacity of the web under eccentric edge loading were derived [15]. Currently, the AISC specification does not address the effect of the eccentricity on the reduction of the web crippling load. Eccentricities can also arise due to moments applied to the top flange in addition to vertical loads. An example would be a beam resting on the top flange of another beam and the two flanges are welded together. Rotation of the supported beam will impose a twisting moment in the flange of the supporting beam and bending of its web, which will reduce its crippling load.
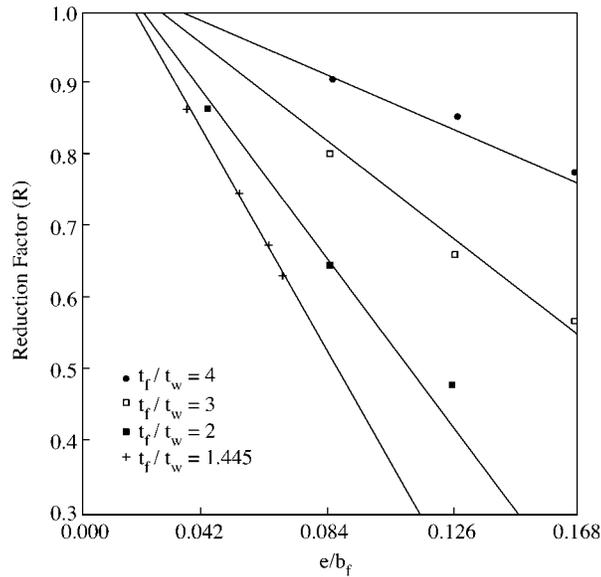
FIGURE 19.11: Eccentricity reduction factor.

19.12 Load-Bearing Stiffeners

Webs of girders are often strengthened with transverse stiffeners at points of concentrated loads and over intermediate and end supports. The AISC specification requires that these stiffeners be double sided, extend at least one-half the beam depth, and either bear on or be welded to the loaded flange. The specification, further, requires that they be designed as axially loaded members with an effective length equals to 0.75 times the web depth; and a strip of the web, with a width equal to 25 times its thickness for intermediate stiffeners and 12 times the thickness for end stiffeners, shall be considered in calculating the geometric properties of the stiffener.

The failure, in cases where the stiffener depth is less than 75% of the depth of the web, can be due to crippling of the web below the stiffener [14, 15], as shown in Figure 19.12. The failure, otherwise,
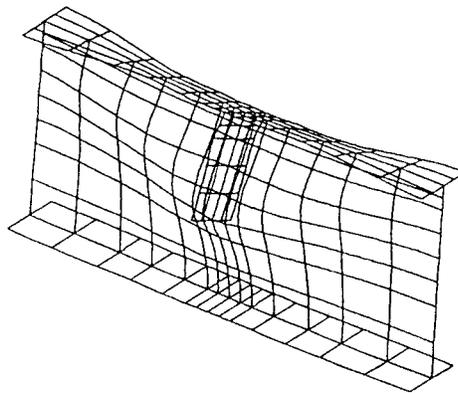
FIGURE 19.12: Web crippling below stiffener.

is due to global buckling of the stiffener, provided that the thickness of the stiffener is adequate to prevent local buckling. The optimum depth of the stiffener is 0.75 times the web depth. The AISC specification does not account for factors such as the stiffener depth and load eccentricity.

In box girders intermediate diaphragms are provided to limit cross-sectional deformation and load-bearing diaphragms are used at the supports to transfer loads to the bridge bearings. Diaphragm design is treated in the BS 5400: Part 3 (1983) [6] and discussed in Chapter 7 of the SSRC guide [22].

19.13 Web Openings

Openings are frequently encountered in the webs of plate and box girders. Research on the buckling and ultimate strength of plates with rectangular and circular openings subjected to in-plane loads has been performed by many investigators. The research has included reinforced and unreinforced openings. A theoretical method of predicting the ultimate capacity of slender webs containing circular and rectangular holes, and subjected to shear, has been developed [34, 35]. The solution is obtained by considering the equilibrium of two tension bands, one above and the other below the opening. These bands have been chosen to conform to the failure pattern observed in tested plate girders with holes. Experimental results showed that the method gives satisfactory and safe predictions. The calculated values were found to be between 5 and 30% below the test results.

Solutions for transversely stiffened webs subjected to shear and bending with centrally located holes are available [28, 30] and are applicable for webs with depth-to-thickness ratios of 120 to 360, panel aspect ratio between 0.7 and 1.5, hole depth greater than $1/10th$ of the web depth, and for circular, elongated circular, and rectangular holes.

19.14 Girders with Corrugated Webs

Corrugated webs can be used in an effort to decrease the weight of steel girders and reduce its fabrication cost. Studies have been conducted in Europe and Japan and girders with corrugated webs have been used in these countries [12]. The results of the studies indicate that the fatigue strength of girders with corrugated webs can be 50% higher compared to girders with flat stiffened webs. In addition to the improved fatigue life, the weight of girders with corrugated webs can be as much as 30 to 60% less than the weight of girders with flat webs and have the same capacity. Due to the weight savings, larger clear spans can be achieved. Beams and girders with corrugated webs are economical to use and can improve the aesthetics of the structure. Beams manufactured and used in Germany for buildings have a web thickness that varies between 2 and 5 mm, and the corresponding web height-to-thickness ratio is in the range of 150 to 260. The corrugated webs of two bridges built in France were 8 mm thick and the web height-to-thickness ratio was in the range of 220 to 375.

Failure in shear is usually due to buckling of the web and the failure in bending is due to yielding of the compression flange and its vertical buckling into the corrugated web, which buckles [19, 20]. The shear buckling mode is global for dense corrugation and local for course corrugation, as shown in Figure 19.13. The load-carrying capacity of the specimens drops after buckling, with some residual load-carrying capacity after failure. In the local buckling mode, the corrugated web acts as a series of flat-plate subpanels that mutually support each other along their vertical (longer) edges and are supported by the flanges at their horizontal (shorter) edges. These flat-plate subpanels are subjected to shear, and the elastic buckling stress is given by

$$\tau_{cre} = k_s \left[\pi^2 E / 12(1 - \mu^2)(w/t)^2 \right] \qquad (19.48)$$
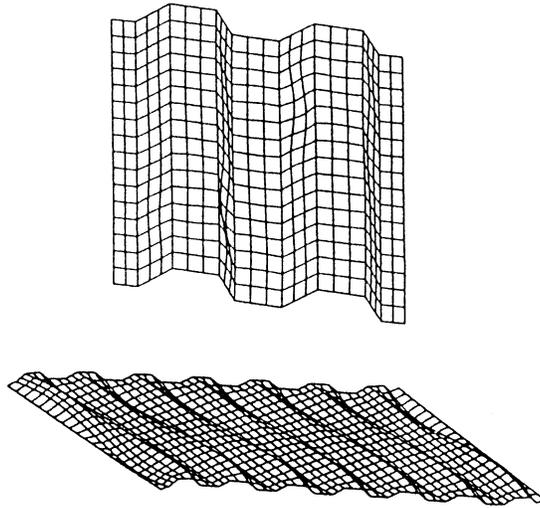
FIGURE 19.13: Local and global buckling.

where

k_s = buckling coefficient, which is a function of the panel aspect ratio, h/w, and the boundary support conditions

h = the web depth

t = the web thickness

w = the flat-plate subpanel width — the horizontal or the inclined, whichever is bigger

E = Young's modulus of elasticity

μ = the Poisson ratio

The buckling coefficient, ks, is given by

k_s = $5.34 + 2.31(w/h) - 3.44(w/h)^2 + 8.39(w/h)^3$, for the longer edges simply supported and the shorter edges clamped

k_s = $8.98 + 5.6(w/h)^2$, in the case where all edges are clamped

An average local buckling stress, $\tau_{av}(= 0.5[\tau_{ssf} + \tau_{fx}])$, is recommended, and in the case of $\tau_{cre} \geq 0.8\tau_y$, inelastic buckling will occur and the inelastic buckling stress, τ_{cri}, can be calculated by $\tau_{cri} = (0.8 * \tau_{cre} * \tau_y)^{0.5}$, where $\tau_{cri} \leq \tau_y$.

As stated earlier, the mode of failure is local and/or global buckling; when global buckling controls, the buckling stress can be calculated for the entire corrugated web panel, using orthotropic-plate buckling theory. The global elastic buckling stress, τ_{cre}, can be calculated from

$$\tau_{cre} = k_s \left[(D_x)^{0.25} (D_y)^{0.75} \right] / th^2 \qquad (19.49)$$

where

D_x = $(q/s)Et^3/12$

D_y = EI_y/q

I_y = $2bt(h_r/2)^2 + \{t(h_r)^3/6\sin\Theta\}$

k_s = Buckling coefficient, equal to 31.6 for simply supported boundaries and 59.2 for clamped boundaries

t = corrugated plate thickness

b, h_r, q, s, and Θ are as shown in Figure 19.14.

In the aforementioned, when $\tau_{cre} \geq 0.8\tau_y$, inelastic buckling will occur and the inelastic buckling stress, τ_{cri}, can be calculated by $\tau_{cri} = (0.8 * \tau_{cre} * \tau_y)^{0.5}$, where $\tau cri \leq \tau_y$. For design, it is recommended that the local and global buckling values be calculated and the smaller value controls.

As stated earlier, the failure in bending is due to compression flange yielding and vertical buckling into the web, as shown in Figure 19.15. The failure is sudden, with no appreciable residual strength. The web offers negligible contribution to the moment carrying capacity of the beam, and for design,
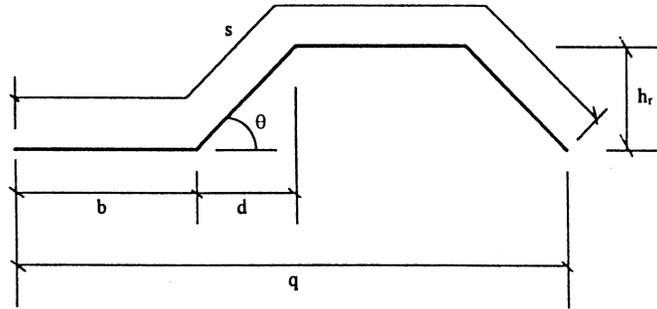
FIGURE 19.14: Dimensions of corrugation profile.

FIGURE 19.15: Bending failure of a beam with corrugate web.

the ultimate moment capacity can be calculated based on the flange yielding, ignoring any contribution from the web. The stresses in the web are equal to zero except near the flanges. This is because the corrugated web has no stiffness perpendicular to the direction of the corrugation, except for a

very small distance which is adjacent to and restrained by the flanges, and the stresses are appreciable only within the horizontal folds of the corrugation.

It must be noted that the common practice is to fillet weld the web to the flanges from one side only; under static loading this welding detail was found to be adequate and there is no need to weld from both sides. Finally, the bracing requirements of the compression flange in beams and girders with corrugated webs are less severe compared to conventional beams and girders with flat webs. Lateral-torsional buckling of beams and girders with corrugated webs has been investigated [20].

19.15 Defining Terms

AASHTO: American Association of State Highway and Transportation Officials.

AISC: American Institute of Steel Construction.

Buckling load: The load at which a compressed element or member assumes a deflected position.

Effective width: Reduced flat width of a plate element due to buckling, the reduced width is termed the effective width.

Corrugated web: A web made of corrugated steel plates, where the corrugations are parallel to the depth of the girder.

Factored load: The nominal load multiplied by a load factor to account for unavoidable deviations of the actual load from the nominal load.

Limit state: A condition at which a structure or component becomes unsafe (strength limit state) or no longer useful for its intended function (serviceability limit state).

LRFD (Load and Resistance Factor Design): A method of proportioning structural components such that no applicable limit state is exceeded when the structure is subjected to all appropriate load combinations.

Shear wall: A wall in a building to carry lateral loads from wind and earthquakes.

Stress: Force per unit area.

Web crippling: Local buckling of the web plate under local loads.

Web slenderness ratio: The depth-to-thickness ratio of the web.

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