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Shell Structures

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11.1 Introduction

11.1.1 Overview

Many steel structures, such as elevated water tanks, oil and water storage tanks, offshore structures, and pressure vessels, are comprised of shell elements that are subjected to compression stresses. The shell elements are subject to instability resulting from the applied loads. The theoretical buckling strength based on linear elastic bifurcation analysis is well known for stiffened as well as unstiffened cylindrical and conical shells and unstiffened spherical and torispherical shells. Simple formulas have been determined for many geometries and types of loads. Initial geometric imperfections and residual stresses that result from the fabrication process, however, reduce the buckling strength of fabricated shells. The amount of reduction is dependent on the geometry of the shell, type of loading (axial compression, bending, external pressure, etc.), size of imperfections, and material properties.

11.1.2 Production Practice

The behavior of a cylindrical shell is influenced to some extent by whether it is manufactured in a pipe or tubing mill or fabricated from plate material. The two methods of production will be referred to as manufactured cylinders and fabricated cylinders. The distinction is important primarily because of the differences in geometric imperfections and residual stress levels that may result from the two different production practices. In general, fabricated cylinders may be expected to have considerably larger magnitudes of imperfections (in out-of-roundness and lack of straightness) than the mill manufactured products. Similarly, fabricated heads are likely to have larger shape imperfections than those produced by spinning. Spun heads, however, typically have a greater variation in thickness and greater residual stresses due to the cold working. The design rules given in this chapter apply to fabricated steel shells.

Fabricated shells are produced from flat plates by rolling or pressing the plates to the desired shape and welding the edges together. Because of the method of construction, the mechanical properties of the shells will vary along the length and around the circumference. Misfit of the edges to be welded together may result in unintentional eccentricities at the joints. In addition, welding tends to introduce out-of-roundness and out-of-straightness imperfections that must be taken into account in the design rules.

11.1.3 Scope

Rules are given for determining the allowable compressive stresses for unstiffened and ring stiffened circular cylinders and cones and unstiffened spherical, ellipsoidal, and torispherical heads. The allowable stress equations are based on theoretical buckling equations that have been reduced by knockdown factors and by plasticity reduction factors that were determined from tests on fabricated shells. The research leading to the development of the allowable stress equations is given in [2, 7, 8, 9, 10].

Allowable compressive stress equations are presented for cylinders and cones subjected to uniform axial compression, bending moment applied over the entire cross-section, external pressure, loads that produce in-plane shear stresses, and combinations of these loads. Allowable compressive stress equations are presented for formed heads that are subjected to loads that produce unequal biaxial stresses as well as equal biaxial stresses.

11.1.4 Limitations

The allowable stress equations are based on an assumed axisymmetric shell with uniform thickness for unstiffened cylinders and formed heads and with uniform thickness between rings for stiffened cylinders and cones. All shell penetrations must be properly reinforced. The results of tests on reinforced openings and some design guidance are given in [6]. The stability criteria of this chapter may be used for cylinders that are reinforced in accordance with the recommendations of this reference if the openings do not exceed 10% of the cylinder diameter or 80% of the ring spacing. Special consideration must be given to the effects of larger penetrations.

The proposed rules are applicable to shells with D/t ratios up to 2000 and shell thicknesses of 3/16 in. or greater. The deviations from true circular shape and straightness must satisfy the requirements stated in this chapter.

Special consideration must be given to ends of members or areas of load application where stress distribution may be nonlinear and localized stresses may exceed those predicted by linear theory. When the localized stresses extend over a distance equal to one half the wave length of the buckling mode, they should be considered as a uniform stress around the full circumference. Additional thickness or stiffening may be required.

Failure due to material fracture or fatigue and failures caused by dents resulting from accidental loads are not considered. The rules do not apply to temperatures where creep may occur.

11.1.5 Stress Components for Stability Analysis and Design

The internal stress field that controls the buckling of a cylindrical shell consists of the longitudinal, circumferential, and in-plane shear membrane stresses. The stresses resulting from a dynamic analysis should be treated as equivalent static stresses.

11.1.6 Materials

Steel

The allowable stress equations apply directly to shells fabricated from carbon and low alloy steel plate materials such as those given in Table 11.1 or Table UCS-23 of [3]. The steel materials in Table 11.1 are designated by group and class. Steels are grouped according to strength level and welding characteristics. *Group I* designates mild steels with specified minimum yield stresses $\leq$ 40 ksi and these steels may be welded by any of the processes as described in [5]. *Group II* designates intermediate strength steels with specified minimum yield stresses $>$ 40 ksi and $\leq$ 52 ksi. These steels require the use of low hydrogen welding processes. *Group III* designates high strength steels with specified minimum yield stresses $>$ 52 ksi. These steels may be used provided that each application is investigated with respect to weldability and special welding procedures that may be required. Consideration should be given to fatigue problems that may result from the use of higher working stresses, and notch toughness in relation to other elements of fracture control such as fabrication, inspection procedures, service stress, and temperature environment.

The steels in Table 11.1 have been classified according to their notch toughness characteristics. *Class C* steels are those that have a history of successful application in welded structures at service temperatures above freezing. Impact tests are not specified. *Class B* steels are suitable for use where thickness, cold work, restraint, stress concentration, and impact loading indicate the need for improved notch toughness. When impact tests are specified, Class B steels should exhibit Charpy V-notch energy of 15 ft-lbs for Group 1 and 25 ft-lbs for Group II at the lowest service temperature. The Class B steels given in Table 11.1 can generally meet the Charpy requirements at temperatures ranging from 50° to 32°F. *Class A* steels are suitable for use at subfreezing temperatures and for critical applications involving adverse combinations of the factors cited above. The steels given in Table 11.1 can generally meet the Charpy requirements for Class B steels at temperatures ranging from $-4°$ to $-40°$F.

Other Materials

The design equations can also be applied to other materials for which a chart or table is provided in Subpart 3 of [4] by substituting the tangent modulus E_t for the elastic modulus E in the allowable stress equations. The method for finding the allowable stresses for shells constructed from these materials is determined by the following procedure.

TABLE 11.1 Steel Plate Materials

Group	Class	Specification	Specified minimum yield stress (ksi)[a]	Specified minimum tensile stress (ksi)[a]
I	C	ASTM A36 (to 2 in. thick)	36	58
		ASTM A131 Grade A (to 1/2 in. thick)	34	58
		ASTM A285 Grade C (to 3/4 in. thick)	30	55
I	B	ASTM A131 Grades B, D	34	58
		ASTM A516 Grade 65	35	65
		ASTM A573 Grade 65	35	65
		ASTM A709 Grade 36T2	36	58
I	A	ASTM A131 Grades CS, E	34	58
II	C	ASTM A572 Grade 42 (to 2 in. thick)	42	60
		ASTM A591 required over 1/2 in. thick		
		ASTM A572 Grade 50 (to 2 in. thick)	50	65
		ASTM A591 required over 1/2 in. thick		
II	B	ASTM A709 Grades 50T2, 50T3	50	65
		ASTM A131 Grade AH32	45.5	68
		ASTM A131 Grade AH36	51	71
II	A	API Spec 2H Grade 42	42	62
		API Spec 2H Grade 50 (to 2 1/2 in. thick)	50	70
		API Spec 2H Grade 50 (over 2 1/2 in. thick)	47	70
		API Spec 2W Grade 42 (to 1 in. thick)	42	62
		API Spec 2W Grade 42 (over 1 in. thick)	42	62
		API Spec 2W Grade 50 (to 1 in. thick)	50	65
		API Spec 2W Grade 50 (over 1 in. thick)	50	65
		API Spec 2W Grade 50T (to 1 in. thick)	50	70
		API Spec 2W Grade 50T (over 1 in. thick)	50	70
		API Spec 2Y Grade 42 (to 1 in. thick)	42	62
		API Spec 2Y Grade 42 (over 1 in. thick)	42	62
		API Spec 2Y Grade 50 (to 1 in. thick)	50	65
		API Spec 2Y Grade 50 (over 1 in. thick)	50	65
		API Spec 2Y Grade 50T (to 1 in. thick)	50	70
		API Spec 2Y Grade 50T (over 1 in. thick)	50	70
		ASTM A131 Grades DH32, EH32	45.5	68
		ASTM A131 Grades DH36, EH36	51	71
		ASTM A537 Class I (to 2 1/2 in. thick)	50	70
		ASTM A633 Grade A	42	63
		ASTM A633 Grades C, D	50	70
		ASTM A678 Grade A	50	70
III	A	ASTM A537 Class II (to 2 1/2 in. thick)	60	80
		ASTM A678 Grade B	60	80
		API Spec 2W Grade 60 (to 1 in. thick)	60	75
		API Spec 2W Grade 60 (over 1 in. thick)	60	75
		ASTM A710 Grade A Class 3 (to 2 in. thick)	75	85
		ASTM A710 Grade A Class 3 (2 in. to 4 in. thick)	65	75
		ASTM A710 Grade A Class 3 (over 4 in. thick)	60	70

[a] 1 ksi = 6.895 MPa

Step 1. Calculate the value of factor A using the following equations. The terms F_{xe}, F_{he}, and F_{ve} are defined in subsequent paragraphs.

$$A = \frac{F_{xe}}{E} \quad A = \frac{F_{he}}{E} \quad A = \frac{F_{ve}}{E}$$

Step 2. Using the value of A calculated in Step 1, enter the applicable material chart in Subpart 3 of [4] for the material under consideration. Move vertically to an intersection with the material temperature line for the design temperature. Use interpolation for intermediate temperature values.

Step 3. From the intersection obtained in Step 2, move horizontally to the right to obtain the value of B. E_t is given by the following equation:

$$E_t = \frac{2B}{A}$$

When values of A fall to the left of the applicable material/temperature line in Step 2, $E_t = E$.

Step 4. Calculate the allowable stresses from the following equations:

$$F_{xa} = \frac{F_{xe}}{FS}\frac{E_t}{E} \quad F_{ba} = F_{xa} \quad F_{ha} = \frac{F_{he}}{FS}\frac{E_t}{E} \quad F_{va} = \frac{F_{ve}}{FS}\frac{E_t}{E}$$

11.1.7 Geometries, Failure Modes, and Loads

Allowable stress equations are given for the following geometries and load conditions.

Geometries

1. Unstiffened Cylindrical, Conical, and Spherical Shells
2. Ring Stiffened Cylindrical and Conical Shells
3. Unstiffened Spherical, Ellipsoidal, and Torispherical Heads

The cylinder and cone geometries are illustrated in Figures 11.1 and 11.3 and the stiffener geometries are illustrated in Figure 11.4. The effective sections for ring stiffeners are shown in Figure 11.2. The maximum cone angle α shall not exceed 60°.

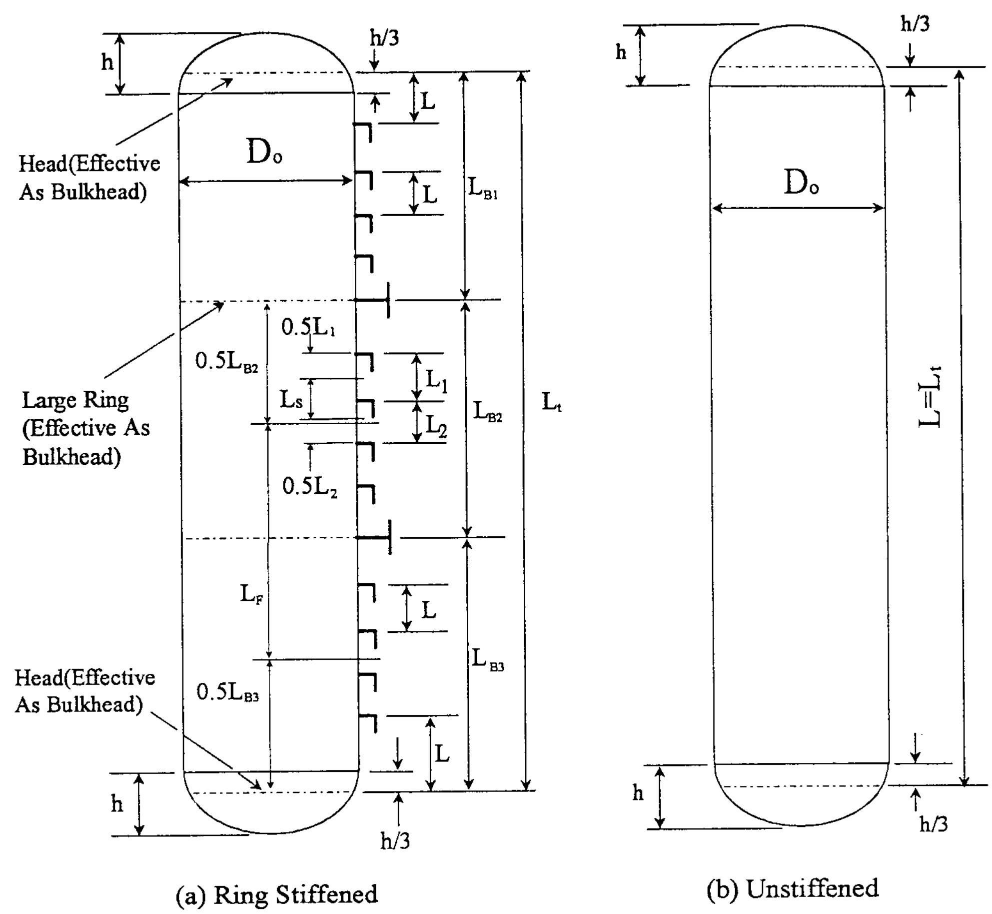

FIGURE 11.1: Geometry of cylinders.

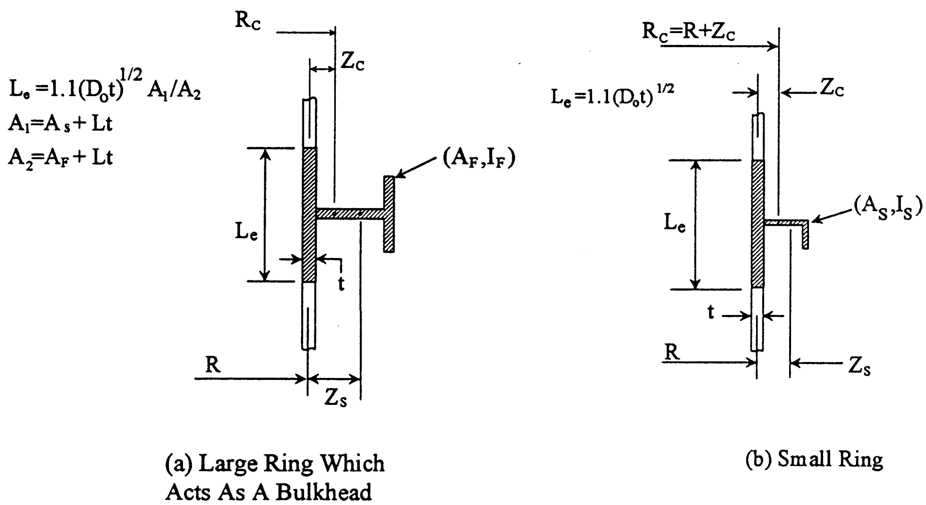

FIGURE 11.2: Sections through rings.

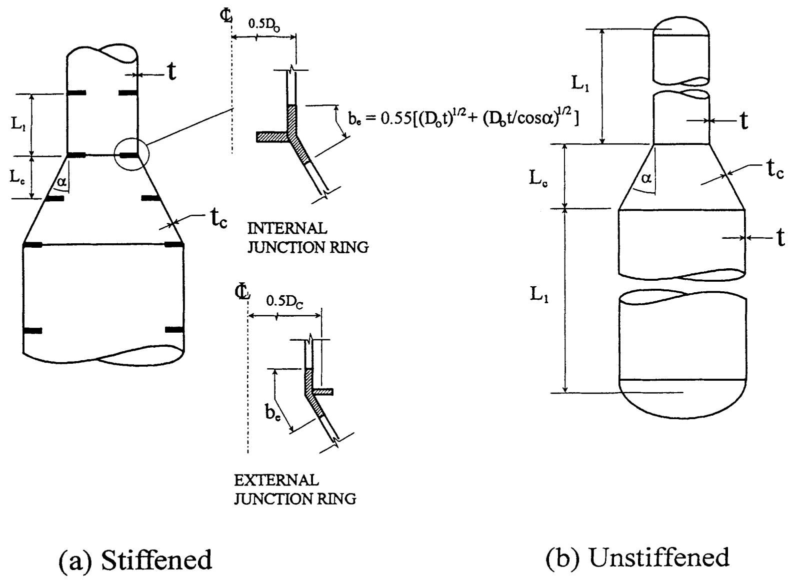

FIGURE 11.3: Geometry of conical sections.

Failure Modes

Buckling stress equations are given herein for four failure modes that are defined below. The buckling patterns are both load and geometry dependent.

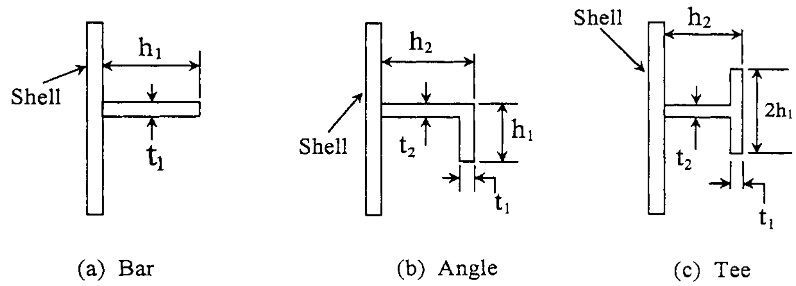

FIGURE 11.4: Stiffener geometry.

1. Local Shell Buckling—This mode of failure is characterized by the buckling of the shell in a radial direction. One or more waves will form in the longitudinal and circumferential directions. The number of waves and the shape of the waves are dependent on the geometry of the shell and the type of load applied. For ring stiffened shells, the stiffening rings are presumed to remain round prior to buckling.
2. General Instability—This mode of failure is characterized by the buckling of one or more rings together with the shell into a circumferential wave pattern with two or more waves.
3. Column Buckling—This mode of failure is characterized by out-of-plane buckling of the cylinder with the shell remaining circular prior to column buckling. The interaction between shell buckling and column buckling is taken into account by substituting the shell buckling stress for the yield stress in the column buckling formula.
4. Local Buckling of Rings—This mode of failure relates to the buckling of the stiffener elements such as the web and flange of a tee type stiffener. Most design rules specify requirements for compact sections to preclude this mode of failure. Very little analytical or experimental work has been done for this mode of failure in association with shell buckling.

Loads and Load Combinations

Allowable stress equations are given for the following types of stresses.

a. Cylinders and Cones

1. Uniform longitudinal compressive stresses
2. Longitudinal compressive stresses due to a bending moment acting across the full circular cross-section
3. Circumferential compressive stresses due to external pressure or other applied loads
4. In-plane shear stresses
5. Any combination of 1, 2, 3, and 4

b. Spherical Shells and Formed Heads

1. Equal biaxial stresses—both stresses are compressive
2. Unequal biaxial stresses—both stresses are compressive
3. Unequal biaxial stresses—one stress is tensile and the other is compressive

11.1.8 Buckling Design Method

The buckling strength formulations presented in this report are based on classical linear theory which is modified by reduction factors that account for the effects of imperfections, boundary conditions, nonlinearity of material properties, and residual stresses. The reduction factors are determined from approximate lower bound values of test data of shells with initial imperfections representative of the tolerance limits specified in this chapter. The validation of the knockdown factors is given in [7], [8], [9], and [10].

11.1.9 Stress Factor

The allowable stresses are determined by applying a stress factor, FS, to the predicted buckling stresses. The recommended values of FS are 2.0 when the buckling stress is elastic and 5/3 when the buckling stress equals the yield stress. A linear variation shall be used between these limits. The equations for FS are given below.

$$FS = 2.0 \quad \text{if } F_{ic} \leq 0.55 F_y \tag{11.1a}$$

$$FS = 2.407 - 0.741 F_{ic}/F_y \quad \text{if } 0.55 F_y < F_{ic} < F_y \tag{11.1b}$$

$$FS = 1.667 \quad \text{if } F_{ic} = F_y \tag{11.1c}$$

F_{ic} is the predicted buckling stress, which is determined by letting $FS = 1$ in the allowable stress equations. For combinations of earthquake load or wind load with other loads, the allowable stresses may be increased by a factor of 4/3.

11.1.10 Nomenclature

Note: The terms not defined here are uniquely defined in the sections in which they are first used.

A = cross-sectional area of cylinder $A = \pi(D_o - t)t$, in.^{2}
A_S = cross-sectional area of a ring stiffener, in.^{2}
A_F = cross-sectional area of a large ring stiffener which acts as a bulkhead, in.^{2}
D_i = inside diameter of cylinder, in.
D_o = outside diameter of cylinder, in.
D_L = outside diameter at large end of cone, in.
D_S = outside diameter at small end of cone, in.
E = modulus of elasticity of material at design temperature, ksi
E_t = tangent modulus of material at design temperature, ksi
f_a = axial compressive membrane stress resulting from applied axial load, Q, ksi
f_b = axial compressive membrane stress resulting from applied bending moment, M, ksi
f_h = hoop compressive membrane stress resulting from applied external pressure, P, ksi
f_q = axial compressive membrane stress resulting from pressure load, Q_p, on the end of a cylinder, ksi.
f_v = shear stress from applied loads, ksi
f_x = $f_a + f_q$, ksi
F_{ba} = allowable axial compressive membrane stress of a cylinder due to bending moment, M, in the absence of other loads, ksi
F_{ca} = allowable compressive membrane stress of a cylinder due to axial compression load with $\lambda_c > 0.15$, ksi
F_{bha} = allowable axial compressive membrane stress of a cylinder due to bending in the presence of hoop compression, ksi

F_{hba} = allowable hoop compressive membrane stress of a cylinder in the presence of longitudinal compression due to a bending moment, ksi

F_{he} = elastic hoop compressive membrane failure stress of a cylinder or formed head under external pressure alone, ksi

F_{ha} = allowable hoop compressive membrane stress of a cylinder or formed head under external pressure alone, ksi

F_{hva} = allowable hoop compressive membrane stress in the presence of shear stress, ksi

F_{hxa} = allowable hoop compressive membrane stress of a cylinder in the presence of axial compression, ksi

F_{ta} = allowable tension stress, ksi

F_{va} = allowable shear stress of a cylinder subjected only to shear stress, ksi

F_{ve} = elastic shear buckling stress of a cylinder subjected only to shear stress, ksi

F_{vha} = allowable shear stress of a cylinder subjected to shear stress in the presence of hoop compression, ksi

F_{xa} = allowable compressive membrane stress of a cylinder due to axial compression load with $\lambda_c \leq 0.15$, ksi

F_{xc} = inelastic axial compressive membrane failure (local buckling) stress of a cylinder in the absence of other loads, ksi

F_{xe} = elastic axial compressive membrane failure (local buckling) stress of a cylinder in the absence of other loads, ksi

F_{xha} = allowable axial compressive membrane stress of a cylinder in the presence of hoop compression, ksi

F_y = minimum specified yield stress of material, ksi

F_u = minimum specified tensile stress of material, ksi

FS = stress factor

I'_s = moment of inertia of ring stiffener plus effective length of shell about centroidal axis of combined section, in.^{4}

$$I'_s = I_s + A_s Z_s^2 \frac{L_e t}{A_s + L_e t} + \frac{L_e t^3}{12}$$

K = effective length factor for column buckling

I_s = moment of inertia of ring stiffener about its centroidal axis, in.^{4}

L = design length of a vessel section between lines of support, in. A line of support is:

1. a circumferential line on a head (excluding conical heads) at one-third the depth of the head from the head tangent line as shown in Figure 11.1
2. a stiffening ring that meets the requirements of Equation 11.17

L_B = length of cylinder between bulkheads or large rings designed to act as bulkheads, in.

L_c = unbraced length of member, in.

L_e = effective length of shell, in. (see Figure 11.2)

L_F = one-half of the sum of the distances, L_B, from the center line of a large ring to the next large ring or head line of support on either side of the large ring, in. (see Figure 11.1)

L_s = one-half of the sum of the distances from the center line of a stiffening ring to the next line of support on either side of the ring, measured parallel to the axis of the cylinder, in. A line of support is described in the definition for L (see Figure 11.1).

L_t = overall length of vessel as shown in Figure 11.1, in.

M = applied bending moment across the vessel cross-section, in.-kips

M_s = $L_s/\sqrt{R_o t}$

M_x = $L/\sqrt{R_o t}$

P = applied external pressure, ksi

P_a = allowable external pressure in the absence of other loads, ksi

Q = applied axial compression load, kips

Q_p = axial compression load on end of cylinder resulting from applied external pressure, kips

R = radius to centerline of shell, in.

R_c = radius to centroid of combined ring stiffener and effective length of shell, in. $R_c = R + Z_c$

R_o = radius to outside of shell, in.

t = thickness of shell, less corrosion allowance, in.

t_c = thickness of cone, less corrosion allowance, in.

Z_c = radial distance from centerline of shell to centroid of combined section of ring and effective length of shell, in. $Z_c = \frac{A_s Z_s}{A_s + L_e t}$

Z_s = radial distance from center line of shell to centroid of ring stiffener (positive for outside rings), in.

S = elastic section modulus of full shell cross-section, in.^{3}

$$S = \frac{\pi \left(D_o^4 - D_i^4\right)}{32 D_o}$$

r = radius of gyration of cylinder, in.

$$r = \frac{\left(D_o^2 + D_i^2\right)^{1/2}}{4}$$

λ_c = slenderness factor

$$\lambda_c = \frac{K L_c}{\pi r} \left(\frac{F_{xa} \cdot FS}{E}\right)^{1/2}$$

11.2 Allowable Compressive Stresses for Cylindrical Shells

The maximum allowable stresses for cylindrical shells subjected to loads that produce compressive stresses are given by the following equations.

11.2.1 Uniform Axial Compression

Allowable longitudinal stress for a cylindrical shell under uniform axial compression is given by F_{xa} for values of $\lambda_c \leq 0.15$ and by F_{ca} for values of $\lambda_c > 0.15$. F_{xa} is the smaller of the values given by Equations 11.3 and Equation 11.4.

$$\lambda_c = \frac{K L_c}{\pi r} \left(\frac{F_{xa} \cdot FS}{E}\right)^{1/2} \tag{11.2}$$

where $K L_c$ is the effective length. L_c is the unbraced length. Recommended values for K [1] are 2.1 for members with one end free and the other end fixed, 1.0 for members with both ends pinned, 0.8 for members with one end pinned and the other end fixed, and 0.65 for members with both ends fixed.

Local Buckling (For $\lambda_c \leq 0.15$)

$$F_{xa} = \frac{F_y}{FS} \quad \text{for} \quad \frac{D_o}{t} \leq 135 \tag{11.3a}$$

$$F_{xa} = \frac{466 F_y}{\left(331 + \frac{D_o}{t}\right) FS} \quad \text{for} \quad 135 < \frac{D_o}{t} < 600 \tag{11.3b}$$

$$F_{xa} = \frac{0.5 F_y}{FS} \quad \text{for} \quad \frac{D_o}{t} \geq 600 \tag{11.3c}$$

or

$$F_{xa} = \frac{F_{xe}}{FS} \tag{11.4}$$

where

$$F_{xe} = \frac{C_x E_t}{D_o} \tag{11.5}$$

$$\begin{aligned}
C_x &= \frac{409\bar{c}}{389 + \frac{D_o}{t}} \quad \text{not to exceed } 0.9 \text{ for } \frac{D_o}{t} < 1247 \\
C_x &= 0.25\bar{c} \quad \text{for} \quad \frac{D_o}{t} \geq 1247 \\
\bar{c} &= 2.64 \quad \text{for} \quad M_x \leq 1.5 \\
\bar{c} &= \frac{3.13}{M_x^{0.42}} \quad \text{for} \quad 1.5 < M_x < 15 \\
\bar{c} &= 1.0 \quad \text{for} \quad M_x \geq 15
\end{aligned}$$

$$M_x = \frac{L}{(R_o t)^{1/2}} \tag{11.6}$$

Column Buckling (For $\lambda_c > 0.15$)

$$F_{ca} = F_{xa} \quad \text{for} \quad \lambda_c \leq 0.15 \tag{11.7a}$$

$$F_{ca} = F_{xa} \left[1 - 0.74 \left(\lambda_c - 0.15\right)\right]^{0.3} \quad \text{for} \quad 0.15 < \lambda_c < \sqrt{2} \tag{11.7b}$$

$$F_{ca} = \frac{0.88 F_{xa}}{\lambda_c^2} \quad \text{for} \quad \lambda_c \geq \sqrt{2} \tag{11.7c}$$

11.2.2 Axial Compression Due to Bending Moment

Allowable longitudinal stress for a cylinder subjected to a bending moment acting across the full circular cross-section is given by F_{ba}.

$$F_{ba} = F_{xa} \quad \text{for} \quad \frac{D_o}{t} \geq 135 \tag{11.8a}$$

$$F_{ba} = \frac{466 F_y}{FS\left(331 + \frac{D_o}{t}\right)} \quad \text{for} \quad 100 \leq \frac{D_o}{t} < 135 \tag{11.8b}$$

$$F_{ba} = \frac{1.081 F_y}{FS} \quad \text{for} \quad \frac{D_o}{t} < 100 \quad \text{and} \quad \gamma \geq 0.11 \tag{11.8c}$$

$$F_{ba} = \frac{(1.4 - 2.9\gamma) F_y}{FS} \quad \text{for} \quad \frac{D_o}{t} < 100 \quad \text{and} \quad \gamma < 0.11 \tag{11.8d}$$

where F_{xa} is the smaller of the values given by Equations 11.3 and 11.4 and $\gamma = \frac{F_y D_o}{Et}$.

11.2.3 External Pressure

The allowable circumferential compressive stress for a cylinder under external pressure is given by F_{ha} and the allowable external pressure is given by the following equations:

$$P_a = 2 F_{ha} \frac{t}{D_o} \tag{11.9}$$

$$F_{ha} = \frac{F_y}{FS} \quad \text{for} \quad \frac{F_{he}}{F_y} \geq 2.439 \tag{11.10a}$$

$$F_{ha} = \frac{0.7 F_y}{FS} \left(\frac{F_{he}}{F_y}\right)^{0.4} \quad \text{for} \quad 0.552 < \frac{F_{he}}{F_y} < 2.439 \tag{11.10b}$$

$$F_{ha} = \frac{F_{he}}{FS} \quad \text{for} \quad \frac{F_{he}}{F_y} \leq 0.552 \tag{11.10c}$$

where

$$F_{he} = \frac{1.6 C_h E t}{D_o} \tag{11.11}$$

$$C_h = 0.55 \frac{t}{D_o} \quad \text{for} \quad M_x \geq 2 \left(\frac{D_o}{t}\right)^{0.94}$$

$$C_h = 1.12 M_x^{-1.058} \quad \text{for} \quad 13 < M_x < 2 \left(\frac{D_o}{t}\right)^{0.94}$$

$$C_h = \frac{0.92}{M_x - 0.579} \quad \text{for} \quad 1.5 < M_x \leq 13$$

$$C_h = 1.0 \quad \text{for} \quad M_x \leq 1.5$$

11.2.4 Shear

Allowable in-plane shear stress for a cylindrical shell is given by F_{va}.

$$F_{va} = \frac{\eta_v F_{ve}}{FS} \tag{11.12}$$

where

$$F_{ve} = \frac{\alpha_v C_v E t}{D_o} \tag{11.13}$$

$$C_v = 4.454 \text{ for } M_x \leq 1.5 \tag{11.14a}$$

$$C_v = \left(\frac{9.64}{M_x^2}\right)\left(1 + 0.0239 M_x^3\right)^{1/2} \text{ for } 1.5 < M_x < 26 \tag{11.14b}$$

$$C_v = \frac{1.492}{M_x^{1/2}} \text{ for } 26 \leq M_x < 4.347\frac{D_o}{t} \tag{11.14c}$$

$$C_v = 0.716\left(\frac{t}{D_o}\right)^{1/2} \text{ for } M_x \geq 4.347\frac{D_o}{t} \tag{11.14d}$$

$$\alpha_v = 0.8 \text{ for } \frac{D_o}{t} \leq 500$$

$$\alpha_v = 1.389 - 0.218\log_{10}\left(\frac{D_o}{t}\right) \text{ for } 500 < \frac{D_o}{t} \leq 1000$$

$$\eta_v = 1.0 \text{ for } \frac{F_{ve}}{F_y} \leq 0.48$$

$$\eta_v = 0.43\frac{F_y}{F_{ve}} + 0.1 \text{ for } 0.48 < \frac{F_{ve}}{F_y} < 1.7$$

$$\eta_v = 0.6\frac{F_y}{F_{ve}} \text{ for } \frac{F_{ve}}{F_y} \geq 1.7$$

11.2.5 Sizing of Rings (General Instability)

Uniform Axial Compression and Axial Compression Due to Bending

When ring stiffeners are used to increase the allowable longitudinal compressive stress, the following equations must be satisfied. If $M_x \geq 15$, stiffener spacing is too large to be effective.

$$A_s \geq \left[\frac{0.334}{M_s^{0.6}} - 0.063\right] L_s t \text{ and } A_s \geq 0.06 L_s t \tag{11.15}$$

$$\text{also } I_s' \geq \frac{5.33 L_s t^3}{M_s^{1.8}} \tag{11.16}$$

External Pressure

(a) Small Rings

$$I_s' \geq \frac{1.5 F_{he} L_s R_c^2 t}{E\left(n^2 - 1\right)} \tag{11.17}$$

F_{he} = stress determined from Equation 11.11 with $M_x = M_s$.

$$n^2 = \frac{2 D_o^{3/2}}{3 L_B t^{1/2}} \text{ and } 4 \leq n^2 \leq 100$$

(b) Large Rings Which Act As Bulkheads

$$I_s' \geq I_F \quad \text{where} \quad I_F = \frac{F_{heF} L_F R_c^2 t}{2E} \tag{11.18}$$

I_F = the value of I_s' which makes a large stiffener act as a bulkhead. The effective length of shell is $L_e = 1.1\sqrt{D_o t}(A_1/A_2)$

A_1 = cross-sectional area of small ring plus shell area equal to $L_s t$, in.^{2}

A_2 = cross-sectional area of large ring plus shell area equal to $L_s t$, in.^{2}

R_c = radius to centroid of combined large ring and effective width of shell, in.

F_{heF} = average value of the hoop buckling stresses, F_{he}, over length L_F where F_{he} is determined from Equation 11.11, ksi

Shear

$$I_s' \geq 0.184 C_v M_s^{0.8} t^3 L_s \tag{11.19}$$

C_v = value determined from Equation 11.14 with $M_x = M_s$.

Local Stiffener Buckling

To preclude local buckling of the stiffener prior to shell buckling, the following stiffener properties shall be met. See Figure 11.4 for stiffener geometry.

(a) Flat Bar Stiffener, Flange of a Tee Stiffener, and Outstanding Leg of an Angle Stiffener

$$\frac{h_1}{t_1} \leq 0.375 \left(\frac{E}{F_y}\right)^{1/2} \tag{11.20}$$

where h_1 is the full width of a flat bar stiffener or outstanding leg of an angle stiffener and one-half of the full width of the flange of a tee stiffener and t_1 is the thickness of the bar, leg of angle, or flange of tee.

(b) Web of Tee Stiffener or Leg of Angle Stiffener Attached to Shell

$$\frac{h_2}{t_2} \leq 1.0 \left(\frac{E}{F_y}\right)^{1/2} \tag{11.21}$$

where h_2 is the full depth of a tee section or full width of an angle leg and t_2 is the thickness of the web or angle leg.

11.3 Allowable Compressive Stresses For Cones

Unstiffened conical transitions or cone sections between rings of stiffened cones with an angle $\alpha \leq 60°$ shall be designed for local buckling as an equivalent cylinder according to the following procedure. See Figure 11.3 for cone geometry.

11.3.1 Uniform Axial Compression and Axial Compression Due to Bending

Allowable Longitudinal and Bending Stresses

Assume an equivalent cylinder with diameter $D_e = D/\cos\alpha$, where D is the outside diameter of the cone at the cross-section under consideration and length equal to L_c. D_e is substituted for

D_o in Equations 11.3 to Equations 11.8 to find F_{xa} and F_{ba} and L_c for L in Equation 11.6. The allowable stress must be satisfied at all cross-sections along the length of the cone.

Unstiffened Cone-Cylinder Junctions

Cone-cylinder junctions are subject to unbalanced radial forces (due to axial load and bending moment) and to localized bending stresses caused by the angle change. The longitudinal and hoop stresses at the junction may be evaluated as follows:

Longitudinal Stress—In lieu of detailed analysis, the localized bending stress at an unstiffened cone-cylinder junction may be estimated by the following equation.

$$f_b' = \frac{0.6t\sqrt{D\,(t+t_c)}}{t_e^2}\,(f_x + f_b)\tan\alpha \tag{11.22}$$

where

D = outside diameter of cylinder at junction to cone
t = thickness of cylinder
t_c = thickness of cone
t_e = t to find stress in cylinder section
t_e = t_c to find stress in cone section
α = cone angle as defined in Figure 11.3
f_x = uniform longitudinal stress in cylinder section at junction resulting from axial loads
f_b = longitudinal stress in cylinder section at junction resulting from bending moment

For strength requirements, the total stress $(f_x + f_b + f_b')$ shall be limited to the minimum tensile strength given in Table 11.1 or Table U, Subpart 1 of [4] for the cone and cylinder material and $f_x + f_b$ shall be less than the allowable tensile stress F_t, where F_t is the smaller of $0.6F_y$ or $F_u/3$.

Hoop Stress—The hoop stress caused by the unbalanced radial line load may be estimated from:

$$f_h' = 0.45\sqrt{D/t}\,(f_x + f_b)\tan\alpha \tag{11.23}$$

For hoop tension, f_h' shall be limited to the tensile allowable. For hoop compression, f_h' shall be limited to F_{ha} where F_{ha} is computed from Equation 11.10 with $F_{he} = 0.4Et/D$.

A cone-cylinder junction that does not satisfy the above criteria may be strengthened either by increasing the cylinder and cone wall thicknesses at the junction, or by providing a stiffening ring at the junction.

Cone-Cylinder Junction Rings

If stiffening rings are required, the section properties shall satisfy the following requirements:

$$A_c \geq \frac{tD}{F_y}(f_x + f_b)\tan\alpha \tag{11.24}$$

$$I_c \geq \frac{tDD_c^2}{8E}(f_x + f_b)\tan\alpha \tag{11.25}$$

where

D = cylinder outside diameter at junction
D_c = diameter to centroid of composite ring section for external rings
D_c = D for internal rings
A_c = cross-sectional area of composite ring section
I_c = moment of inertia of composite ring section

In computing A_c and I_c the effective length of the shell wall acting as a flange for the composite ring section shall be computed from:

$$b_e = 0.55\left(\sqrt{D/t} + \sqrt{Dt_c/\cos\alpha}\right) \tag{11.26}$$

11.3.2 External Pressure

Allowable Circumferential Compression Stresses

Assume an equivalent cylinder with diameter $D_e = 0.5(D_L + D_S)$ and length $L_e = L_c/\cos\alpha$. This length and diameter shall be substituted into Equations 11.10 and 11.11 to determine F_{ha}.

Intermediate Stiffening Rings

If required, circumferential stiffening rings within cone transitions shall be sized using Equation 11.17 with $R_c = D/2$ where D is the cone diameter at the ring, t is the cone thickness, L_s is the average distance to adjacent rings along the cone axis, and F_{he} is the average of the elastic hoop buckling stress values computed for the two adjacent bays by the method given in the preceding paragraph.

Cone-Cylinder Junction Rings

A junction ring is not required for buckling due to external pressure if $f_h < F_{ha}$ where F_{ha} is determined from Equation 11.10 with F_{he} computed using C_h equal to 0.55 $(\cos\alpha)(t/D)$ in Equation 11.11. D is the cylinder diameter at the junction.

Circumferential stiffening rings required at the cone-cylinder junctions shall be sized such that the moment of inertia of the composite ring section satisfies the following equation:

$$I_c \geq \frac{D^2}{16E}\left\{tL_1F_{he} + \frac{t_cL_cF_{hec}}{\cos^2\alpha}\right\} \tag{11.27}$$

where

D	=	cylinder outside diameter at junction
L_c	=	distance to first stiffening ring in cone section along cone axis
L_1	=	distance to first stiffening ring in cylinder section or line of support
F_{he}	=	elastic hoop buckling stress for cylinder (see Equation 11.11)
F_{hec}	=	F_{he} for cone section treated as an equivalent cylinder
t	=	cylinder thickness
t_c	=	cone thickness

11.3.3 Shear

Allowable In-Plane Shear Stress

Assume an equivalent cylinder with a length equal to the slant length of the cone between rings $(L_c/\cos\alpha)$ and a diameter $D_e = D/\cos\alpha$, where D is the outside diameter of the cone at the cross-section under consideration. This length and diameter shall be substituted into Equations 11.12 to 11.14 to determine F_{va}.

Intermediate Stiffening Rings

If required, circumferential stiffening rings within cone transition shall be sized using Equation 11.19 where L_s is the average distance to adjacent rings along the cone axis.

11.3.4 Local Stiffener Buckling

To preclude local buckling of a stiffener, the requirements of Equations 11.20 and 11.21 must be met.

11.4 Allowable Stress Equations For Unstiffened and Ring-Stiffened Cylinders and Cones Under Combined Loads

11.4.1 For Combination of Uniform Axial Compression and Hoop Compression

For $\lambda_c \leq 0.15$

The allowable stress in the longitudinal direction is given by F_{xha} and the allowable stress in the circumferential direction is given by F_{hxa}.

$$F_{xha} = \left(\frac{1}{F_{xa}^2} - \frac{C_1}{C_2 F_{xa} F_{ha}} + \frac{1}{C_2^2 F_{ha}^2} \right)^{-0.5} \tag{11.28}$$

where

$$C_1 = \frac{F_{xa} \cdot FS + F_{ha} \cdot FS}{F_y} - 1.0 \text{ and } C_2 = \frac{f_x}{f_h}$$

$$f_x = f_a + f_q = \frac{Q}{A} + \frac{Q_p}{A} \text{ and } f_h = \frac{P D_o}{2t}$$

$F_{xa} \cdot FS$ is given by the smaller of Equation 11.3 or 11.4, and $F_{ha} \cdot FS$ is given by Equation 11.10.

$$F_{hxa} = \frac{F_{xha}}{C_2} \tag{11.29}$$

For $0.15 < \lambda_c < 1.2$

F_{xha} is the smaller of F_{ah1} and F_{ah2} where $F_{ah1} = F_{xha}$ given by Equation 11.28 with $f_x = f_a$ and F_{ah2} is given by the following equation.

$$F_{ah2} = F_{ca} \left(1 - \frac{f_q}{F_y} \right) \tag{11.30}$$

F_{ca} is given by Equation 11.7.

11.4.2 For Combination of Axial Compression Due to Bending Moment, M, and Hoop Compression

The allowable stress in the longitudinal direction is given by F_{bha} and the allowable stress in the circumferential direction is given by F_{hba}.

$$F_{bha} = C_3 C_4 F_{ba} \tag{11.31}$$

where C_3 and C_4 are given by the following equations and F_{ba} is given by Equation 11.8.

$$C_4 = \frac{f_b}{f_h} \frac{F_{ha}}{F_{ba}}$$

$$C_3^2 \left(C_4^2 + 0.6 C_4 \right) + C_3^{2n} - 1 = 0 \tag{11.32}$$

$$f_b = \frac{M}{S} \quad f_h = \frac{P D_o}{2t} \quad n = 5 - 4 \frac{F_{ha} \cdot FS}{F_y}$$

Solve for $C3$ from Equation 11.31 by iteration. $F_{ha} \cdot FS$ is given by Equation 11.10.

$$F_{hba} = F_{bha} \frac{f_h}{f_b} \tag{11.33}$$

11.4.3 For Combination of Hoop Compression and Shear

The allowable shear stress is given by F_{vha} and the allowable circumferential stress is given by F_{hva}.

$$F_{vha} = \left[\left(\frac{F_{va}^2}{2C_5 F_{ha}} \right)^2 + F_{va}^2 \right]^{1/2} - \frac{F_{va}^2}{2C_5 F_{ha}} \tag{11.34}$$

where $C_5 = \frac{f_v}{f_h}$ and F_{va} is given by Equation 11.12 and F_{ha} is given by Equation 11.10.

$$F_{hva} = \frac{F_{vha}}{C_5} \tag{11.35}$$

11.4.4 For Combination of Uniform Axial Compression, Axial Compression Due to Bending Moment, M, and Shear, in the Presence of Hoop Compression, $(f_h \neq 0)$

$$\text{Let } K_s = 1 - \left(\frac{f_v}{F_{va}} \right)^2 \tag{11.36}$$

For $\lambda_c \leq 0.15$

$$\left(\frac{f_a}{K_s F_{xha}} \right)^{1.7} + \frac{f_b}{K_s F_{bha}} \leq 1.0 \tag{11.37}$$

F_{xha} is given by Equation 11.28, F_{bha} is given by Equation 11.30 and F_{va} is given by Equation 11.12.
For $0.15 < \lambda_c < 1.2$

$$\frac{f_a}{F_{xha}} + \frac{8}{9} \frac{\Delta f_b}{F_{bha}} \leq 1.0 \quad \text{for} \quad \frac{f_a}{F_{xha}} \geq 0.2 \tag{11.38}$$

where

$$\Delta = \frac{C_m}{1 - f_a \cdot FS / F_e} \qquad F_e = \frac{\pi^2 E}{(K L_c / r)^2}$$

See Equation 11.2 for KL_c and Equation 11.30 for F_{xha}. F_{bha} is given by Equation 11.31. FS is determined from Equation 11.1 where $F_{ic} = F_{xa} \cdot FS$ (see Equations 11.3 and 11.4). C_m is a coefficient whose value shall be taken as follows [1]:

1. For compression members in frames subject to joint translation (sidesway),

$$C_m = 0.85.$$

2. For rotationally restrained compression members in frames braced against joint translation and not subject to transverse loading between their supports in the plane of bending,

$$C_m = 0.6 - 0.4(M_1/M_2)$$

where M_1/M_2 is the ratio of the smaller to larger moments at the ends of that portion of the member that is unbraced in the plane of bending under consideration. M_1/M_2 is positive when the member is bent in reverse curvature and negative when bent in single curvature.

3. For compression members in frames braced against joint translation and subjected to transverse loading between their supports the following apply:
   a. for members whose ends are restrained against rotation in the plane of bending,

$$C_m = 0.85$$

   b. for members whose ends are unrestrained against rotation in the plane of bending,

$$C_m = 1.0$$

11.4.5 For Combination of Uniform Axial Compression, Axial Compression Due to Bending Moment, M, and Shear, in the Absence of Hoop Compression, $(f_h = 0)$

For $\lambda_c \leq 0.15$

$$\left(\frac{f_a}{K_s F_{xa}}\right)^{1.7} + \frac{f_b}{K_s F_{ba}} \leq 1.0 \tag{11.39}$$

F_{xa} is given by the smaller of Equations 11.3 or 11.4, F_{ba} is given by Equation 11.8 and K_s is given by Equation 11.36.

For $0.15 < \lambda_c < 1.2$

$$\frac{f_a}{K_s F_{ca}} + \frac{8}{9}\frac{\Delta f_b}{K_s F_{ba}} \leq 1.0 \quad \text{for} \quad \frac{f_a}{K_s F_{ca}} \geq 0.2 \tag{11.40}$$

$$\frac{f_a}{2K_s F_{ca}} + \frac{\Delta f_b}{K_s F_{ba}} \leq 1.0 \quad \text{for} \quad \frac{f_a}{K_s F_{ca}} < 0.2 \tag{11.41}$$

F_{ca} is given by Equation 11.7, F_{ba} is given by Equation 11.31, and K_s is given by Equation 11.36. See Equation 11.38 for definition of Δ.

11.5 Tolerances for Cylindrical and Conical Shells

11.5.1 Shells Subjected to Uniform Axial Compression and Axial Compression Due to Bending Moment

The difference between the maximum and minimum diameters at any cross-section shall not exceed 1% of the nominal diameter at the cross-section under consideration. Additionally, the local deviation from a straight line, e, measured along a meridian over a gauge length L_x shall not exceed the maximum permissible deviation e_x given below.

$e_x = 0.002R$
$L_x = 4\sqrt{Rt}$ but not greater than L for cylinders
$L_x = 4\sqrt{Rt/\cos\alpha}$ but not greater than $L_c/\cos\alpha$ for cones
$L_x = 25t$ across circumferential welds

Also L_x is not greater than 95% of the meridianal distance between circumferential welds.

11.5.2 Shells Subjected to External Pressure

The difference between the maximum and minimum diameters at any cross-section shall not exceed 1% of the nominal diameter at the cross-section under consideration. Additionally, the maximum deviation from a true circular form, e, shall not exceed the value given by Figure 11.5 or by the following equations.

$$e = 0.0165t(M_x + 3.25)^{1.069} \qquad 0.1t \leq e \leq 0.0242R \tag{11.42}$$

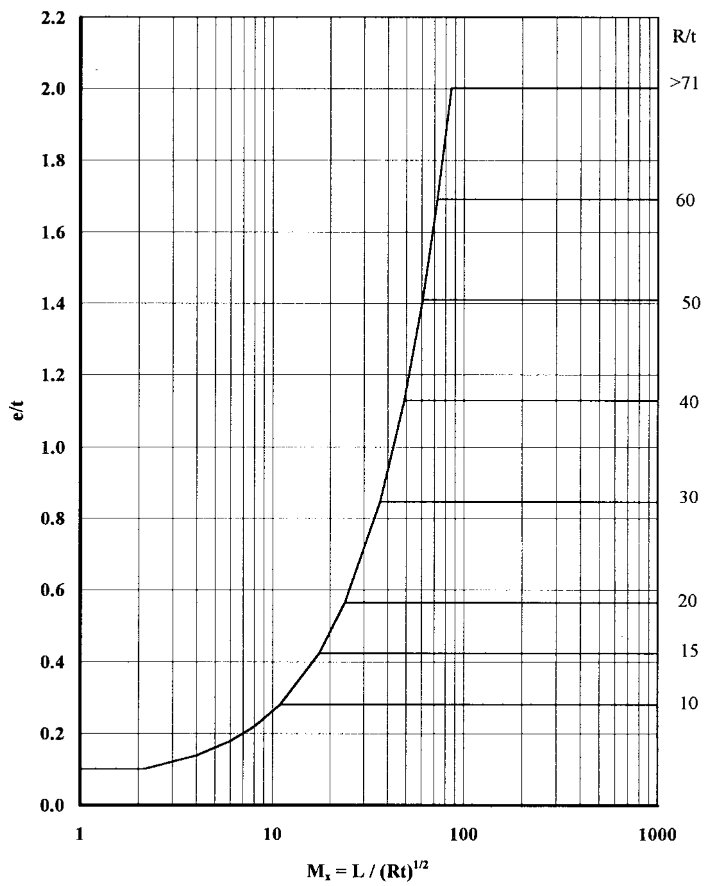

FIGURE 11.5: Values of e/t which give a buckling pressure of 80% of the theoretical buckling pressure.

Also, e shall not exceed $2t$. Measurements to determine e are made with a gauge or template with the chord length L_c given by the following equation.

$$L_c = 2R\sin(\pi/2n) \tag{11.43}$$

$$n = c\left(\frac{\sqrt{R/t}}{L/R}\right)^d \qquad 2 \leq n \leq 1.41(R/t)^{0.5} \tag{11.44}$$

where

$$c = 2.28(R/t)^{0.54} \leq 2.80$$
$$d = 0.38(R/t)^{0.044} \leq 0.485$$

11.5.3 Shells Subjected to Shear

The difference between the maximum and minimum diameters at any cross-section shall not exceed 1% of the nominal diameter at the cross-section under consideration.

11.6 Allowable Compressive Stresses for Spherical Shells and Formed Heads, With Pressure on Convex Side

11.6.1 Spherical Shells

With Equal Biaxial Stresses

The allowable compressive stress for a spherical shell under uniform external pressure is given by F_{ha} and the allowable external pressure is given by P_a.

$$F_{ha} = \frac{F_y}{FS} \qquad \text{for } \frac{F_{he}}{F_y} \geq 6.25 \tag{11.45a}$$

$$F_{ha} = \frac{1.31F_y}{FS\left(1.15 + \frac{F_y}{F_{he}}\right)} \qquad \text{for } 1.6 < \frac{F_{he}}{F_y} < 6.25 \tag{11.45b}$$

$$F_{ha} = \frac{0.18F_{he} + 0.45F_y}{FS} \qquad \text{for } 0.55 < \frac{F_{he}}{F_y} \leq 1.6 \tag{11.45c}$$

$$F_{ha} = \frac{F_{he}}{FS} \qquad \text{for } \frac{F_{he}}{F_y} \leq 0.55 \tag{11.45d}$$

$$F_{he} = 0.075E\frac{t}{R_o} \tag{11.46}$$

$$P_a = 2F_{ha}\frac{t}{R_o} \tag{11.47}$$

where R_o is the radius to the outside of the spherical shell and F_{ha} is given by Equation 11.45.

With Unequal Biaxial Stresses—Both Stresses Are Compressive

The allowable compressive stresses for a spherical shell subjected to unequal biaxial stresses, σ_1 and σ_2, where both σ_1 and σ_2 are compression stresses resulting from applied loads, are given by the following equations.

$$F_{1a} = \frac{0.6}{1 - 0.4k}F_{ha} \tag{11.48}$$

$$F_{2a} = kF_{1a} \tag{11.49}$$

where $k = \sigma_2/\sigma_1$ and F_{ha} is given by Equation 11.45. F_{1a} is the allowable stress in the direction of σ_1 and F_{2a} is the allowable stress in the direction of σ_2. The larger of the compression stresses is σ_1.

With Unequal Biaxial Stresses—One Stress Is Compressive and the Other Is Tensile

The allowable compressive stress for a spherical shell subjected to unequal biaxial stresses σ_1 and σ_2, where σ_1 is a compression stress and σ_2 is a tensile stress, is given by F_{1a} where F_{1a} is the value of F_{ha} determined from Equation 11.45 with F_{he} given by Equation 11.50.

$$F_{he} = \left(C_o + C_p\right) E \frac{t}{R_o} \tag{11.50}$$

$$\begin{aligned}
C_o &= \frac{102.2}{195 + R_o/t} \quad \text{for} \quad \frac{R_o}{t} < 622 \\
C_o &= 0.125 \quad \text{for} \quad \frac{R_o}{t} \geq 622 \\
C_p &= \frac{1.06}{3.24 + \frac{1}{\bar{p}}} \\
\bar{p} &= \frac{\sigma_2}{E} \frac{R_o}{t}
\end{aligned}$$

Shear

When shear is present, the principal stresses shall be calculated and used for σ_1 and σ_2.

11.6.2 Toroidal and Ellipsoidal Heads

The allowable compressive stresses for formed heads is determined by the equations given for spherical shells where R_o is defined below.

R_o = the outside radius of the crown portion of the head for torispherical heads, in.
R_o = the equivalent outside spherical radius taken as $K_o D_o$ for ellipsoidal heads, in.
K_o = factor depending on the ellipsoidal head proportions $D_o/2h_o$ (see Table 11.2)
h_o = outside height of the ellipsoidal head measured from the tangent line (head-bend line), in.

TABLE 11.2 Factor K_o

$D_o/2h_o$	...	3.0	2.8	2.6	2.4	2.2
K_o	...	1.36	1.27	1.18	1.08	0.99
$D_o/2h_o$	2.0	1.8	1.6	1.4	1.2	1.0
K_o	0.90	0.81	0.73	0.65	0.57	0.50

Note: Use interpolation for intermediate values.

11.7 Tolerances for Formed Heads

The inner surface of a spherical shell or formed head shall not deviate from the specified shape more than 1.25% of the nominal diameter of the vessel. Such deviations shall be measured perpendicular to the specified shape. Additionally, the maximum local deviation from a true circular form, e, for spherical shells and any spherical portion of a formed head designed for external pressure shall not exceed the shell thickness. Measurements to determine e are made with a gauge or template with the chord length L_c given by the following equation:

$$L_c = 3.72\sqrt{Rt}$$

References

[1] AISC. 1989. *Manual of Steel Construction, Allowable Stress Design*, 9th ed., Section C-C2, American Institute of Steel Construction, Chicago, IL.

[2] API 2U. 1987. API Bulletin 2U, "Bulletin on Stability Design of Cylindrical Shells," 1st ed., American Petroleum Institute, Washington, D.C.

[3] ASME. 1992. ASME Boiler and Pressure Vessel Code, Section VIII, Rules for Construction of Pressure Vessels, Division 1, American Society of Mechanical Engineers, New York.

[4] ASME. 1992. ASME Boiler and Pressure Vessel Code, Section II, Materials, Part D-Properties, American Society of Mechanical Engineers, New York.

[5] AWS. 1992, Structural Welding Code, AWS D1.1-92, American Welding Society.

[6] Miller, C.D. 1982. "Experimental Study of the Buckling of Cylindrical Shells With Reinforced Openings," ASME/ANS Nuclear Engineering Conference, Portland, OR.

[7] Miller, C.D. 1991. "ASME Code Case N-284: Metal Containment Shell Buckling Design Methods," Revision 1, in *Code Cases: Nuclear Components, ASME Boiler and Pressure Vessel Code*, American Society of Mechanical Engineers, New York, March 14, 1995.

[8] Miller, C.D. 1995. *An Evaluation of Codes and Standards Related to Buckling of Cylindrical Shells Subjected to Axial Compression, Bending and External Pressure*, UMI Dissertation Services, Ann Arbor, MI.

[9] Miller, C.D. and Saliklis, E.P. 1993. "Analysis of Cylindrical Shell Database and Validation of Design Formulations," API Project 90-56, Chicago Bridge & Iron Technical Services Co., Plainfield, IL.

[10] Miller, C.D. and Saliklis, E.P. 1995. "Analysis of Cylindrical Shell Database and Validation of Design Formulations, Phase 2: For D/t Values > 300," API Project 92-56, Chicago Bridge & Iron Technical Services Co., Plainfield, IL.

Further Reading

Additional information on the design of shell structures can be found in the following references:

[1] American Iron and Steel Institute, 1992. Steel Plate Engineering Data, Volume 1—Steel Tanks for Liquid Storage and Volume 2—Useful Information on the Design of Plate Structures.

[2] Wozniak, R.S. 1990. Steel Tanks, in *Structural Engineering Handbook*, Gaylord, E.H. and Gaylord, C.S. Eds., 3rd ed., McGraw-Hill, New York, 27-1 to 27-29.

The following is a list of codes, specifications, and standards that provide rules for the design of shell structures subject to instability from loads which produce compressive stresses in the shell elements. A comparison was made by Miller and Saliklis [8, 9, 10] of the predicted failure stresses given by each of these sets of rules with the test data obtained from over 600 tests on steel models representative of fabricated shells. The best fit equations were determined for each shell type and load. These equations were then modified to obtain a better fit with the test database. The equations given in this chapter are the results of these studies.

[3] API BUL 2U. 1987. *Bulletin on Stability Design of Cylindrical Shells*, 1st ed., American Petroleum Institute, Washington, D.C.

[4] API RP 2A-LRFD. 1993. *Recommended Practice for Planning, Designing and Constructing Fixed Offshore Platforms—Load and Resistance Factor Design*, 1st ed., American Petroleum Institute, Washington, D.C.

[5] API RP 2A-WSD. 1993. *Recommended Practice for Planning, Designing and Constructing Fixed Offshore Platforms—Working Stress Design*, 20th ed., American Petroleum Institute, Washington, D.C.

[6] API STD 620. 1990. *Design and Construction of Large, Welded Low-Pressure Storage Tanks,* 8th ed., American Petroleum Institute, Washington, D.C.
[7] API STD 650. 1993. *Welded Steel Tanks for Oil Storage,* 9th ed., American Petroleum Institute, Washington, D.C.
[8] ASME VIII. 1992. *Pressure Vessels, Division 2, ASME Boiler and Pressure Code,* American Society of Mechanical Engineers, Washington, D.C.
[9] AWWA D100. 1984. *AWWA Standard for Welded Steel Tanks for Water Storage,* American Water Works Association, Denver, CO.
[10] DIN 18800. 1990. *Stability of Shell Type Steel Structures,* German Code DIN 18800, Part 4.
[11] ECCS No. 56. 1988. *Buckling of Steel Shells, European Recommendations,* European Convention for Constructional Steelwork, Publication No. 56, 4th ed., Brussels, Belgium.
[12] NPD. 1990. *Buckling Criteria for Cylindrical Shells,* Norwegian Petroleum Directorate, Oslo, Norway.