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Preface

This book grew out of my lecture notes for graduate courses on the theory of plasticity and nonlinear continuum mechanics that I taught at several universities in the USA and former Yugoslavia during the past two decades. The book consists of three parts. The first part is an introduction to nonlinear continuum mechanics. After tensor preliminaries in Chapter 1, selected topics of kinematics and kinetics of deformation are presented in Chapters 2 and 3. Hill's theory of conjugate stress and strain measures is used. Chapter 4 is a brief treatment of the thermodynamics of deformation, with an accent given to formulation with internal state variables. Part 2 of the book is devoted to nonlinear elasticity. Constitutive theory of finite strain elasticity is presented in Chapter 5, and its rate-type formulation in Chapter 6. An analysis of elastic stability at finite strain is given in Chapter 7. Nonlinear elasticity is included in the book because it illustrates an application of many general concepts from Part 1, and because it is combined in Part 3 with finite deformation plasticity to derive general constitutive structure of finite deformation elastoplasticity. Part 3 is the largest part of the book, consisting of seven chapters on plasticity. Chapter 8 is an analysis of the constitutive framework for rate-independent and rate-dependent plasticity. The postulates of Drucker and Ilyushin are discussed in the context of finite strain. Derivation of elastoplastic constitutive equations for various phenomenological models of material response is presented in Chapter 9. Formulations in stress and strain space, using the yield surfaces with and without vertices, are given. Isotropic, kinematic, combined isotropic–kinematic and multisurface hardening models are introduced. Pressure-dependent plasticity and non-associative flow rules are then discussed. Fundamental aspects of thermoplasticity, rate-dependent plasticity and deformation theory of plasticity are also included. Hill's theory of uniqueness and plastic stability is presented in Chapter 10, together with an analysis of eigenmodal deformations and acceleration waves in elastoplastic solids. Rice's treatment of plastic flow localization in pressure-insensitive and pressure-sensitive materials is then given. Chapter 11 is devoted to formulation of the constitutive theory of elastoplasticity in the framework of Lee's multiplicative decomposition of deformation gradient into its elastic and plastic parts. Isotropic and orthotropic materials are considered, with an introductory treatment of damage-elastoplasticity. The theory of monocrystalline plasticity is presented in Chapter 12. Crystallographic slip is assumed to be the only mechanism of plastic deformation. Hardening rules and uniqueness of slip rates are examined. Specific forms of constitutive equations for rate-independent and rate-dependent crystals are derived. Chapter 13 covers some fundamental topics of micro-to-macro transition in the constitutive description. The analysis is aimed toward the derivation of constitutive equations for a polycrystalline aggregate from known constitutive equations of single crystals. The fourteenth, and final chapter of the book is devoted to approximate models of polycrystalline plasticity. The classical model of Taylor and the analysis of Bishop and Hill are presented. The main theme is the self-consistent method, introduced in polycrystalline plasticity by Kröner, Budiansky and Wu. Hill's formulation of the method is used in the finite deformation presentation. Calculations of the polycrystalline stress-strain curve and polycrystalline yield surface, development of the crystallographic texture, and effects of the grain-size on the aggregate response are discussed.

This book is an advanced treatment of finite deformation elastoplasticity and is intended for graduate students and other interested readers who are familiar with an introductory treatment of plasticity. Such treatment is usually given in an infinitesimal strain context and with a focus on the geometry of admissible yield surfaces, von Mises and Tresca yield conditions, derivation of the Levy-Mises and Prandtl-Reuss equations, and the analysis of some elementary elastoplastic problems. Familiarity with basic concepts of crystallography and the dislocation theory from an undergraduate course in materials science is also assumed. Important topics of the slip-line theory and limit analysis are not discussed, since they have been repeatedly well covered in a number of existing plasticity books. Numerical treatments of boundary value problems and experimental techniques are not included either, as they require books on their own. A recent text by Simo and Hughes can be consulted as a reference to computational plasticity.

I began to study plasticity as a graduate student of Professor Erastus Lee at Stanford University in the late seventies. His research work and teaching of plasticity was a great inspiration to all his students. I am indebted to him for his guidance during our research on the rate-type constitutive theory of elastoplasticity based on the multiplicative decomposition of deformation gradient. The influence of Rodney Hill's development of the theory of plasticity on my writing is evident from the contents of this book. Large parts of all chapters are based on his research papers from 1948 to 1993. Communications with Professor Hill in 1994 were most inspirational. Two years spent in the solid mechanics group at Brown University in the late eighties and collaborations with Alan Needleman and Fong Shih were rewarding to my understanding of plasticity. Much of the first two parts of this book I wrote in the mid-nineties while teaching and conducting research in the Mechanical and Aerospace Engineering Department of Arizona State University. Collaboration with Dusan Krajcinovic on damage-elastoplasticity was a beneficial experience. A major part of the book was written while I was an Adjunct Professor in the Department of Applied Mechanics and Engineering Sciences of the University of California in San Diego. Professors Xanthippi Markenscoff and Marc Meyers repeatedly encouraged me to write a book on plasticity, and I express my gratitude to them for their support. Collaboration with David Benson on viscoplasticity and dynamic plasticity is also acknowledged. The books by Ray Ogden and Kerry Havner were in many aspects exemplary to my writing in chapters devoted to nonlinear elasticity and crystalline plasticity. I am indebted to Dr. Owen Richmond from Alcoa Laboratories for his continuing support of my research work at Brown, ASU and UCSD. The research support from NSF and the US Army is also acknowledged. Several chapters of this book were written while I was visiting the University of Montenegro during summers of the last two years. Docent Borko Vujičić from the Physics Department was always available to help with Latex related issues in the preparation of the manuscript. I thank him for that. Computer specialists Todd Porteous and Andres Burgos from UCSD were also of help. My appreciation finally extends to Cindy Renee Carelli, acquisitions editor, and Bill Heyward, project editor from CRC Press, for their assistance in publishing this book.

Vlado A. Lubarda
San Diego, April 2001

Professor Vlado A. Lubarda received his Ph.D. degree from Stanford University in 1980. He has been a Docent and an Associate Professor at the University of Montenegro, and a Fulbright Fellow and a Visiting Associate Professor at Brown University and the Arizona State University. Currently, he is an Adjunct Professor of Applied Mechanics in the Department of Mechanical and Aerospace Engineering at the University of California, San Diego. Dr. Lubarda has done extensive research in the constitutive theory of large deformation elastoplasticity, damage mechanics, and dislocation theory. He is the author of 75 journal and conference publications and the textbook *Strength of Materials* (in Serbo-Croatian). He has served as a reviewer to numerous international journals, and was elected in 2000 to the Montenegrin Academy of Sciences and Arts.