The Finite Element Method
Fifth edition
Volume 2: Solid Mechanics

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Dedication

This book is dedicated to our wives Helen and Mary Lou and our families for their support and patience during the preparation of this book, and also to all of our students and colleagues who over the years have contributed to our knowledge of the finite element method. In particular we would like to mention Professor Eugenio Oñate and his group at CIMNE for their help, encouragement and support during the preparation process.
Professor O.C. Zienkiewicz, CBE, FRS, FREng is Professor Emeritus and Director of the Institute for Numerical Methods in Engineering at the University of Wales, Swansea, UK. He holds the UNESCO Chair of Numerical Methods in Engineering at the Technical University of Catalunya, Barcelona, Spain. He was the head of the Civil Engineering Department at the University of Wales Swansea between 1961 and 1989. He established that department as one of the primary centres of finite element research. In 1968 he became the Founder Editor of the International Journal for Numerical Methods in Engineering which still remains today the major journal in this field. The recipient of 24 honorary degrees and many medals, Professor Zienkiewicz is also a member of five academies – an honour he has received for his many contributions to the fundamental developments of the finite element method. In 1978, he became a Fellow of the Royal Society and the Royal Academy of Engineering. This was followed by his election as a foreign member to the U.S. Academy of Engineering (1981), the Polish Academy of Science (1985), the Chinese Academy of Sciences (1998), and the National Academy of Science, Italy (Academia dei Lincei) (1999). He published the first edition of this book in 1967 and it remained the only book on the subject until 1971.

Professor R.L. Taylor has more than 35 years’ experience in the modelling and simulation of structures and solid continua including two years in industry. In 1991 he was elected to membership in the U.S. National Academy of Engineering in recognition of his educational and research contributions to the field of computational mechanics. He was appointed as the T.Y. and Margaret Lin Professor of Engineering in 1992 and, in 1994, received the Berkeley Citation, the highest honour awarded by the University of California, Berkeley. In 1997, Professor Taylor was made a Fellow in the U.S. Association for Computational Mechanics and recently he was elected Fellow in the International Association of Computational Mechanics, and was awarded the USACM John von Neumann Medal. Professor Taylor has written several computer programs for finite element analysis of structural and non-structural systems, one of which, FEAP, is used world-wide in education and research environments. FEAP is now incorporated more fully into the book to address non-linear and finite deformation problems.

Preface to Volume 2

The first volume of this edition covered basic aspects of finite element approximation in the context of linear problems. Typical examples of two- and three-dimensional elasticity, heat conduction and electromagnetic problems in a steady state and transient state were dealt with and a finite element computer program structure was introduced. However, many aspects of formulation had to be relegated to the second and third volumes in which we hope the reader will find the answer to more advanced problems, most of which are of continuing practical and research interest.

In this volume we consider more advanced problems in solid mechanics while in Volume 3 we consider applications in fluid dynamics. It is our intent that Volume 2 can be used by investigators familiar with the finite element method in general terms and will introduce them here to the subject of specialized topics in solid mechanics. This volume can thus in many ways stand alone. Many of the general finite element procedures available in Volume 1 may not be familiar to a reader introduced to the finite element method through different texts. We therefore recommend that the present volume be used in conjunction with Volume 1 to which we make frequent reference.

Two main subject areas in solid mechanics are covered here:

1. **Non-linear problems** (Chapters 1–3 and 10–12) In these the special problems of solving non-linear equation systems are addressed. In the first part we restrict our attention to non-linear behaviour of materials while retaining the assumptions on small strain used in Volume 1 to study the linear elasticity problem. This serves as a bridge to more advanced studies later in which geometric effects from large displacements and deformations are presented. Indeed, non-linear applications are today of great importance and practical interest in most areas of engineering and physics. By starting our study first using a small strain approach we believe the reader can more easily comprehend the various aspects which need to be understood to master the subject matter. We cover in some detail problems in viscoelasticity, plasticity, and viscoplasticity which should serve as a basis for applications to other material models. In our study of finite deformation problems we present a series of approaches which may be used to solve problems including extensions for treatment of constraints (e.g. near incompressibility and rigid body motions) as well as those for buckling and large rotations.
2. **Plates and shells (Chapters 4–9)** This section is of course of most interest to those engaged in 'structural mechanics' and deals with a specific class of problems in which one dimension of the structure is small compared to the other two. This application is one of the first to which finite elements were directed and which still is a subject of continuing research. Those with interests in other areas of solid mechanics may well omit this part on first reading, though by analogy the methods exposed have quite wide applications outside structural mechanics.

Volume 2 concludes with a chapter on Computer Procedures, in which we describe application of the basic program presented in Volume 1 to solve non-linear problems. Clearly the variety of problems presented in the text does not permit a detailed treatment of all subjects discussed, but the 'skeletal' format presented and additional information available from the publisher's web site\(^1\) will allow readers to make their own extensions.

We would like at this stage to thank once again our collaborators and friends for many helpful comments and suggestions. In this volume our particular gratitude goes to Professor Eric Kasper who made numerous constructive comments as well as contributing the section on the mixed-enhanced method in Chapter 10. We would also like to take this opportunity to thank our friends at CIMNE for providing a stimulating environment in which much of Volume 2 was conceived.

OCZ and RLT

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\(^1\) Complete source code for all programs in the three volumes may be obtained at no cost from the publisher's web page: http://www.bh.com/companions/fem