Introduction

I.1 Introduction

Stress analysis is an important part of engineering science, as failure of most engineering components is usually due to stress. The component under a stress investigation can vary from the legs of an integrated circuit to the legs of an offshore drilling rig, or from a submarine pressure hull to the fuselage of a jumbo jet aircraft.

The present chapter will commence with elementary trigonometric definitions and show how elementary trigonometry can be used for analysing simple pin-jointed frameworks (or trusses). The chapter will then be extended to define couples and show the reader how to take moments.

I.2 Trigonometrical definitions

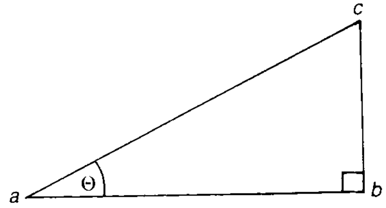


Figure I.1 Right-angled triangle.

With reference to Figure I.1,

$$\sin\theta = bc/ac$$

$$\cos\theta = ab/ac \qquad \text{(I.1)}$$

$$\tan\theta = bc/ab$$

For a triangle without a right angle in it, as shown in Figure I.2, the *sine* and *cosine* rules can be used to determine the lengths of unknown sides or the value of unknown angles.

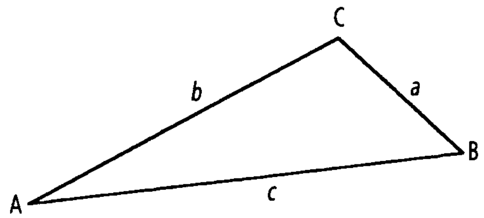


Figure I.2. Triangle with no right angle.

The *sine rule* states that:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \tag{I.2}$$

where

a = length of side BC; opposite the angle A

b = length of side AC; opposite the angle B

c = length of side AB; opposite the angle C

The cosine rule states that:

$$a^2 = b^2 + c^2 - 2bc \cos \mathrm{A}$$

I.3 Vectors and scalars

A scalar is a quantity which has magnitude but no direction, such as a mass, length and time. A vector is a quantity which has magnitude and direction, such as weight, force, velocity and acceleration.

NB It is interesting to note that the moment of a couple, (Section I.6) and energy (Chapter 17), have the same units; but a moment of a couple is a vector quantity and energy is a scalar quantity.

I.4 Newton's laws of motion

These are very important in engineering mechanics, as they form the very fundamentals of this topic.

Newton's three laws of motion were first published by Sir Isaac Newton in *The Principia* in 1687, and they can be expressed as follows:

(1) Every body continues in its state of rest or uniform motion in a straight line, unless it is compelled by an external force to change that state.

(2) The rate of change of momentum of a body with respect to time, is proportional to the resultant force, and takes place in a direction of which the resultant force acts.

(3) Action and reaction are equal and opposite.

I.5 Elementary statics

The trigonometrical formulae of I.2 can be used in statics. Consider the force F acting on an angle θ to the horizontal, as shown by Figure I.3(a). Now as the force F is a vector, (i.e. it has magnitude and direction), it can be represented as being equivalent to its horizontal and vertical components, namely F_H and F_V, respectively, as shown by Figure I.3(b). These horizontal and vertical components are also vectors, as they have magnitude and direction.

NB If F is drawn to scale, it is possible to obtain F_H and F_V from the scaled drawing.

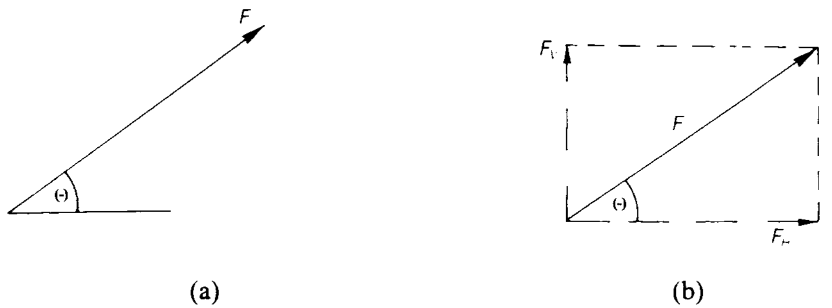


Figure I.3 Resolving a force.

From elementary trigonometry

$$\frac{F_H}{F} = \cos\theta$$

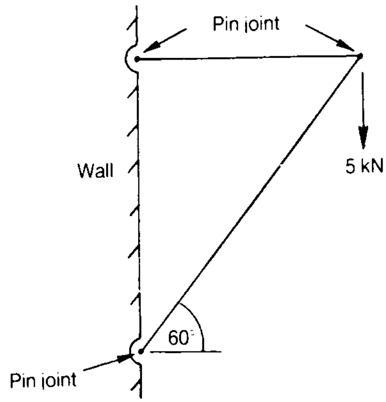
$$\therefore F_H = F\cos\theta \text{—horizontal component of } F$$

Similarly,

$$\frac{F_V}{F} = \sin\theta$$

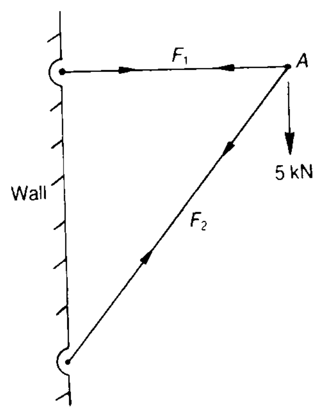
$$\therefore F_V = F\sin\theta \text{—vertical component of } F$$

Problem I.1 Determine the forces in the plane pin-jointed framework shown below.

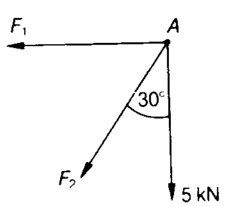


Solution

Assume all unknown forces in each member are in tension, i.e. the internal force in each member is pulling away from its nearest joint, as shown below.



Isolate joint A and consider equilibrium around the joint,



Resolving forces vertically

From Section I.7

$$\text{upward forces} = \text{downward forces}$$
$$0 = 5 + F_2 \cos 30$$

or
$$F_2 = -\frac{5}{\cos 30} = -5.77 \text{ kN (compression)}$$

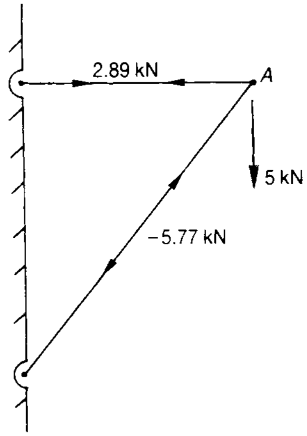
The negative sign for F_2 indicates that this member is in compression.

Resolving forces horizontally

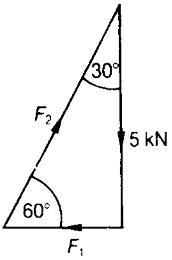
From Section I.7

$$\text{forces to the left} = \text{forces to the right}$$
$$F_1 + F_2 \sin 30 = 0$$
$$F_1 = -F_2 \sin 30 = 5.77 \sin 30$$
$$F_1 = 2.887 \text{ kN (tension)}$$

The force diagram is as follows:



Another method of determining the internal forces in the truss shown on page 4 is through the use of the triangle of forces. For this method, the magnitude and the direction of the known force, namely the 5kN load in this case, must be drawn to scale.



To complete the triangle, the directions of the unknown forces, namely F_1 and F_2 must be drawn, as shown above. The directions of these forces can then be drawn by adding the arrowheads to the triangle so that the arrowheads are either all in a clockwise direction or, alternatively, all in a counter-clockwise direction.

Applying the sine rule to the triangle of forces above,

$$\frac{5}{\sin 60} = \frac{F_1}{\sin 30}$$

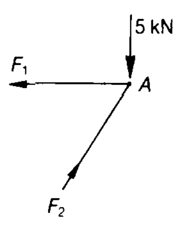
$$\therefore\ F_1 = \frac{5 \times 0.5}{0.866} = 2.887 \text{ kN}$$

Similarly by applying the sine rule:

$$\frac{5}{\sin 60} = \frac{F_2}{\sin 90}$$

$$\therefore\ F_2 = \frac{5}{0.866} = 5.77 \text{ kN}$$

These forces can now be transferred to the joint A of the pin-jointed truss below, where it can be seen that the member with the load F_1 is in tension, and that the member with the load F_2 is in compression.



This is known as a free body diagram.

I.6 Couples

A couple can be described as the moment produced by two equal and opposite forces acting together, as shown in Figure I.4 where,

the moment at the couple = $M = F \times l$ (N.m)

F = force (N)

l = lever length (m)

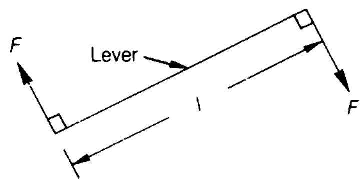


Figure I.4 A clockwise couple.

For the counter-clockwise couple of Figure I.5,

$$M = F \cos \theta \times l$$

where $F \cos \theta$ = the force acting perpendicularly to the lever of length l.

NB The components of force $F \sin \theta$ will simply place the lever in tension, and will not cause a moment.

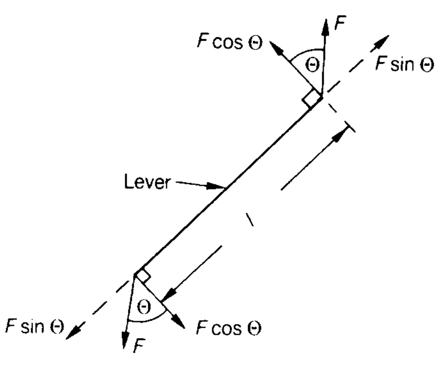


Figure I.5 A counter-clockwise couple.

It should be noted from Figure I.4 that the lever can be described as the perpendicular distance between the line of action of the two forces causing the couple.

Furthermore, in Figure I.5, although the above definition still applies, the same value of couple can be calculated, if the lever is chosen as the perpendicular distance between the components of the force that are perpendicular to the lever, and the forces acting on this lever are in fact those components of force.

I.7 Equilibrium

This section will be limited to one- or two-dimensional systems, where all the forces and couples will be acting in on plane; such a system of forces is called a coplanar system.

In two dimensions, equilibrium is achieved when the following laws are satisfied:

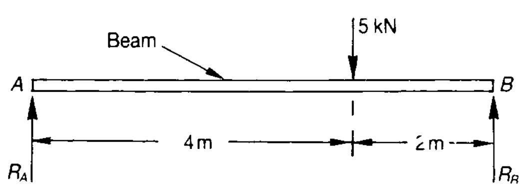
(1) upward forces = downward forces

(2) forces to the left = forces to the right

(3) clockwise couples = counter-clockwise couples.

To demonstrate the use of these two-dimensional laws of equilibrium, the following problems will be considered.

Problem I.2 Determine the values of the reactions R_A and R_B, when a beam is simply-supported at its ends and subjected to a downward force of 5 kN.



<u>*Solution*</u>

For this problem, it will be necessary to take moments. By taking moments, it is meant that the values of the moments must be considered about a suitable position.

Suitable positions for taking moments on this beam are A and B. This is because, if moments are taken about A, the unknown section R_A will have no lever and hence, no moment about A, thereby simplifying the arithmetic. Similarly, by taking moments about B, the unknown R_B will have no lever and hence, no moment about B, thereby simplifying the arithmetic.

Taking moments about B

clockwise moments = counter-clockwise moments

$$R_A \times (4 + 2) = 5 \times 2$$

or

$$R_A = 10/6$$

$$R_A = 1.667 \text{ kN}$$

Resolving forces vertically

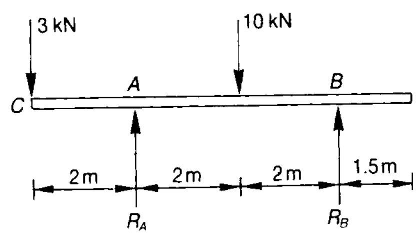
$$\text{upward forces} = \text{downward forces}$$

$$R_A + R_B = 5$$

or $$R_B = 5 - R_A = 5 - 1.667$$

$$R_B = 3.333 \text{ kN}$$

Problem I.3 Determine the values of the reactions of R_A and R_B for the simply-supported beam shown.



<u>*Solution*</u>

Taking moments about B

$$\text{clockwise couples} = \text{counter-clockwise couples}$$

$$R_A \times 4 = 3 \times 6 + 10 \times 2$$

$$R_A = \frac{18 + 20}{4}$$

$$R_A = 9.5 \text{ kN}$$

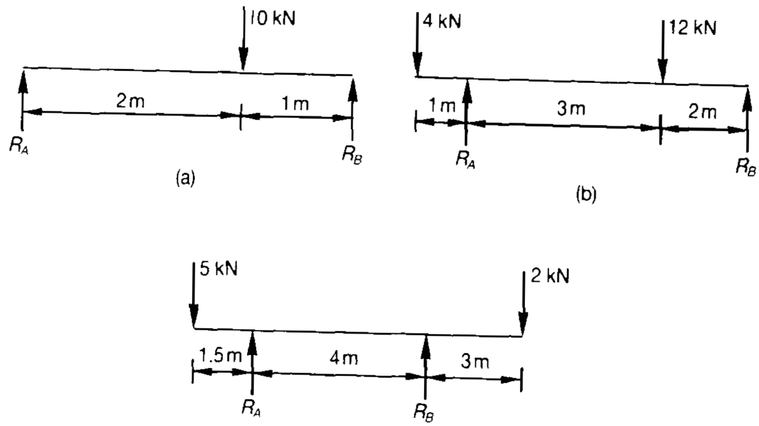
Resolving forces vertically

$$R_A + R_B = 3 + 10$$

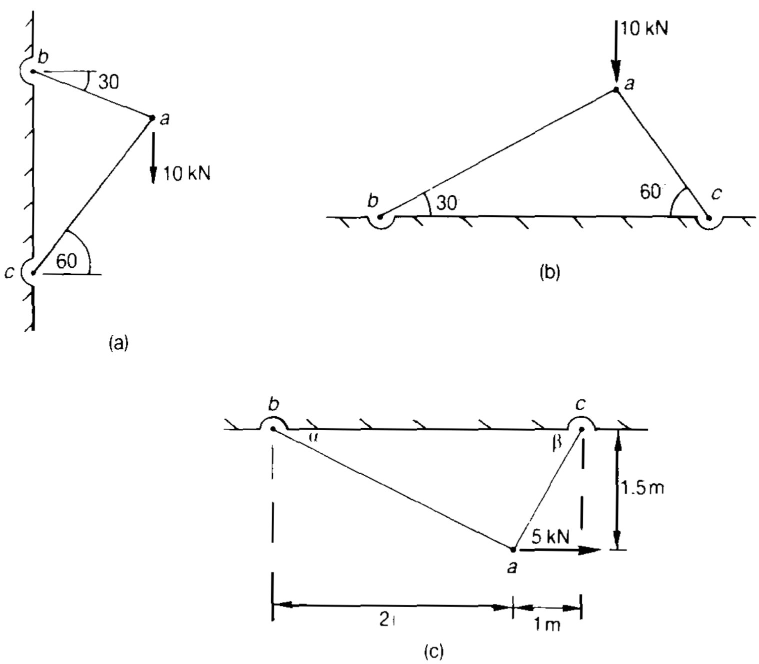
or $$R_B = 13 - 9.5 = 3.5 \text{ kN}$$

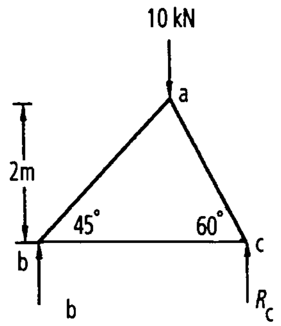
Further problems ***(answers on page 691)***

Problem I.4 Determine the reactions R_A and R_B for the simply-supported beams.



Problem I.5 Determine the forces the pin-jointed trusses shown.





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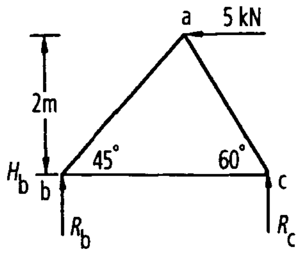

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