Answers to further problems

1.4 (a) $R_A = 3.333$ kN $R_B = 6.667$ kN
(b) $R_A = 9.6$ kN $R_B = 6.4$ kN
(c) $R_A = 4.625$ kN $R_B = 3.375$ kN

1.5 (a) $F_{ab} = 5$ kN $F_{ac} = -8.66$ kN
(b) $F_{ab} = -5$ kN $F_{ac} = -8.66$ kN
(c) $F_{ab} = 4.17$ kN $F_{ac} = -3$ kN
(d) $F_{ab} = -4.71$ kN $F_{ac} = -7.454$ kN $F_{bc} = 3.333$ kN
(e) $F_{ab} = -4.71$ kN $F_{ac} = 3.727$ kN $F_{bc} = 3.333$ kN

1.17 31.0 MN/m^2 (compressive); 0.098 cm.

1.18 0.902 cm.

1.19 0.865 cm.

1.21 51.0 kN.

1.22 65.5 MN/m^2 tensile in steel; 41.0 MN/m^2 compressive in copper; increase of length 0.611 cm; force to prevent expansion 135.5 kN.

2.2 $F_{ab} = -0.46$ W $F_{ad} = 0.763$ W $F_{bc} = -0.651$ W
$F_{cd} = 0.46$ W $F_{de} = -0.008$ W $F_{bd} = -0.54$ W

2.3 $F_{ac} = 4$ kN $F_{ad} = 2.829$ kN $F_{bd} = -10$ kN $F_{cd} = -4$ kN
$F_{df} = -2$ kN $F_{ce} = 5.66$ kN $F_{ed} = -8$ kN $F_{ef} = 5.66$ kN

2.4 $F_{ac} = 7.8$ kN $F_{bc} = -1.45$ kN $F_{bd} = -8.8$ kN $F_{ce} = 6.8$ kN
$F_{cd} = 1$ kN $F_{ed} = -1.45$ kN $F_{df} = -7.8$ kN $F_{eg} = 5.8$ kN
$F_{ef} = 1$ kN $F_{fg} = -1.45$ kN $F_{fh} = -6.8$ kN $F_{gj} = 6.35$ kN
$F_{gk} = 0.5$ kN $F_{gh} = -3.9$ kN $F_{hk} = -7.9$ kN $F_{jk} = -4.5$ kN
$F_{jl} = 4.5$ kN $F_{kl} = -4$ kN

2.5 $R_A = 1.333$ kN $R_B = 1.667$ kN
$F_{ac} = -1.54$ kN $F_{ac} = 0.77$ kN $F_{ec} = 1.54$ kN $F_{ef} = -1.54$ kN
$F_{cf} = -0.38$ kN $F_{cd} = 1.732$ kN $F_{fd} = 0.38$ kN $F_{fg} = -1.92$ kN
$F_{dg} = 1.92$ kN $F_{db} = 0.96$ kN $F_{gb} = -1.92$ kN

3.4 480 MN/m^2.

3.5 521 kW.

3.6 295 m/s^2.

3.7 188.5 Nm.

3.8 $d = 6.29$ cm; cotter thickness 1.57 cm; mean width of cotter 7.98 cm; distance of cotter hole from end of left-hand rod 2.97 cm; diameter of right-hand rod through cotter pin 8.28 cm; maximum diameter of right-hand rod 12.58 cm; distance of end of right-hand rod from cotter hole 2.97 cm.

3.9 8.93 cycles/sec.

3.10 0.6 MN/m^2.

4.7 376 kN/m.

5.7 50.0 MN/m^2 tensile; 28.9 MN/m^2 shearing.
5.8 greatest tensile stress 86.6 MN/m^2, on plane at 34° 44′ to cross-section; greatest shearing stress 64.0 MN/m^2, on planes at 10° 16′ and 79° 44′ to cross-section.
5.9 30 MN/m^2 tensile; 120 MN/m^2 compressive.
5.10 81.0 MN/m^2, inclined at 23° 27′ to horizontal.
5.11 90 MN/m^2 tensile; 60 MN/m^2 compressive; 5.40×10^{-4} tensile; 4.35×10^{-4} compressive.
5.12 7.5 MN/m^2 normal; 51.9 MN/m^2, shearing.

6.8 11.0 MN/m^2
6.9 1.03 kg/m.
6.10 0.114 per cent.
6.11 (a) copper: 38.2 MN/m^2; wire: 83.9 MN/m^2;
(b) copper: 28.6 MN/m^2 (compressive); wire: 230 MN/m^2.
6.12 1.19 MN/m^2.
6.13 171 MN/m^2.

7.10 489 kNM.
7.13 238 kNm; 0.75 m from *A*.

8.5 (a) 2.779×10^{-5} m^4.
(b) 10.83×10^{-6} m^4.
(c) 5.334×10^{-5} m^4.
8.6 (a) 1.419×10^{-5} m^4.
(b) 3.942×10^{-5} m^4.
8.7 H/3 BH^3 /36

9.12 40.6 kN.m.
9.13 69.9 MN/m^2.
9.14 15.9 MN/m^2.
9.15 86.0 MN/m^2.
9.16 6.17 cm.

10.4 1 cm thickness; 5 cm spacing of rivets, assuming one rivet at any cross-section.
10.5 maximum tensile stress of 124 MN/m^2 is greater than the allowable stress; maximum shearing stress of 18 MN/m^2 is less than the allowable stress.
10.6 96 per cent of shearing force carried by web; 88 per cent of bending moment carried by flanges.
10.7 web thickness 0.67 cm; weld throats 0.33 cm.
10.8 $\tau = 2450\,(RL/t) \sin\theta$, where θ is the angular position of any section from the vertical line through the centre of the tube.

10.9 bending is limiting, and gives an allowable superimposed load of 45 kN/m; required welds 0.26 cm throat thickness.

10.10 (a) 1.273 R.

(b) 1.72 R.

11.8 0.378×10^{-3} m^2; 13.02 kN/m.

11.9 114 kN.

11.10 wood 4.56 MN/m^2; steel 52.9 MN/m^2; glue 0.21 MN/m^2.

11.11 (i) 120 MN/m^2.

(ii) 1.00 MN/m^2.

(iii) 100 kN/m.

(iv) 0.75 cm.

12.3 tensile 155 MN/m^2, compressive 147 MN/m^2; neutral axis 0.365 m from outside of box-section.

12.4 17.68 kN; 11.8 MN/m^2 compressive.

12.6 maximum tensile 38.0 MN/m^2; maximum compressive 46.0 MN/m^2.

12.7 161 kN.

12.8 13.8 MN/m^2; 5.94 cm from tip of T.

13.2 1.80 cm and 2.48 cm.

13.3 3.06 cm.

14.2 maximum bending moment 105 kNm; points of inflexion at 1.75 m from each end.

14.3 169.7 kNm at left-hand end; 150.0 kNm at right-hand end; 1.52 m from left-hand end; 1.69 m from right-hand end.

15.2 217 kN.

15.3 62.4 kN/m.

15.4 required elastic section modulus 791 cm^3.

15.5 required elastic section modulus 2030 cm^3.

15.6 84.2 kN/m, with collapse in the end spans.

15.7 3.26 cm.

16.7 38.1 MN/m^2; 1.09°; 39.2 cm.

16.8 40.3 MN/m^2; 3.83°.

16.9 Shearing stress 37.7 MN/m^2; maximum tensile stress 37.7 MN/m^2; angle of twist 4.31°.

16.10 0.644: 1.

16.11 38.7 Nm.

16.12 147 MN/m^2 (tensile) at 34.8° to axis, 70.5 MN/m^2 (compressive) at 57.2° to axis.

17.15 No horizontal deflection.

17.16 609 kNm and 423 kNm.

17.17 $3WR/4\pi$, at the support, where W is the weight of the ring.

17.18 12.45 PR^3/EI.

17.19 2.89×10^{-4} m^3.

17.20 1.288×10^{-3} m^3.

17.21 6.324×10^{-4} m^3 (verticals); 9.486×10^{-4} m^3 (top left); 1.997×10^{-3} m^3 (top right).

18.2 970 N.

18.3 0.10 cm.

18.4 1.65 kN.

18.5 24.5 kN.

19.6
$$\left\{ w = \frac{W}{8\pi D}\left[\frac{(3+\nu)}{2(1+\nu)}\left(R^2 - r^2\right) + r^2 \ln\left(\frac{r}{R}\right)\right]\right\}$$

19.7
$$\left\{ \hat{w} = \frac{pR^4}{64D}\left[\frac{(5+\nu)}{(1+\nu)} - \frac{(6+2\nu)}{(1+\nu)}\left(\frac{r}{R}\right)^2 + \left(\frac{r}{R}\right)^4\right];\right.$$

$$M_r = -\frac{(3+\nu)}{16} pR^2\left[1 - \left(\frac{r}{R}\right)^2\right];$$

$$\left. M_t = \frac{pR^2}{16}\left[-(3+\nu) + (1+3\nu)\left(\frac{r}{R}\right)^2\right]\right\}$$

19.8
$$\left\{ \hat{w} = \frac{W}{16\pi D}\left[\frac{(3+\nu)}{(1+\nu)}\left(R_2^2 - R_1^2\right) + 2R_1^2 \ln\left(\frac{R_1}{R_2}\right)\right]\right\}$$

19.9 (a)
$$\left\{ \hat{w} = \frac{P}{16\pi D}\left[\frac{(3+\nu)}{(1+\nu)} R_2^2 - \frac{(7+3\nu)}{4(1+\nu)} R_1^2 + R_1^2 \ln\left(\frac{R_1}{R_2}\right)\right];\right.$$

$$\left. \hat{M} = \frac{P}{4\pi}\left[1 - \frac{(1-\nu)}{4}\left(\frac{R_1}{R_2}\right)^2 - (1+\nu) \ln\left(\frac{R_1}{R_2}\right)\right];\right.$$

(b)
$$\hat{w} = \frac{P}{16\pi D}\left[R_2^2 - 0.75R_1^2 + R_1^2 \ln\left(\frac{R_1}{R_2}\right)\right];$$

$$\hat{M} = \frac{P}{4\pi}\left[1 - 0.5\left(\frac{R_1}{R_2}\right)^2\right] \text{ for } R_1 / R_2 > 0.57$$

and $$\hat{M} = \frac{P}{4\pi}(1+\nu)\left[0.25\left(\frac{R_1}{R_2}\right)^2 - 1\ln\left(\frac{R_1}{R_2}\right)\right] \text{ for } R_1\ R_2 < 0.57;$$

where $$P = p * \pi R_1^2\Big\}$$

19.10 $$\left\{0.126pR^4 / \left(Et^3\right); p\left(\frac{R}{t}\right)^2\left[-1.238\left(\frac{r}{R}\right)^2 + 0.507 + 0.0105\left(\frac{R}{r}\right)^2\right]\right\}$$

19.11 $$\left\{0.115WR^2 / \left(Et^3\right); \frac{W}{t^2}\left[0.621\ln\left(\frac{R}{r}\right) - 0.436 + 0.0224\left(\frac{R}{r}\right)^2\right]\right\}$$

22.1 $$\begin{bmatrix} 3 & 1 \\ 4 & -1 \end{bmatrix}$$

22.2 $$\begin{bmatrix} 5 & 1 \\ 0 & 7 \end{bmatrix}$$

22.3 $$\begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}$$

22.4 $$\begin{bmatrix} -1 & 2 \\ 0 & -4 \end{bmatrix}$$

22.5 $$\begin{bmatrix} -2 & -4 \\ 4 & -12 \end{bmatrix}$$

22.6 $$\begin{bmatrix} -4 & -1 \\ 0 & -10 \end{bmatrix}$$

22.7 10.

22.8 4.

22.9 $\begin{bmatrix} 0.3 & -0.1 \\ -0.2 & 0.4 \end{bmatrix}$

22.10 $\begin{bmatrix} -1 & 0 \\ -0.5 & -0.25 \end{bmatrix}$

22.11 $\begin{bmatrix} 10 & -1 & -2 \\ -3 & 9 & 1 \\ -4 & -2 & 7 \end{bmatrix}$

22.12 $\begin{bmatrix} -8 & -3 & 2 \\ -1 & -7 & -5 \\ 4 & -2 & -5 \end{bmatrix}$

22.13 $\begin{bmatrix} 1 & -2 & 0 \\ -2 & 1 & -2 \\ 0 & -2 & 1 \end{bmatrix}$

22.14 $\begin{bmatrix} 9 & -1 & -4 \\ 1 & 8 & 0 \\ -2 & 3 & 6 \end{bmatrix}$

22.15 $\begin{bmatrix} 11 & -15 & -8 \\ -11 & 6 & -5 \\ -2 & -16 & 0 \end{bmatrix}$

22.16 $\begin{bmatrix} 7 & -13 & -4 \\ -17 & 4 & -13 \\ -4 & -4 & 6 \end{bmatrix}$

22.17 −7.

22.18 362.

22.19 $\begin{bmatrix} 0.429 & -0.286 & -0.571 \\ -0.268 & -0.143 & -0.286 \\ -0.571 & -0.286 & 0.429 \end{bmatrix}$

22.20
$$\begin{bmatrix} 0.133 & -1.66\times10^{-2} & -5.25\times10^{-2} \\ -1.66\times10^{-2} & 0.127 & -6.91\times10^{-2} \\ -8.84\times10^{-2} & -1.10\times10^{-2} & 0.202 \end{bmatrix}$$

23.8 (a) $u_4^\circ = 2.828/AE$ $v_4^\circ = 0$
$F_{1\text{-}4} = 1.414$ kN $F_{2\text{-}4} = 0$ $F_{3\text{-}4} = -1.414$ kN
(b) $u_4^\circ = 2.626/AE$ $v_4^\circ = 0$
$F_{1\text{-}4} = 1.313$ kN $F_{2\text{-}4} = 0$ $F_{3\text{-}4} = 1.313$ kN

23.9 (a) $u_5^\circ = 2.178/AE$ $v_5^\circ = -0.243/AE$
(b) $u_5^\circ = 2.065/AE$ $v_5^\circ = -0.224/AE$ $\theta_5 = 2.528/AE$

23.10 (a) $v_1 = -Wl^3/3EI$ $\theta = -Wl^2/2EI$
$v_2 = \theta_2 = 0$
$M_1 = 0$ $M_2 = Wl$
(b) $v_1 = \theta_1 = v_3 = \theta_3 = 0$; $\theta_2 = 0$
$v_2 = -Wl^3/192EI$
$M_1 = -M_3 = -Wl/8$
$M_2 = \pm Wl/8$

23.11 $v_1 = \theta_1 = v_2 = v_3 = v_4 = \theta_4 = 0$
$\theta_2 = -1.136/EI$
$\theta_3 = 0.2/EI$
$M_1 = -0.345$ $M_2 = \pm 0.311$
$M_3 = \pm 1.45$ $M_4 = 2.65$

23.12 $v_1 = \theta_1 = v_2 = v_3 = v_4 = \theta_4 = 0$
$\theta_2 = 0.386/EI$ $\theta_3 = 0.193/EI$
$M_1 = -0.643$ $M_2 = \pm 1.864$
$M_3 = \pm 4.629$ $M_4 = 6.236$

23.13 (a) $u_4^\circ = 220.59/AE$ $v_4^\circ = 209.67/AE$ $w_4^\circ = -77.21/AE$
$F_{1\text{-}4} = 4.59$ kN $F_{2\text{-}4} = -2.75$ kN $F_{3\text{-}4} = -7.28$ kN
(b) $u_5^\circ = 2.287/AE$ $v_5^\circ = -8.591/AE$ $w_5^\circ = -1.904/AE$
$F_{1\text{-}5} = -1.685$ kN $F_{2\text{-}5} = 1.179$ kN $F_{3\text{-}5} = -2.927$ kN
$F_{4\text{-}5} = -0.054$ kN

23.13 (c) $u_5^\circ = 13.48/AE$ $v_5^\circ = 41.72/AE$ $w_5^\circ = -39.46/AE$
$F_{1\text{-}5} = 1.05$ MN $F_{2\text{-}5} = -4.51$ MN $F_{3\text{-}5} = -9.51$ MN
$F_{4\text{-}5} = -2.15$ MN

23.14 (a) $u_2^\circ = 1.257\times10^{-2} = u_3^\circ$
$\theta_2 = 0.162\times10^{-2}$ rads $\theta_3 = 0.162\times10^{-3}$ rads
$M_1 = -10.47$ kNm $M_2 = \pm 3.52$ kNm
$M_3 = \pm 6.188$ kNm $M_4 = -7.81$ kNm

(b) $u_2^{\circ} = 12.134/EI$ $\theta_2 = 3.777/EI$

$\theta_3 = 1.132/EI$

$M_1 = -9.32$ kNm $M_2 = \pm 0.7$ kNm

$M_3 = \pm 6.58$ kNm $M_4 = -7.33$ kNm

24.2 $$\frac{AE}{l}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

24.3 $$\frac{(A_1 + A_2)E}{2l}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

24.4 $$\frac{GJ}{l}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

24.5 $$EI\begin{bmatrix} 12/l^3 & -6/l^2 & -12/l^3 & -6/l^2 \\ -6/l^2 & 4/l & 6/l^2 & 2/l \\ -12/l^3 & 6/l^2 & 12/l^3 & 6/l^2 \\ -6/l^2 & 2/l & 6/l^2 & 4/l \end{bmatrix}$$

25.7 7.00 Hz, 28Hz, etc., 11.85 Hz, 47.4 Hz etc.

25.8 4.73 Hz.

25.9 $(3EI/ML^3)^{1/2}/2\pi$.

25.10 404.1 Hz, 598.5 Hz.

25.11 294.8 Hz, 361.6 Hz, 485.4 Hz.

25.12 53.1 Hz, 164.1 Hz.

25.13 40.56 Hz.

25.14 191.8 Hz, 354.3 Hz, 907.8 Hz.