10 Shearing stresses in beams

10.1 Introduction

We referred earlier to the existence of longitudinal direct stresses in a cantilever with a lateral load at the free end; on a closer study we found that these stresses are distributed linearly over the cross-section of a beam carrying a uniform bending moment. In general we are dealing with bending problems in which there are shearing forces present at any cross-section, as well as bending moments. In practice we find that the longitudinal direct stresses in the beam are almost unaffected by the shearing force at any section, and are governed largely by the magnitude of the bending moment at that section. Consider again the bending of a cantilever with a concentrated lateral load F, at the free end, Figure 10.1; Suppose the beam is of rectangular cross-section. If we cut the beam at any transverse cross-section, we must apply bending moments M and shearing forces F at the section to maintain equilibrium. The bending moment M is distributed over the cross-section in the form of longitudinal direct stresses, as already discussed.

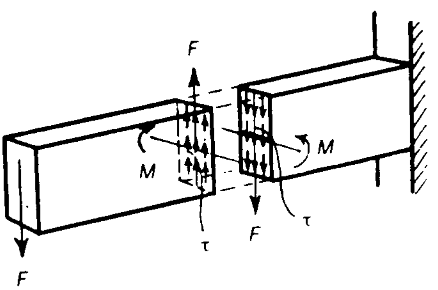


Figure 10.1 Shearing actions in a cantilever carrying an end load.

The shearing force F is distributed in the form of shearing stresses τ, acting tangentially to the cross-section of the beam; the form of the distribution of τ is dependent on the shape of the cross-section of the beam, and on the direction of application of the shearing force F. An interesting feature of these shearing stresses is that, as they give rise to complementary shearing stresses, we find that shearing stresses are also set up in longitudinal planes parallel to the axis of the beam.

10.2 Shearing stresses in a beam of narrow rectangular cross-section

We consider first the simple problem of a cantilever of *narrow* rectangular cross-section, carrying a concentrated lateral load F at the free end, Figure 10.2; h is the depth of the cross-section, and t is the thickness, Figure 10.3; the depth is assumed to be large compared with the thickness. The load is applied in a direction parallel to the longer side h.

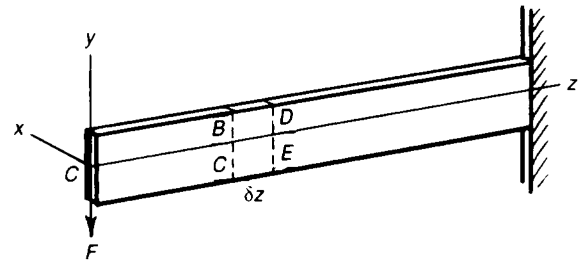


Figure 10.2 Shearing stresses in a cantilever of narrow rectangular cross-section under end load.

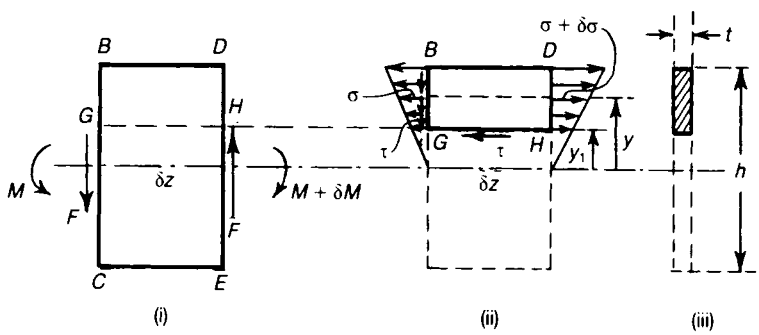


Figure 10.3 Shearing actions on an elemental length of a beam of narrow rectangular cross-section.

Consider an elemental length δz of the beam at a distance z from the loaded end. On the face BC of the element the hogging bending moment is

$$M = Fz$$

We suppose the longitudinal stress σ at a distance y from the centroidal axis Cx is the same as that for uniform bending of the element. Then

$$\sigma = \frac{My}{I_x} = \frac{Fyz}{I_x}$$

Where I_x is the second moment of area about the centroidal axis of bending, Cx, which is also a neutral axis . On the face DE of the element the bending moment has increased to

$$M + \delta M = F(z + \delta z)$$

The longitudinal bending stress at a distance y from the neutral axis has increased correspondingly to

$$\sigma + \delta\sigma = \frac{F(z + \delta z)y}{I_x}$$

Now consider a depth of the beam contained between the upper extreme fibre *BD*, given by $y = \frac{1}{2}h$, and the fibre *GH*, given by $y = y_1$, Figure 10.3(ii). The total longitudinal force on the face *BG* due to bending stresses σ is

$$\int_{y_1}^{h/2} \sigma t dy = \frac{Fzt}{I_x} \int_{y_1}^{\frac{1}{2}h} y dy + \frac{Fzt}{2I_x}\left[\frac{h^2}{4} - y_1^2\right]$$

By a similar argument we have that the total force on the face *DH* due to bending stresses σ + δσ is

$$\frac{Ft}{2I_x}\left(\frac{h^2}{4} - y_1^2\right)(z + \delta z)$$

These longitudinal force, which act in opposite directions, are not quite in balance; they differ by a small amount

$$\frac{Ft}{2I_r}\left(\frac{h^2}{4} - y_1^2\right)\delta z$$

Now the upper surface *BD* is completely free of shearing stress, and this out-of-balance force can only be equilibrated by a shearing force on the face *GH*. We suppose this shearing force is distributed uniformly over the face *GH*; the shearing stress on this face is then

$$\begin{aligned}\tau &= \frac{Ft}{2I_x}\left(\frac{h^2}{4} - y_1^2\right)\delta z \,/\, t\delta z \\ &= \frac{F}{2I_x}\left(\frac{h^2}{4} - y_1^2\right)\end{aligned} \qquad (10.1)$$

This shearing stress acts on a plane parallel to the neutral surface of the beam; it gives rise therefore to a complementary shearing stress τ at a point of the cross-section a distance y_1 from the neutral axis, and acting tangentially to the cross-section. Our analysis gives then the variation of shearing stress over the depth of the cross-section. For this simple type of cross-section

$$I_x = \frac{1}{12} h^3 t$$

and so

$$\tau = \frac{6F}{h^3 t}\left(\frac{h^2}{4} - y_1^2\right) = \frac{6F}{ht}\left[\frac{1}{4} - \left(\frac{y_1}{h}\right)^2\right] \tag{10.2}$$

We note firstly that τ is independent of z; this is so because the resultant shearing force is the same for all cross-sections, and is equal to F. The resultant shearing force implied by the variation of τ is

$$\int_{-h/2}^{+h/2} \tau t dy_1 = \frac{6F}{h}\int_{-h/2}^{+h/2}\left[\frac{1}{4} - \left(\frac{y_1}{h}\right)^2\right] dy_1 = F$$

The shearing stresses τ are sufficient then to balance the force F applied to every cross-section of the beam.

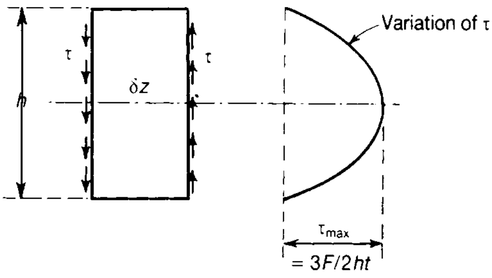


Figure 10.4 Variation of shearing stresses over the depth of a beam of rectangular cross-section.

The variation of τ over the cross-section of the beam is parabolic, Figure 10.4; τ attains a maximum value on the neutral axis of the beam, where $y_1 = 0$, and

$$\tau_{max} = \frac{3F}{2ht} \tag{10.3}$$

The shearing stresses must necessarily be zero at the extreme fibres as there can be no complementary shearing stresses in the longitudinal direction on the upper and lower surfaces of the beam.

In the case of a cantilever with a single concentrated load F at the free end the shearing force is the same for all cross-sections, and the distribution of shearing stresses is also the same for all cross-sections. In a more general case the shearing force is variable from one cross-section to another: in this case the value of F to be used is the shearing force at the section being considered.

10.3 Beam of any cross-section having one axis of symmetry

We are concerned generally with more complex cross-sectional forms than narrow rectangles. Consider a beam having a uniform cross-section which is symmetrical about Cy, Figure 10.5. Suppose, as before, that the beam is a cantilever carrying an end load F acting parallel to Cy and passing through the centroid C of the cross-section. Then Cx is the axis of bending.

Consider an elemental length δz of the beam; on the near face of this element, which is at a distance z from the free end of the cantilever, the bending moment is

$$M = Fz$$

This gives rise to bending stresses in the cross-section; the longitudinal bending stress at a point of the cross-section a distance y from the neutral axis Cx is

$$\sigma = \frac{My}{I_x} = \frac{Fyz}{I_x}$$

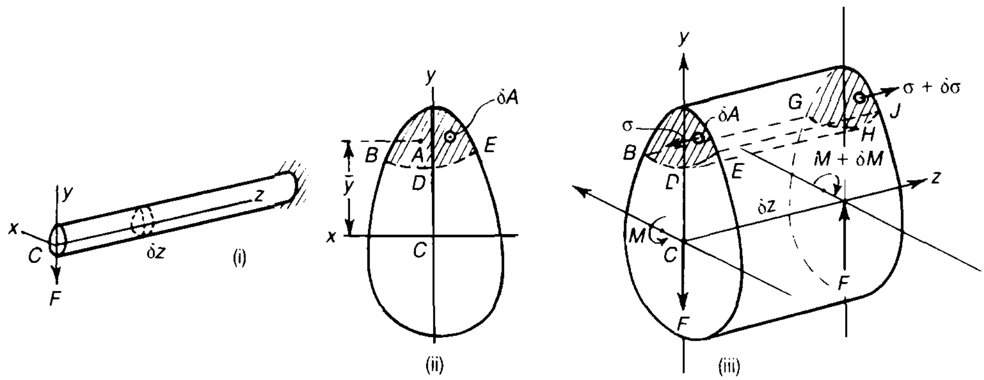


Figure 10.5 Shearing stresses in a bent beam having one axis of symmetry.

Now consider a section of the element cut off by the cylindrical surface $BDEGHJ$, Figure 10.5(iii), which is parallel to Cz. Suppose A is the area of each end of this cylindrical element; then the total longitudinal force on the end BDE due to bending stresses is

$$\int_A \sigma dA = \frac{Fz}{I_x} \int_A y dA$$

where δA is an element of the area A, and y is the distance of this element from the neutral axis Cx. The total longitudinal force on the remote end GHJ due to bending stresses is

$$\int_A (\sigma + \delta\sigma)\, dA = \frac{F}{I_x}(z + \delta z) \int_A y dA$$

as the bending moment at this section is

$$M + \delta M = F(z + \delta z)$$

The tension loads at the ends of the element $BDEGHJ$ differ by an amount

$$\frac{F\delta z}{I_x} \int_A y dA$$

If $\bar{y}$ is the distance of the centroid of the area A from Cx, then

$$\int_A y dA = A\bar{y}$$

The out-of-balance tension load is equilibrated by a shearing force over the cylindrical surface *BDEGHJ*.

This shearing force is then

$$\frac{F\delta z}{I_x} \int_A y dA = \frac{F\delta z}{I_x} A\bar{y}$$

and acts along the surface *BDEGHJ* and parallel to Cz. The total shearing force per unit length of the beam is

$$q = \frac{F\delta z}{I_x} A\bar{y} / \delta z = \frac{FA\bar{y}}{I_x} \tag{10.4}$$

If b is the length of the curve *BDE*, or *GHJ*, then the average shearing stress over the surface *BDEGHJ* is

$$\bar{\tau} = \frac{FA\bar{y}}{bI_x} \tag{10.5}$$

When b is small compared with the other linear dimensions of the cross-section we find that the shearing stress is nearly uniformly distributed over the surfaces of the type *BDEGHJ*. This is the case in thin-walled beams, such as I-sections and channel sections.

10.4 Shearing stresses in an I-beam

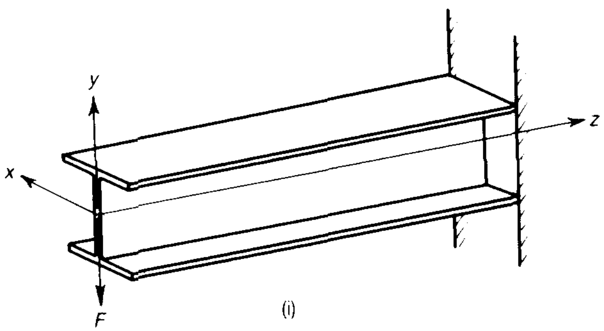
As an application of the general method developed in the preceding paragraph, consider the shearing stresses induced in a thin-walled I-beam carrying a concentrated load F at the free end, acting parallel to Cy, Figure 10.6. The cross-section has two axes of symmetry Cx and Cy; the flanges are of breadth b, and the distance between the centres of the flanges is h; the flanges and web are assumed to be of uniform thickness t.

Equation (10.4) gives the shearing force q per unit length of beam at any region of the cross-section. Consider firstly a point *1* of the flange at a distance s_1 from a free edge, Figure 10.6(iii); the area of flange cut off by a section through the point *1* is

$$A = s_1 t$$

The distance of the centroid of this area from the neutral axis Cx is

$$\bar{y}_1 = \frac{1}{2}h$$



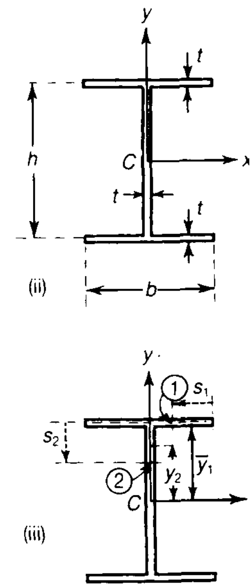


Figure 10.6 Flexural shearing stresses in an I-beam.

Then from equation (10.4), the shearing force at point *1* of the cross-section is

$$q = \frac{F s_1 t h}{2 I_x} \tag{10.6}$$

If the wall thickness t is small compared with the other linear dimensions of the cross-section, we may assume that q is distributed uniformly over the wall thickness t; the shearing stress is then

$$\tau = \frac{q}{t} = \frac{F s_1 h}{2 I_x} \tag{10.7}$$

at point *1*. At the free edge, given by $s_1 = 0$, we have $\tau = 0$, since there can be no longitudinal shearing stress on a free edge of the cross-section. The shearing stress τ increases linearly in intensity as s_1 increases from zero to $\frac{1}{2}b$; at the junction of web and flanges $s_1 = \frac{1}{2}b$, and

$$\tau = \frac{Fbh}{4 I_x} \tag{10.8}$$

As the cross-section is symmetrical about Cy, the shearing stress in the adjacent flange also increases linearly from zero at the free edge.

Consider secondly a section through the web at the point *2* at a distance s_2 from the junctions of the flanges and web. In evaluating $A\bar{y}$ for this section we must consider the total area cut off by the section through the point *2*. However, we can evaluate $A\bar{y}$ for the component areas cut off by the section through the point *2*; we have

$$A\bar{y} = (bt)\,\frac{1}{2}h + (s_2 t)\left(\frac{1}{2}h - \frac{1}{2}s_2\right)$$

$$= \frac{1}{2}t\,[bh + s_2(h - s_2)]$$

The from equation (10.4),

$$q = \frac{Ft}{2I_x}[bh + s_2(h - s_2)]$$

If this shearing force is assumed to be uniformly distributed as a shearing stress, then

$$\tau = \frac{q}{t} = \frac{F}{2I_x}[bh + s_2(h - s_2)] \qquad (10.9)$$

At the junction of web and flanges $s_2 = 0$, and

$$\tau = \frac{Fbh}{2I_x} \qquad (10.10)$$

At the neutral axis, $s_2 = \frac{1}{2}h$, and

$$\tau = \frac{Fbh}{2I_x}\left[1 + \frac{h}{4b}\right] \qquad (10.11)$$

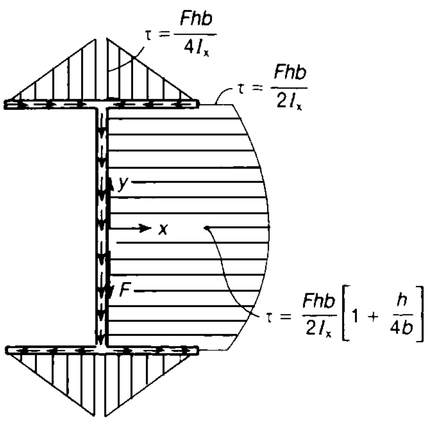


Figure 10.7 Variation of shearing stresses in an I-beam.

We notice that τ varies parabolically throughout the depth of the web, attaining a maximum value at $s_2 = \frac{1}{2}h$, the neutral axis, Figure 10.7. In any cross-section of the beam the shearing stresses vary in the form shown; in the flanges the stresses are parallel to Cx, and contribute nothing to the total force on the section parallel to Cy.

At the junctions of the web and flanges the shearing stress in the web is twice the shearing stresses in the flanges. The reason for this is easily seen by considering the equilibrium conditions at this junction. Consider a unit length of the beam along the line of the junction, Figure 10.8; the shearing stresses in the flanges are

$$\tau_f = \frac{Fbh}{4I_x} \tag{10.12}$$

while the shearing stress in the web we have estimated to be

$$\tau_w = \frac{Fbh}{2I_x} \tag{10.13}$$

For longitudinal equilibrium of a unit length of the junction of web and flanges, we have

$$2[\tau_f \times (t \times 1)] = \tau_w \times (t \times 1)$$

which gives

$$\tau_w = 2\tau_f \tag{10.14}$$

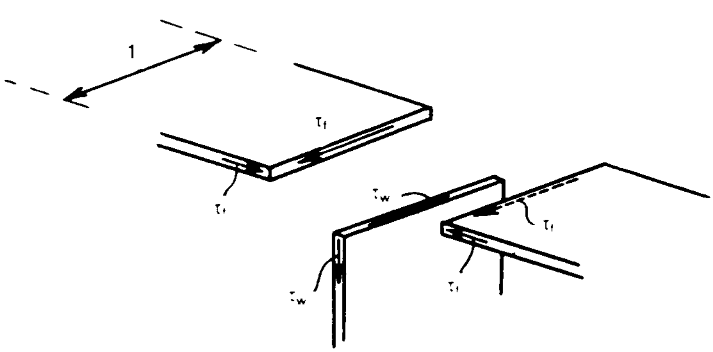


Figure 10.8 Equilibrium of shearing forces at the junction of the web and flanges of an I-beam.

This is true, in fact, for the relations we have derived above; longitudinal equilibrium is ensured at any section of the cross-section in our treatment of the problem. If the flanges and web were of different thicknesses, t_f and t_w, respectively, the equilibrium condition at the junction would be

$$2\tau_f t_f = \tau_w t_w$$

Then

$$\frac{\tau_w}{\tau_f} = \frac{2t_f}{t_w} \tag{10.15}$$

The implication of this equilibrium condition is that at a junction, such as that of the flanges and web of an I-section, the sum of the shearing forces per unit length for the components meeting at that junction is zero when account is taken of the relevant directions of these shearing forces. For a junction

$$\sum \tau t = 0 \tag{10.16}$$

where τ is the shearing stress in an element at the junction, and t is the thickness of the element; the summation is carried out for all elements meeting at the junction.

For an I-section carrying a shearing force acting parallel to the web we see that the maximum shearing stress occurs at the middle of the web, and is given by equation (10.11). Now, I_x for the section is given approximately by

$$I_x = \frac{1}{12} h^3 t + \frac{1}{2} h^2 bt = \frac{h^3 t}{12}\left(1 + \frac{6b}{h}\right) \tag{10.17}$$

Then

$$\tau_{\max} = \frac{6Fb}{h^2 t}\left[\frac{1 + h/4b}{1 + 6b/h}\right] \tag{10.18}$$

The total shearing force in the web of the beam parallel to Cy is F; if this were distributed uniformly over the depth of the web the average shearing stress would be

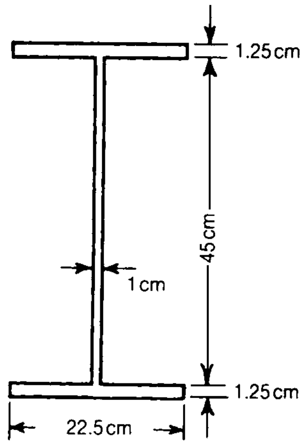
$$\tau_{av} = \frac{F}{ht} \tag{10.19}$$

Then for the particular case when $h = 3b$, we have

$$\tau_{\max} = \frac{7}{6}\left(\frac{F}{ht}\right) \tag{10.20}$$

Then $\tau_{\max}$ is only one-sixth or about 17% greater than the mean shearing stress over the web.

Problem 10.1 The web of a girder of I-section is 45 cm deep and 1 cm thick; the flanges are each 22.5 cm wide by 1.25 cm thick. The girder at some particular section has to withstand a total shearing force of 200 kN. Calculate the shearing stresses at the top and middle of the web. *(Cambridge)*



<u>*Solution*</u>

The second moment of area of the web about the centroidal axis is

$$\frac{1}{12}\,(0.010)\,(0.45)^3 \;=\; 0.0760 \times 10^{-3}\ \mathrm{m}^4$$

The second moment of are of each flange about the centroidal axis is

$$(0.225)\,(0.0125)\,(0.231)^2 \;=\; 0.150 \times 10^{-3}\ \mathrm{m}^4$$

The total second moment of area is then

$$I_x \;=\; [0.076 + 2(0.150)]\,10^{-3} \;=\; 0.376 \times 10^{-3}\ \mathrm{m}^4$$

At a distance y above the neutral axis, the shearing stress from equation (10.9) is

$$\tau \;=\; \frac{F}{2I_x}\left[\left(bh + \frac{1}{4}\,h^2\right) - y^2\right]$$

$$= \frac{200 \times 10^3}{2 \times 0.376 \times 10^{-3}}\left[(0.225)\,(0.4625) + \frac{1}{4}\,(0.4625)^2 - y^2\right]$$

where $s_2 = h/2 - y$

At the top of the web, we have $y = 0.231$ m, and

$$\tau \;=\; 27.7\ \mathrm{MN/m^2}$$

While at the middle of the web, where $y = 0$, we have

$$\tau = 41.9 \text{ MN/m}^2$$

10.5 Principal stresses in beams

We have shown how to find separately the longitudinal stress at any point in a beam due to bending moment, and the mean horizontal and vertical shearing stresses, but it does not follow that these are the greatest direct or shearing stresses. Within the limits of our present theory we can employ the formulae of Sections 5.7 and 5.8 to find the principal stresses and the maximum shearing stress.

We can draw, on a side elevation of the beam, lines showing the direction of the principal stresses. Such lines are called the *lines of principal stress*; they are such that the tangent at any point gives the direction of principal stress. As an example, the lines of principal stress have been drawn in Figure 10.9 for a simply-supported beam of uniform rectangular cross-section, carrying a uniformly distributed load. The stresses are a maximum where the tangents to the curves are parallel to the axis of the beam, and diminish to zero when the curves cut the faces of the beam at right angles. On the neutral axis, where the stress is one of shear, the principal stress curves cut the axis at 45°.

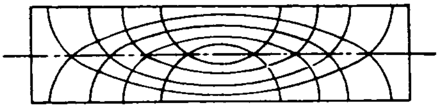


Figure 10.9 Principal stress lines in a simply-supported rectangular beam carrying a uniformly distributed load.

Problem 10.2 The flanges of an I-girder are 30 cm wide by 2.5 cm thick and the web is 60 cm deep by 1.25 cm thick. At a particular section the sagging bending moment is 500 kNm and the shearing force is 500 kN. Consider a point in the section at the top of the web and calculate for this point; (i) the longitudinal stress, (ii) the shearing stress, (iii) the principal stresses. *(Cambridge)*

<u>*Solution*</u>

First calculate the second moment of area about the neutral axis; the second moment of area of the web is

$$\frac{1}{12}(0.0125)(0.6)^3 = 0.225 \times 10^{-3} \text{ m}^4$$

The second moment of area of each flange is

$$(0.3)(0.025)(0.3125)^2 = 0.733 \times 10^{-3} \text{ m}^4$$

The total second moment of area is then

$$I_x = [0.225 + 2(0.733)]10^{-3} = 1.691 \times 10^{-3} \text{ m}^4$$

Next, for a point at the top of the web,

$$A\bar{y} = (0.3 \times 0.025)(0.3125) = 2.34 \times 10^{-3} \text{ m}^4$$

Then, for this point, with $M = 500$ kNm we have

$$\sigma = \frac{My}{I_x} = \frac{(500 \times 10^3)(0.3)}{1.691 \times 10^{-3}} = 88.6 \text{ MN/m}^2 \text{ (compressive)}$$

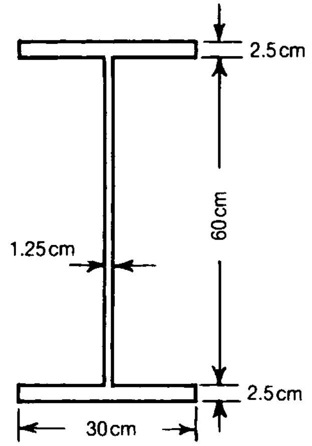
$$\tau = \frac{FA\bar{y}}{I_x t} = \frac{(500 \times 10^3)(2.34 \times 10^{-3})}{(1.691 \times 10^{-3})(0.0125)} = 55.3 \text{ MN.m}^2$$

The principal stresses are then

$$\frac{1}{2}\sigma \pm \left[\frac{1}{4}\sigma^2 + \tau^2\right]^{\frac{1}{2}} = (-44.3 \pm 70.9) \text{ MN/m}^2$$

$$= 26.6 \text{ and } -115.2 \text{ MN/m}^2$$

It should be noticed that the greater principal stress is about 30% greater than the longitudinal stress. At the top of the flange the longitudinal stress is -96 MN/m^2, so the greatest principal stress at the top of the web is 20% greater than the maximum longitudinal stress.



10.6 Superimposed beams

If we make a beam by placing one member on the top of another, Figure 10.10, there will be a tendency, under the action of lateral loads for the two members to slide over each other along the plane of contact AB, Figure 10.10. Unless this sliding action is prevented in some way, the one beam will act independently of the other; when there is no shearing connection between the beams along AB, the strength of the compound beam is the sum of the strengths of the separate beams.

However, if the sliding action is resisted, the compound beam behaves more nearly as a solid member; for elastic bending the permissible moment is proportional to the elastic section modulus.

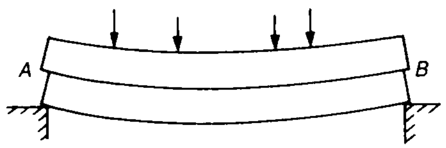


Figure 10.10 Sliding action between two beams superimposed without shearing connections.

In the case of two equal beams of rectangular cross-section, the elastic section modulus of each beam is

$$\frac{bh^2}{6}$$

where b is the breadth and h is the depth of each beam. For two such beams, placed one on the other, without shearing connection, the elastic section modulus is

$$2 \times \frac{1}{6} bh^2 = \frac{1}{3} bh^2$$

If the two beams have a rigid shearing connection, the effective depth is $2h$, and the elastic section modulus is

$$\frac{1}{6} b(2h)^2 = \frac{2}{3} bh^2$$

The elastic section modulus, and therefore the permissible bending moment, is doubled by providing a shearing connection between the two beams. In the case of steel beams, the flanges along the plane of contact AB, may be riveted, bolted, or welded together.

10.7 Shearing stresses in a channel section; shear centre

We have discussed the general case of shearing stresses in the bending of a beam having an axis of symmetry in the cross-section; we assumed that the shearing forces were applied parallel to this axis of symmetry. This is a relatively simple problem to treat because there can be no twisting of the beam when a shearing force is applied parallel to the axis of symmetry. We consider now the case when the shearing force is applied at right angles to an axis of symmetry of the cross-section. Consider for example a channel section having an axis Cx of symmetry in the cross-section, Figure 10.11; the section is of uniform wall-thickness t, b is the total breadth of each flange, and h is the distance between the flanges; C is the centroid of the cross-section. Suppose the beam is supported at one end, and that a shearing force F is applied at the free end in a direction parallel to Cy. We apply this shearing force at a point O on Cx such that no torsion of the channel occurs, Figure 10.12; if F is applied considerably to the left of C, twisting obviously will occur in a counter-clockwise direction; if F is applied considerably to the right then twisting occurs in a clockwise direction. There is some intermediate position of O for which no twisting occurs; as we shall see this position is not coincident with the centroid C.

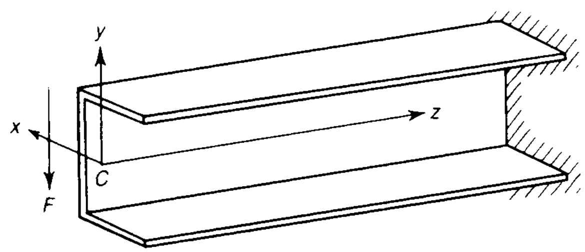


Figure 10.11 Shearing of a channel cantilever.

The problem is greatly simplified if we assume that F is applied at a point O on Cx to give no torsion of the channel; suppose O is a distance e from the centre of the web, Figure 10.12.

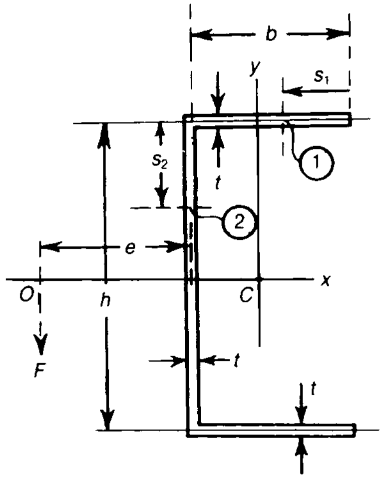


Figure 10.12 Shearing stress at any point of a channel beam.

At any section of the beam there are only bending actions present; therefore, we can again use the relation

$$q = \frac{FA\bar{y}}{I_x} \tag{10.21}$$

At a distance s_1 from the free edge of a flange

$$q_1 = \frac{Fht}{2I_x} s_1$$

At a distance s_2 along the web from the junction of web and flange

$$q_2 = \frac{Ft}{2I_x}\left[bh + s_2 \left(h - s_2\right)\right]$$

The shearing stress in flange is

$$\tau_1 = \frac{q_1}{t} = \frac{Fh}{2I_x} s_1$$

and in the web is

$$\tau_2 = \frac{q_2}{t} = \frac{F}{2I_x}\left[bh + s_2 \left(h - s_2\right)\right]$$

The shearing stress τ_1 in the flanges increases linearly from zero at the free edges to a maximum at the corners; the variation of shearing stress τ_2 in the web is parabolic in form, attaining a maximum value

$$\tau_{max} = \frac{Fbh}{2I_x}\left(1 + \frac{h}{4b}\right) \tag{10.22}$$

at the mid-depth of the web, Figure 10.13. The shearing stresses τ_1 in the flanges imply total shearing forces of amounts

$$\frac{Fht}{2I_x}\int_0^b s_1 \, ds_1 = \frac{Fb^2ht}{4I_x} \tag{10.23}$$

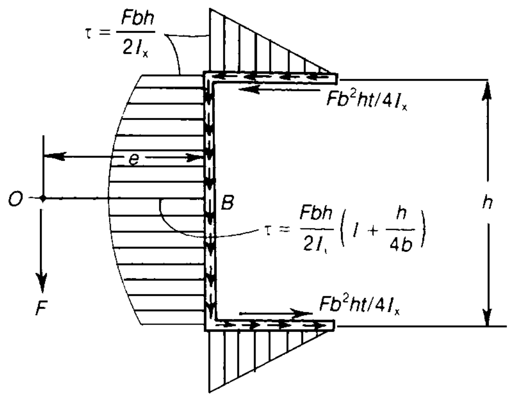


Figure 10.13 Variation of shearing stresses over the cross-section of a channel beam; e is the distance to the stress centre O.

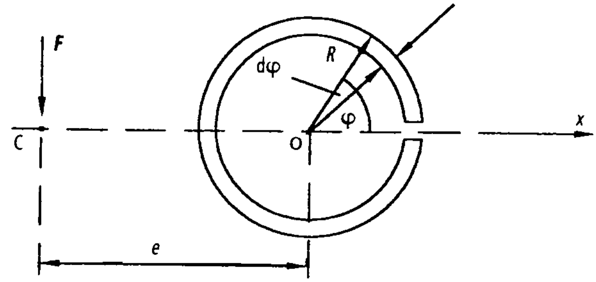
acting parallel to the centre lines of the flanges; the total shearing forces in the two flanges are in opposite directions. If the distribution of shearing stresses τ_1 and τ_2 is statically equivalent to the applied shearing force F, we have, on taking moments about B— the centre of the web— that

$$Fe = \frac{Fb^2ht}{4I_x}.h = \frac{Fb^2h^2t}{4I_x}$$

Then $$e = \frac{b^2h^2t}{4I_x} \qquad (10.24)$$

which, as we should expect, is independent of F. We note that O is remote from the centroid C of the cross-section; the point O is usually called the *shear centre;* it is the point of the cross-section through which the resultant shearing force must pass if bending is to occur without torsion of the beam.

Problem 10.3 Determine the maximum value of the shearing stress and the shear centre position 'e' for the thin-walled split tube in the figure below.



<u>Solution</u>

Consider an infinitesimally small element of the tube wall at an angle φ

$$\int y\, dA = \int_0^{\varphi} R \sin\varphi \,.\, (t \,.\, R \,.\, d\varphi)$$

$$= R^2\, t \left[-\cos\varphi\right]_0^{\varphi}$$

$$= R^2\, t \left[-\cos\varphi - (-1)\right]$$

$$= R^2\, t\, (1 - \cos\varphi)$$

At φ, the shearing stress τ_φ is given by

$$\tau_\varphi = \frac{F}{bI_x} \int y\, dA$$

or $$\tau_\varphi = \frac{F}{tI_x} R^2\, t\, (1 - \cos\varphi) \qquad (10.25)$$

or $$\tau_\varphi = \frac{FR^2}{I_x} (1 - \cos\varphi)$$

Now $$I_x = \int y^2\, dA$$

$$= \int_0^{2\pi} (R\sin\varphi)^2 (Rt d\varphi)$$

$$= R^3\, t \int_0^{2\pi} \sin^2\varphi\, d\varphi$$

but $$\sin^2\varphi = \frac{1 - \cos 2\varphi}{2}$$

$$\therefore I_x = R^3\, t \int_0^{2\pi} \frac{(1 - \cos 2\varphi)}{2}\, d\varphi$$

$$= \frac{R^3\, t}{2} \left[\varphi - \frac{\sin 2\varphi}{2}\right]_o^{2\pi}$$

$$= \frac{R^3\, t}{2} \left[(2\pi - 0) - (0 - 0)\right]$$

or $I_x = \pi R^3 t$ (10.26)

Substituting equation (10.26) into (10.25), we get

$$\tau_\varphi = \frac{FR^2}{\pi R^3 t}(1 - \cos\varphi) = \frac{F}{\pi R t}(1 - \cos\varphi) \qquad (10.27)$$

$\tau_{\varphi(max)}$ occurs when $\varphi = \pi$

$$\tau_{\varphi(max)} = \frac{F}{\pi R t}(1 + 1) = \frac{2F}{\pi R t} \qquad (10.28)$$

To determine the shear centre position, take moments about the point 'O'.

i.e.
$$Fe = \int_0^{2\pi} \tau_\varphi \; (t \; R \; d\varphi) \; R$$

$$= \frac{R^2 t \, . \, F}{\pi R t} \int_0^{2\pi} (1 - \cos\varphi) \, . \, d\varphi$$

$$= \frac{FR}{\pi} [\varphi - \sin\varphi]_0^{2\pi}$$

$$= \frac{FR}{\pi} [(2\pi - 0) - (0 - 0)]$$

$$= \frac{FR}{\pi} \; 2\pi$$

$$\therefore e = 2R$$

Further problems *(answers on page 692)*

10.4 A plate web girder consists of four plates, in each flange, of 30 cm width. The web is 60 cm deep, 2 cm thick and is connected to the flanges by 10 cm by 10 cm by 1.25 cm angles, riveted with 2 cm diameter rivets. Assuming the maximum bending moment to be 1000 kNm, and the shearing force to be 380 kN, obtain suitable dimensions for (i) the thickness of the flange plates, (ii) the pitch of the rivets. Take the tensile stress as 100 MN/m^2, and the shearing stress in the rivets as 75 MN/m^2. *(RNEC)*

10.5 In a small gantry for unloading goods from a railway waggon, it is proposed to carry the lifting tackle on a steel joist, 24 cm by 10 cm, of weight 320 N/m, supported at the ends, and of effective length 5 m. The equivalent dead load on the joist due to the load to be raised is 30 kN, and this may act at any point of the middle 4 m. By considering the fibre stress and the shear, examine whether the joist is suitable. The flanges are 10 cm by 1.2 cm, and the web is 0.75 cm thick. The allowable fibre stress is 115 MN/m^2, and the allowable shearing stress 75 MN/m^2. *(Cambridge)*

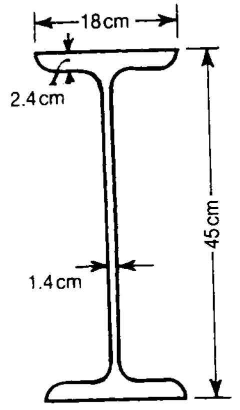
10.6 A girder of I-section has a web 60 cm by 1.25 cm and flanges 30 cm by 2.5 cm. The girder is subjected at a bending moment of 300 kNm and a shearing force of 1000 kN at a particular section. Calculate how much of the shearing force is carried by the web, and how much of the bending moment by the flanges. *(Cambridge)*

10.7 The shearing force at a given section of a built-up I-girder is 1000 kN and the depth of the web is 2 m. The web is joined to the flanges by fillet welds. Determine the thickness of the web plate and the thickness of the welds, allowing a shearing stress of 75 MN/m^2 in both the web and welds.

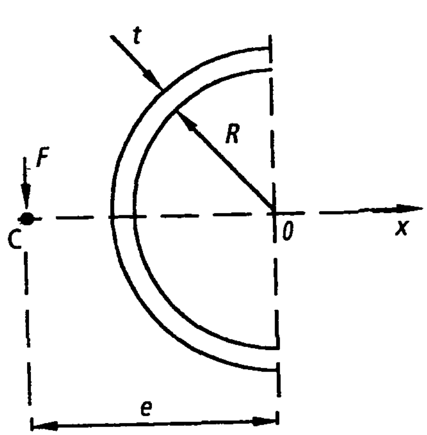
10.8 A thin metal pipe of mean radius R, thickness t and length L, has its ends closed and is full of water. If the ends are simply-supported, estimate the form of the distribution of shearing stresses over a section near one support, ignoring the intrinsic weight of the pipe.

10.9 A compound girder consists of a 45 cm by 18 cm steel joist, of weight 1000 N/m, with a steel plate 25 cm by 3 cm welded to each flange. If the ends are simply-supported and the effective span is 10 m, what is the maximum uniformly distributed load which can be supported by the girder? What weld thicknesses are required to support this load?

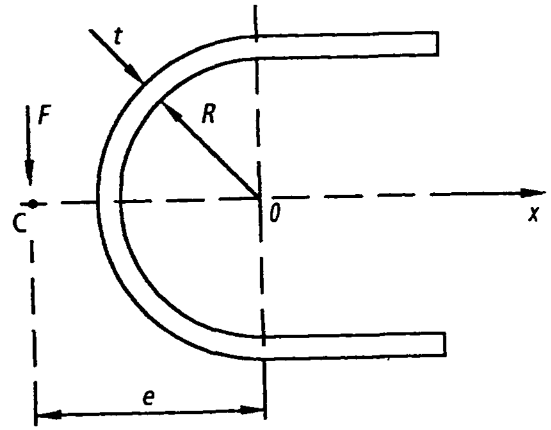
Allowable longitudinal stress in plates = 110 MN/m^2
Allowable shearing stress in welds = 60 MN/m^2
Allowable shearing stress in web of girder = 75 MN/m^2



10.10 Determine the maximum value of the shearing stresses and the positions of the shear centres for the thin-walled tubes shown in the figures below.



(a)



(b)