4 Joints and connections

4.1 Importance of connections

Many engineering structures and machines consist of components suitably connected through carefully designed joints. In metallic materials, these joints may take a number of different forms, as for example welded joints, bolted joints and riveted joints. In general, such joints are stressed in complex ways, and it is not usually possible to calculate stresses accurately because of the geometrical discontinuities in the region of a joint. For this reason, good design of connections is a mixture of stress analysis and experience of the behaviour of actual joints; this is particularly true of connections subjected to repeated loads.

Bolted joints are widely used in structural steel work and recently the performance of such joints has been greatly improved by the introduction of high-tensile, friction-grip bolts. Welded joints are widely used in steel structures, as for example, in ship construction. Riveted joints are still widely used in aircraft-skin construction in light-alloy materials. Epoxy resin glues are often used in the aeronautical field to bond metals.

4.2 Modes of failure of simple bolted and riveted joints

One of the simplest types of joint between two plates of material is a bolted or riveted lap joint, Figure 4.1.

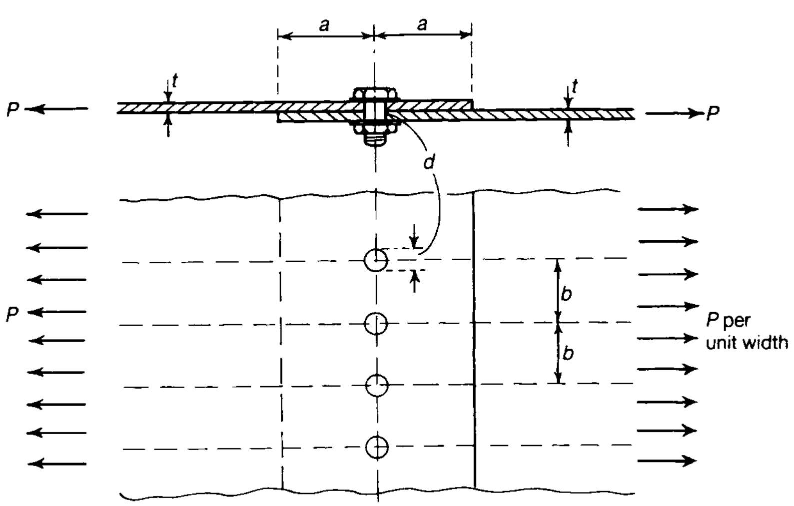


Figure 4.1 Single-bolted lap joint under tensile load.

We shall discuss the forms of failure of the joint assuming it is bolted, but the analysis can be extended in principle to the case of a riveted connection. Consider a joint between two wide plates, Figure 4.1; suppose the plates are each of thickness t, and that they are connected together with a single line of bolts, giving a total overlap of breadth $2a$. Suppose also that the bolts are each of diameter d, and that their centres are a distance b apart along the line of bolts; the line of bolts is a distance a from the edge of each plate. It is assumed that a bolt fills a hole, so that the holes in the plates are also of diameter d.

We consider all possible simple modes of failure when each plate carries a tensile load of P per unit width of plate:

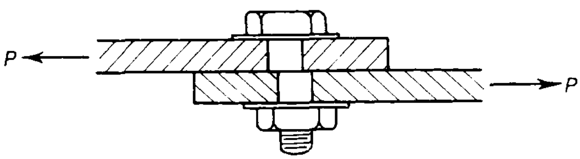


Figure 4.2 Failure by shearing of the bolts.

(1) The bolts may fail by shearing, as shown in Figure 4.2; if τ_1 is the maximum shearing stress the bolts will withstand, the total shearing force required to shear a bolt is

$$\tau_1 \times \left(\frac{\pi d^2}{4}\right)$$

Now, the load carried by a single bolt is Pb, so that a failure of this type occurs when

$$Pb = \tau_1 \left(\frac{\pi d^2}{4}\right)$$

This gives

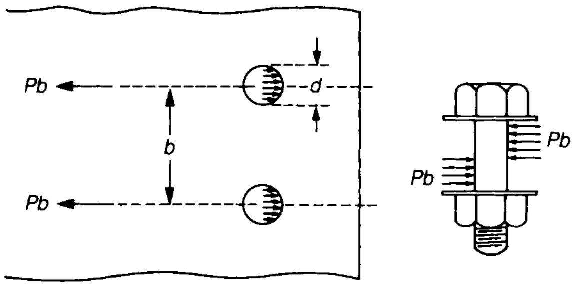
$$P = \frac{\pi d^2 \tau_1}{4b} \tag{4.1}$$

(2) The bearing pressure between the bolts and the plates may become excessive; the total bearing load taken by a bolt is Pb, Figure 4.3, so that the average bearing pressure between a bolt and its surrounding hole is

$$\frac{Pb}{td}$$

If P_b is the pressure at which either the bolt or the hole fails in bearing, a failure of this type occurs when:

$$P = \frac{P_b td}{b} \tag{4.2}$$



(i) (ii)

Figure 4.3 (i) Bearing pressure on the holes of the upper plate.
(ii) Bearing pressures on a bolt.

(3) Tensile failures may occur in the plates; clearly the most heavily stressed regions of the plates are on sections such as *ee*, Figure 4.4, through the line of bolts. The average tensile stress on the reduced area of plate through this section is

$$\frac{Pb}{(b-d)t}$$

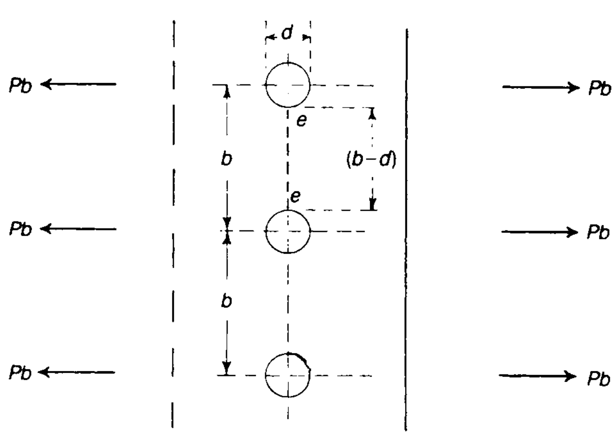


Figure 4.4 Tensile failures in the plates.

If the material of the plate has an ultimate tensile stress of σ_{ult}, then a tensile failure occurs when

$$P = \frac{\sigma_{ult}\, t(b - d)}{b} \tag{4.3}$$

(4) Shearing of the plates may occur on planes such as *cc*, Figure 4.5, with the result that the whole block of material *cccc* is sheared out of the plate. If τ_2 is the maximum shearing stress of the material of the plates, this mode of failure occurs when

$$Pb = \tau_2 \times 2at$$

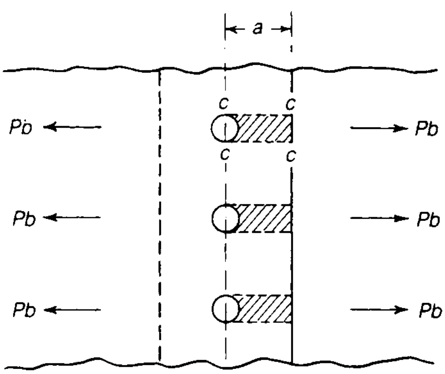


Figure 4.5 Shearing failure in the plates.

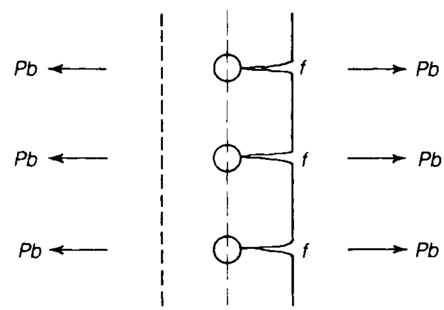


Figure 4.6 Tensile failures at the free edges of the plates.

This gives

$$P = \frac{2_{at}\, \tau_2}{b} \tag{4.4}$$

(5) The plates may fail due to the development of large tensile stresses in the regions of points such as *f*, Figure 4.6. The failing load in this condition is difficult to estimate, and we do not attempt the calculation at this stage.

In riveted joints it is found from tests on mild-steel plates and rivets that if the centre of a rivet hole is not less than 1½ times the rivet hole diameter from the edge of the plate, then failure of the plate by shearing, as discussed in (4) and (5), does not occur. Thus, if for mild-steel plates and rivets,

$$a \geq 1.5d \tag{4.5}$$

we can disregard the modes of failure discussed in (4) and (5). In the case of wrought aluminium alloys, the corresponding value of a is

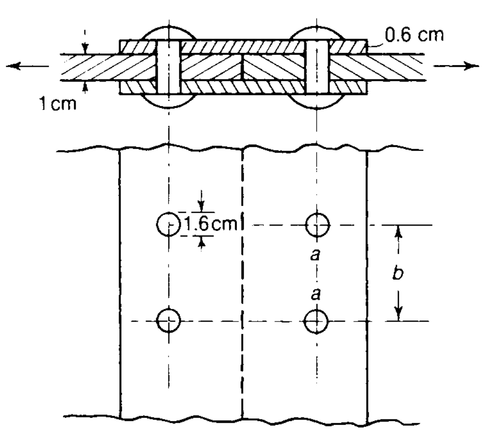
$$a \geq 2d \tag{4.6}$$

We have assumed, in discussing the modes of failure, that all load applied to the two plates of Figure 4.1 is transmitted in shear through the bolts or rivets. This is so only if there is a negligible frictional force between the two plates. If hot-driven rivets are used, appreciable frictional forces are set up on cooling; these forces play a vital part in the behaviour of the connection. With cold-driven rivets the frictional force is usually small, and may be neglected.

Problem 4.1 Two steel plates, each 1 cm thick, are connected by riveting them between cover plates each 0.6 cm thick. The rivets are 1.6 cm diameter. The tensile stress in the plates must not exceed 140 MN/m^2, and the shearing stress in the rivets must not exceed 75 MN/m^2. Find the proportions of the joint so that it shall be equally strong in shear and tension, and estimate the bearing pressure between the rivets and the plates.

<u>*Solution*</u>

Suppose b is the rivet pitch, and that P is the tensile load per metre carried by the connection. Then the tensile load on one rivet is Pb. The cover plates, taken together, are thicker than the main plates, and may be disregarded therefore, in the strength calculations. We imagine there is no restriction on the distance from the rivets to the extreme edges of the main plates and cover plates; we may disregard then any possibility of shearing or tensile failure on the free edges of the plates.



There are then two possible modes of failure:

(1) Tensile failure of the main plates may occur on sections such as aa. The area resisting tension is

$$0.010\,(b - 0.016)\ \mathrm{m}^2$$

The permissible tensile load is, therefore,

$$Pb = (140 \times 10^6)\,[0.010\,(b - 0.016)] \text{ N per rivet}$$

(2) The rivets may fail by shearing. The area of each rivet is

$$\frac{\pi}{4}(0.016)^2 = 0.201 \times 10^{-3} \text{ m}^2$$

The permissible load per rivet is then

$$Pb = 2(75 \times 10^6)\,(0.201 \times 10^{-3}) \text{ N}$$

as each rivet is in double shear.

If the joint is equally strong in tension and shear, we have, from (1) and (2),

$$(140 \times 10^6)\,[0.010\,(b - 0.016)] = 2(75 \times 10^6)\,(0.201 \times 10^{-3})$$

This gives

$$b = 0.038 \text{ m}$$

Now

$$Pb = 2(75 \times 10^6)\,(0.201 \times 10^{-3}) = 30.2 \text{ kN}$$

The average bearing pressure between the main plates and rivets is

$$\frac{30.2 \times 10^3}{(0.016)\,(0.010)} = 189 \text{ MN/m}^2$$

4.3 Efficiency of a connection

After analysing the connection of Figure 4.1, suppose we find that in the weakest mode of failure the carrying capacity of the joint is P_0. If the two plates were continuous through the connection, that is, if there were no overlap or bolts, the strength of the plates in tension would be

$$P_{ult} = \sigma_{ult} t$$

where σ_{ult} is the ultimate tensile stress of the material of the plates. The ratio

$$\eta = \frac{P_0}{P_{ult}} = \frac{P_0}{\sigma_{ult}\, t} \tag{4.7}$$

is known as the *efficiency* of the connection; clearly, η defines the extent to which the strength of the connection attains the full strength of the continuous plates. Joint efficiencies are also described in Chapter 6.

Problem 4.2 What is the efficiency of the joint of Problem 4.1?

<u>Solution</u>

The permissible tensile load per rivet is 30.2 kN. For a continuous joint the tensile load which could be carried by a 3.8 cm width of main plate is

$$(0.038)\,(0.010)\,(140 \times 10^6) \;=\; 53.2 \text{ kN}$$

Then

$$\eta \;=\; \frac{30.2}{53.2} \;=\; 0.57, \quad \text{or} \quad 57\%$$

4.4 Group-bolted and -riveted joints

When two members are connected by cover plates bolted or riveted in the manner shown in Figure 4.7, the joint is said to be *group-bolted* or *-riveted.*

The greatest efficiency of the joint shown in Figure 4.7 is obtained when the bolts or rivets are re-arranged in the form shown in Figure 4.8, where it is supposed six bolts or rivets are required each side of the join. The loss of cross-section in the main members, on the line *a*, is that due to one bolt or rivet hole. If the load is assumed to be equally distributed among the bolts or rivets, the bolt or rivet on the line *a* will take one-sixth of the total load, so that the tension in the main plates, across *b*, will be 5/6ths of the total.

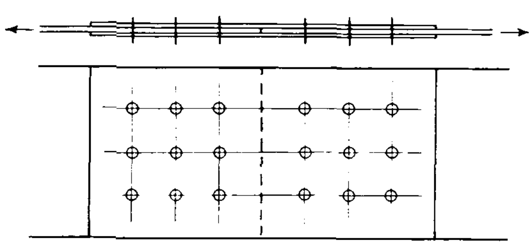


Figure 4.7 A group-bolted or -riveted joint.

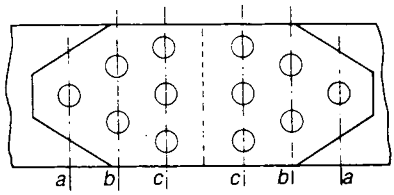


Figure 4.8 Joint with tapered cover plates.

But this section is reduced by two bolt or rivet holes, so that, relatively, it is as strong as the section a, and so on: the reduction of the nett cross-section of the main plates increases as the load carried by these plates decreases. Thus a more efficient joint is obtained than when the bolts or rivets are arranged as in Figure 4.7.

4.5 Eccentric loading of bolted and riveted connections

Structural connections are commonly required to transmit moments as well as axial forces. Figure 4.9 shows the connection between a bracket and a stanchion; the bracket is attached to the stanchion through a system of six bolts or rivets, a vertical load P is applied to the bracket. Suppose the bolts or rivets are all of the same diameter. The load P is then replaced by a parallel load P applied to the centroid C of the rivet system, together with a moment Pe about the centroid Figure 4.9(ii); e is the perpendicular distance from C onto the line of action of P.

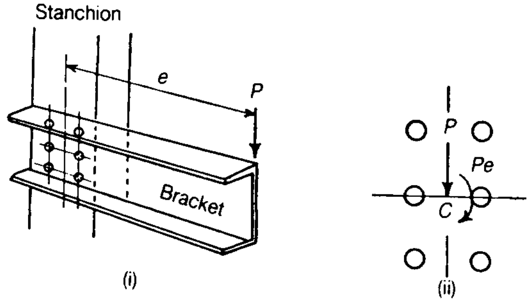


Figure 4.9 Eccentrically loaded connection leading to a bending action on the group of bolts, as well as a shearing action.

Consider separately the effects of the load P at C and the moment Pe. We assume that P is distributed equally amongst the bolts or rivets as a shearing force parallel to the line of action of P.

The moment Pe is assumed to induce a shearing force F in any bolt or rivet perpendicular to the line joining C to the bolt or rivet; moreover the force F is assumed to be proportional to the distance r from the bolt or rivet to C, (Figure 4.10).

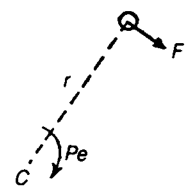


Figure 4.10 Assumed forces on the bolts.

For equilibrium we have

$$Pe = \Sigma Fr$$

If $F = kr$, where k is constant for all rivets, then

$$Pe = k \Sigma r^2$$

Thus, we have

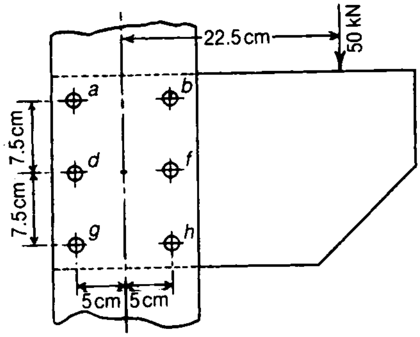
$$k = \frac{Pe}{\Sigma r^2}$$

The force on a rivet is

$$F = kr = \frac{Pe}{\Sigma r^2} r \tag{4.8}$$

The resultant force on a bolt or rivet is then the vector sum of the forces due to P and Pe.

Problem 4.3 A bracket is bolted to a vertical stanchion and carries a vertical load of 50 kN. Assuming that the total shearing stress in a bolt is proportional to the relative displacement of the bracket and the stanchion in the neighbourhood of the bolt, find the load carried by each of the bolts. *(Cambridge)*



<u>*Solution*</u>

The centroid of the bolt system is at the point *C*. For bolt *a*

$$r = aC = [(0.050)^2 + (0.075)^2]^{½} = 0.0902 \text{ m}$$

For bolt *b*,

$$r = bC = aC = 0.0902 \text{ m}$$

For bolts *d* and *f*,

$$r = 0.050 \text{ m}$$

For bolts *g* and *h*,

$$r = gC = aC = 0.0902 \text{ m}$$

Then

$$\Sigma r^2 = 4(0.0902)^2 + 2(0.050)^2 = 0.0376 \text{ m}^2$$

Now

$$e = 0.225 \text{ m} \quad \text{and} \quad P = 50 \text{ kN}$$

Then

$$Pe = (0.225)(50 \times 10^3) = 11.25 \times 10^3 \text{ Nm}$$

The loads on the bolts *a, b g, h*, due to the couple *Pe* alone, are then

$$\frac{Pe}{\Sigma r^2} r = \frac{11.25 \times 10^3}{0.0376} (0.0902) = 28.0 \text{ kN}$$

These loads are at right-angles to *Ca, Cb, Cg* and *Ch*, respectively. The corresponding loads on the bolts *d* and *f* are

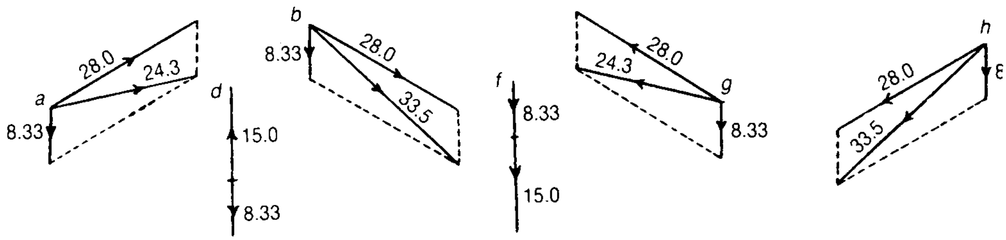
$$\frac{Pe}{\Sigma r^2} r = \frac{11.25 \times 10^3}{0.0376} (0.050) = 15.0 \text{ kN}$$

perpendicular to *Cd* and *Cf*, respectively.

The load on each bolt due to the vertical shearing force of 50 kN alone is

$$\frac{50 \times 10^3}{6} = 8.33 \times 10^3 \text{ N} = 8.33 \text{ kN}$$

This force acts vertically downwards on each bolt. The resultant loads on all the rivets are found by drawing parallelograms of forces as follows:



All force vectors are in kN. The resultant loads on the bolts are then as follows:

Bolts	Resultant Load
a and *g*	24.3 kN
b and *h*	33.5 kN
d	6.7 kN
f	23.3 kN

4.6 Welded connections

Some metals used in engineering—such as steel and aluminium—can be deposited in a molten state between two components to form a joint, which is then called a welded connection. The metal deposited to form the joint is called the weld. Two types of weld are in common use, the *butt weld* and the *fillet weld*; Figure 4.11 shows two plates connected by a butt weld; the plates are tapered at the joint to give sufficient space for the weld material. If the plates carry a tensile load the weld material carries largely tensile stresses. Figure 4.12 shows two plates connected by fillet welds; if the joint carries a tensile load the welds carry largely shearing stresses, although the state of stress in the welds is complex, and tensile stresses may also be present. Fillet welds of the type indicated in Figure 4.12 transmit force between the two plates by shearing actions within the welds; if the weld has the triangular cross-section shown in Figure 4.13(i), the shearing stress is greatest across the narrowest section of the weld, having a thickness $t/\sqrt{2}$. This section is called the *throat* of the weld. In Figure 4.13(ii), the weld has the same thickness t at all sections. To estimate approximately the strength of the welds in Figure 4.13 it is assumed that failure of the welds takes place by shearing across the throats of the welds.

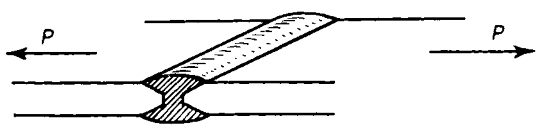


Figure 4.11 Butt weld between two plates.

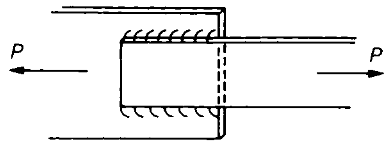


Figure 4.12 Fillet welds in a plate connection.

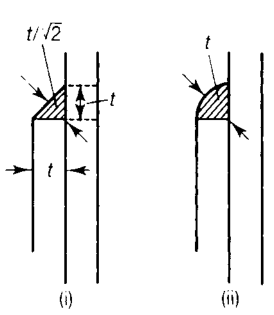
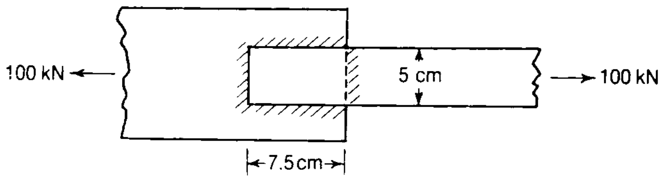


Figure 4.13 Throat of a fillet weld.

Problem 4.4 A steel strip 5 cm wide is fillet-welded to a steel plate over a length of 7.5 cm and across the ends of the strip. The connection carries a tensile load of 100 kN. Find a suitable size of the fillet weld if longitudinal welds can be stressed to 75 MN/m^2 and the transverse welds to 100 MN/m^2.



Solution

Suppose the throat thickness of the fillet-welds is t. Then the longitudinal welds carry a shearing force

$$\tau A = (75 \times 10^6)(0.075 \times 2t) = (11.25 \times 10^6)\, t \text{ N}$$

The transverse welds carry a shearing force

$$\tau A = (100 \times 10^6)(0.050 \times 2t) = (10 \times 10^6)\, t \text{ N}$$

Then

$$(11.25 \times 10^6)\, t + (10 \times 10^6)\, t = 100 \times 10^3$$

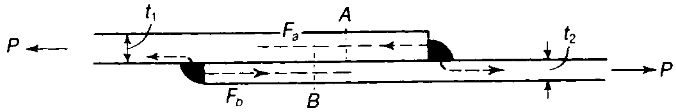
and therefore,

$$t = \frac{100}{21.25} \times 10^{-3} = 4.71 \times 10^{-3} \text{ m} = 0.471 \text{ cm}$$

The fillet size is then

$$t\sqrt{2} = 0.67 \text{ cm}$$

Problem 4.5 Two metal plates of the same material and of equal breadth are fillet welded at a lap joint. The one plate has a thickness t_1 and the other a thickness t_2. Compare the shearing forces transmitted through the welds, when the connection is under a tensile force P.



Solution

The sections of the plates between the welds will stretch by approximately the same amounts; thus, these sections will suffer the same strains and, as they are the same materials, they will also suffer the same stresses. If a shearing force F_a is transmitted by the one weld and a shearing force F_b by the other, then the tensile force over the section A in the one plate is F_a and over the section B in the other plate is F_b. If the plates have the same breadth and are to carry equal tensile stresses over the sections A and B, we have

$$\frac{F_a}{t_1} = \frac{F_b}{t_2}$$

and thus

$$\frac{F_a}{F_b} = \frac{t_1}{t_2}$$

We also have

$$F_a + F_b = P$$

and so

$$F_a = \frac{P}{1 + \frac{t_2}{t_1}} \quad \text{and} \quad F_b = \frac{P}{1 + \frac{t_1}{t_2}}$$

4.7 Welded connections under bending actions

Where a welded connection is required to transmit a bending moment we adopt a simple empirical method of analysis similar to that for bolted and riveted connections discussed in Section 4.5. We assume that the shearing stress in the weld is proportional to the distance of any part of the weld from the centroid of the weld. Consider, for example, a plate which is welded to a stanchion and which carries a bending moment M in the plane of the welds, Figure 4.14. We suppose the fillet-welds are of uniform thickness t around the parameter of a rectangle of sides a and b. At any point of the weld we take the shearing stress, τ, as acting normal to the line joining that point to the centroid C of the weld. If δA is an elemental area of weld at any point, then

$$M = \int \tau r \, dA$$

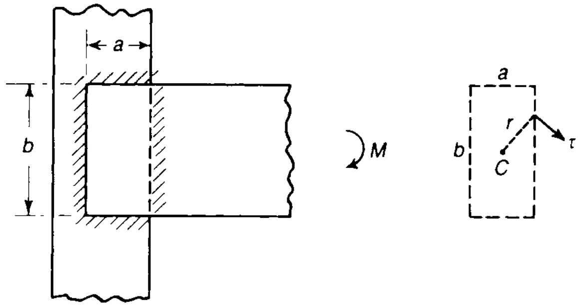


Figure 4.14 A plate fillet welded to a column, and transmitting a bending moment M.

If

$$\tau = kr$$

then

$$M = \int kr^2 \, dA = kJ$$

where J is the polar second moment of area of the weld about the axis through C and normal to the plane of the weld. Thus

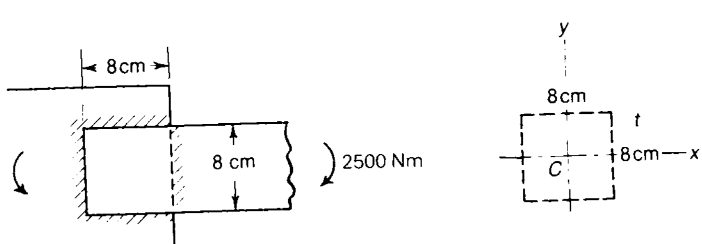
$$k = \frac{M}{J}$$

and

$$\tau = \frac{Mr}{J} \tag{4.9}$$

According to this simple empirical theory, the greatest stresses occur at points of the weld most remote from the centroid C.

Problem 4.6 Two steel plates are connected together by 0.5 cm fillet welds. Estimate the maximum shearing stress in the welds if the joint carries a bending moment of 2500 Nm.



<u>Solution</u>

The centroid of the welds is at the centre of an 8 cm square. Suppose t is the throat or thickness of the welds. The second moment of area of the weld about Cx or Cy is

$$I_x = I_y = 2\left[\frac{1}{12}(t)(0.08)^3\right] + 2\left[(t)(0.08)(0.04)^2\right]$$

$$= (0.341 \times 10^{-3})\, t \text{ m}^4$$

The polar second moment of area about an axis through C is then

$$J = I_x + I_y = 2(0.341 \times 10^{-3})\, t = (0.682 \times 10^{-3})\, t \text{ m}^4$$

Now $t = 0.005/\sqrt{2}\, m$, and so

$$J = 2.41 \times 10^{-6} \text{ m}^4$$

The shearing stress in the weld at any radius r is

$$\tau = \frac{Mr}{J}$$

This is greatest at the corners of the square where it has the value

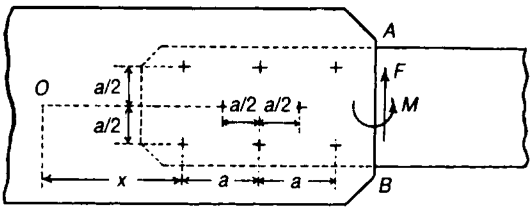
$$\tau_{max} = \frac{M}{J}\left(\frac{0.08}{\sqrt{2}}\right) = \frac{2500}{2.41 \times 10^{-6}}\left(\frac{0.08}{\sqrt{2}}\right)$$

$$= 58.6 \text{ MN/m}^2$$

Further problems *(answers on page 692)*

4.7 Two plates, each 1 cm thick are connected by riveting a single cover strap to the plates through two rows of rivets in each plate. The diameter of the rivets is 2 cm, and the distance between rivet centres along the breadth of the connection is 12.5 cm. Assuming the other unstated dimensions are adequate, calculate the strength of the joint per metre breadth, in tension, allowing 75 MN/m^2 shearing stress in the rivets and a tensile stress of 90 MN/m^2 in the plates. *(Cambridge)*

4.8 A flat steel bar is attached to a gusset plate by eight bolts. At the section AB the gusset plate exerts on the flat bar a vertical shearing force F and a counter-clockwise couple M.



Assuming that the gusset plate, relative to the flat bar, undergoes a minute rotation about a point O on the line of the two middle rivets, also that the loads on the rivets are due to and proportional to the relative movement of the plates at the rivet holes, prove that

$$x = -a \times \frac{4M + 3aF}{4M + 6aF}$$

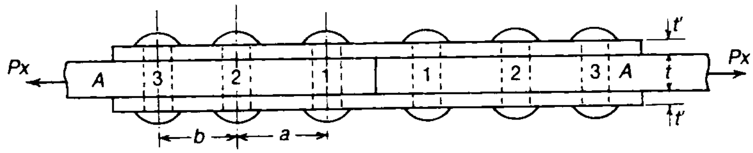
Prove also that the horizontal and vertical components of the load on the top right-hand rivet are

$$\frac{2M + 3aF}{24a} \quad \text{and} \quad \frac{4M + 9aF}{24a}$$

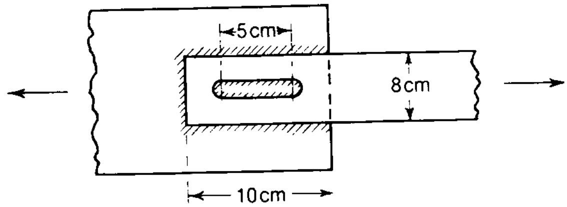
respectively.

4.9 A steel strip of cross-section 5 cm by 1.25 cm is bolted to two copper strips, each of cross-section 5 cm by 0.9375 cm, there being two bolts on the line of pull. Show that, neglecting friction and the deformation of the bolts, a pull applied to the joint will be shared by the bolts in the ratio 3 to 4. Assume that E for steel is twice E for copper.

4.10 Two flat bars are riveted together using cover plates, x being the pitch of the rivets in a direction at right angles to the plane of the figure. Assuming that the rivets themselves do not deform, show that the load taken by the rivets (1) is $tPx\,/\,(t + 2t^1)$ and that the rivets (2) are free from load.



4.11 Two tie bars are connected together by 0.5 cm fillet welds around the end of one bar, and around the inside of a slot machined in the same bar. Estimate the strength of the connection in tension if the shearing stresses in the welds are limited to 75 MN/m^2.



4.12 A bracket plate is welded to the face of a column and carries a vertical load P. Determine the value of P such that the maximum shearing stress in the 1 cm weld is 75 MN/m^2. *(Bristol)*