3 Shearing stress

3.1 Introduction

In Chapter 1 we made a study of tensile and compressive stresses, which we called direct stresses. There is another type of stress which plays a vital role in the behaviour of materials, especially metals.

Consider a thin block of material, Figure 3.1, which is glued to a table; suppose a thin plate is now glued to the upper surface of the block. If a horizontal force F is applied to the plate, the plate will tend to slide along the top of the block of material, and the block itself will tend to slide along the table. Provided the glued surfaces remain intact, the table resists the sliding of the block, and the block resists the sliding of the plate on its upper surface. If we consider the block to be divided by any imaginary horizontal plane, such as ab, the part of the block above this plane will be trying to slide over the part below the plane. The material on each side of this plane will be trying to slide over the part below the plane. The material on each side of this plane is said to be subjected to a *shearing* action; the stresses arising from these actions are called *shearing stresses*. Shearing stresses act *tangential* to the surface, unlike direct stresses which act perpendicular to the surface.

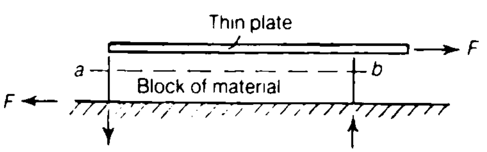


Figure 3.1 Shearing stresses caused by shearing forces.

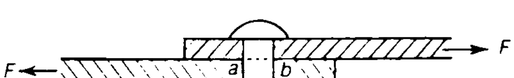


Figure 3.2 Shearing stresses in a rivet; shearing forces F is transmitted over the face ab of the rivet.

In general, a pair of garden shears cuts the stems of shrubs through shearing action and not bending action. Shearing stresses arise in many other practical problems. Figure 3.2 shows two flat plates held together by a single rivet, and carrying a tensile force F. We imagine the rivet divided into two portions by the plane ab; then the upper half of the rivet is tending to slide over the lower half, and a shearing stress is set up in the plane ab. Figure 3.3 shows a circular shaft a, with a collar c, held in bearing b, one end of the shaft being pushed with a force F; in this case there is, firstly, a tendency for the shaft to be pushed bodily through the collar, thereby inducing shearing stresses over the cylindrical surfaces d of the shaft and the collar; secondly, there is a tendency for the collar to push through the bearing, so that shearing stresses are set up on cylindrical surfaces such as e in the bearing. As a third example, consider the case of a steel bolt in the end of a bar of wood, Figure 3.4, the bolt being pulled by forces F; suppose the grain of the wood runs parallel to the length of the bar; then if the forces F are large enough the block $abcd$ will be pushed out, shearing taking place along the planes ab and cd.

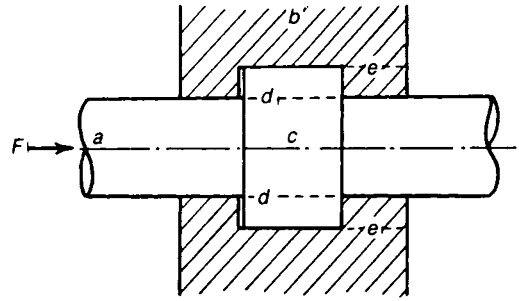


Figure 3.3 Thrust on the collar of a shaft, generating shearing stress over the planes d.

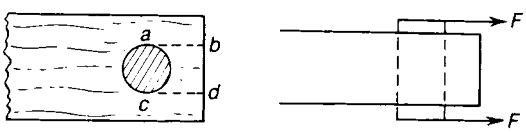


Figure 3.4 Tearing of the end of a timber member by a steel bolt, generating a shearing action on the planes ab and cd.

3.2 Measurement of shearing stress

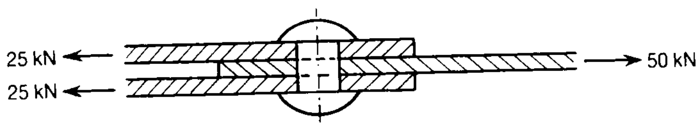
Shearing stress on any surface is defined as the intensity of shearing force tangential to the surface. If the block of material of Figure 3.1 has an area A over any section such as ab, the average shearing stress τ over the section ab is

$$\tau = \frac{F}{A} \tag{3.1}$$

In many cases the shearing force is not distributed uniformly over any section; if δF is the shearing force on any elemental area δA of a section, the shearing stress on that elemental area is

$$\tau = \lim_{\delta A \to 0} \frac{\delta F}{\delta A} = \frac{dF}{dA} \tag{3.2}$$

Problem 3.1 Three steel plates are held together by a 1.5 cm diameter rivet. If the load transmitted is 50 kN, estimate the shearing stress in the rivet.



Solution

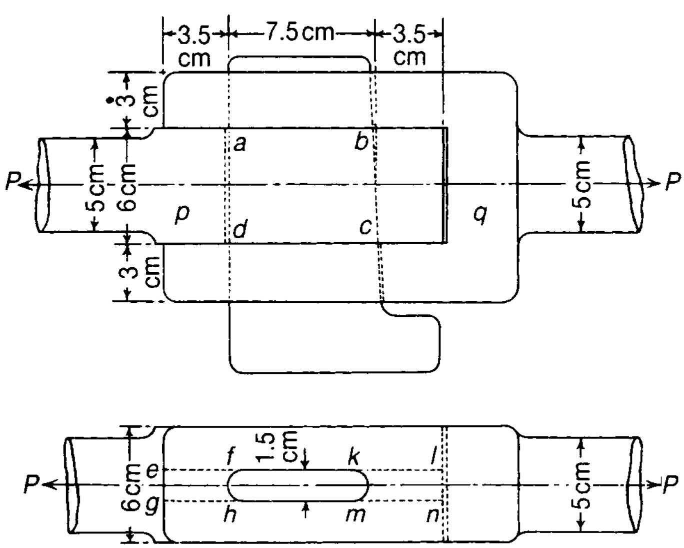
There is a tendency to shear across the planes in the rivet shown by broken lines. The area resisting shear is twice the cross-sectional area of the rivet; the cross-sectional area of the rivet is

$$A = \frac{\pi}{4}(0.015)^2 = 0.177 \times 10^{-3} \ \mathrm{m}^2$$

The average shearing stress in the rivet is then

$$\tau = \frac{F}{A} = \frac{25 \times 10^3}{0.177 \times 10^{-3}} = 141 \ \mathrm{MN/m^2}$$

Problem 3.2 Two steel rods are connected by a cotter joint. If the shearing strength of the steel used in the rods and the cotter is 150 $\mathrm{MN/m^2}$, estimate which part of the joint is more prone to shearing failure.



Solution

Shearing failure may occur in the following ways:

(i) Shearing of the cotter in the planes *ab* and *cd*.
The area resisting shear is $2(fkmh) = 2(0.075\ (0.015) = 2.25 \times 10^{-3}\ \mathrm{m}^2$

For a shearing failure on these planes, the tensile force is

$$P = \tau A = (150 \times 10^6)(2.25 \times 10^{-3}) = 338 \text{ kN}$$

(ii) By the cotter tearing through the ends of the socket q, i.e. by shearing the planes ef and gh. The total area resisting shear is

$$A = 4(0.030)(0.035) = 4.20 \times 10^{-3} \text{ m}^2$$

For a shearing failure on these planes

$$P = \tau A = (150 \times 10^6)(4.20 \times 10^{-3}) = 630 \text{ kN}$$

(iii) By the cotter tearing through the ends of the rod p, i.e. by shearing in the planes kl and mn. The total area resisting shear is

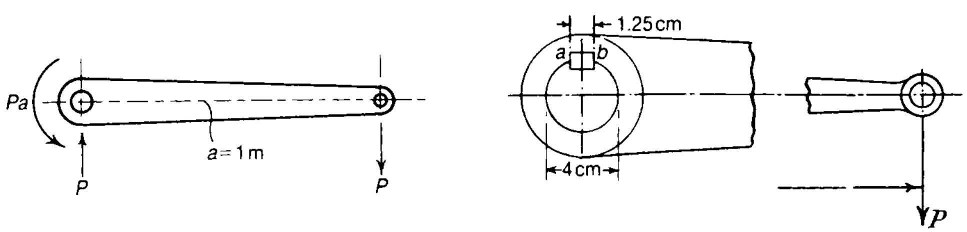
$$A = 2(0.035)(0.060) = 4.20 \times 10^{-3} \text{ m}^2$$

For a shearing failure on these planes

$$P = \tau A = (150 \times 10^6)(4.20 \times 10^{-3}) = 630 \text{ kN}$$

Thus, the connection is most vulnerable to shearing failure in the cotter itself, as discussed in (i); the tensile load for shearing failure is 338 kN.

Problem 3.3 A lever is keyed to a shaft 4 cm in diameter, the width of the key being 1.25 cm and its length 5 cm. What load P can be applied at an arm of $a = 1$ m if the average shearing stress in the key is not to exceed 60 MN/m^2?



Solution

The torque applied to the shaft is Pa. If this is resisted by a shearing force F on the plane ab of the key, then

$$Fr = Pa$$

where r is the radius of the shaft. Then

$$F = \frac{Pa}{r} = \frac{P(1)}{(0.02)} = 50P$$

The area resisting shear in the key is

$$A = 0.0125 \times 0.050 = 0.625 \times 10^{-3}\ \text{m}^2$$

The permissible shearing force on the plane *ab* of the key is then

$$F = \tau A = (60 \times 10^6)(0.625 \times 10^{-3}) = 37.5\ \text{kN}$$

The permissible value of P is then

$$P = \frac{F}{50} = 750\ \text{N}$$

3.3 Complementary shearing stress

Let us return now to the consideration of the block shown in Figure 3.1. We have seen that horizontal planes, such as *ab*, are subjected to shearing stresses. In fact the state of stress is rather more complex than we have supposed, because for rotational equilibrium of the whole block an external couple is required to balance the couple due to the shearing forces F. Suppose the material of the block is divided into a number of rectangular elements, as shown by the full lines of Figure 3.5. Under the actions of the shearing forces F, which together constitute a couple, the elements will tend to take up the positions shown by the broken lines in Figure 3.5. It will be seen that there is a tendency for the vertical faces of the elements to slide over each other. Actually the ends of the elements do not slide over each other in this way, but the tendency to so do shows that the shearing stress in horizontal planes is accompanied by shearing stresses in vertical planes perpendicular to the applied shearing forces. This is true of all cases of shearing action: a given shearing stress acting on one plane is always accompanied by a *complementary shearing stress* on planes at right angles to the plane on which the given stress acts.

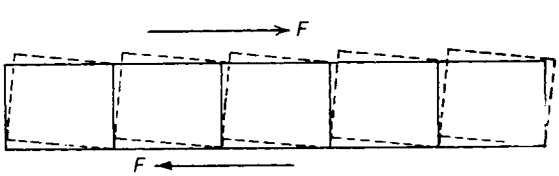


Figure 3.5 Tendency for a set of disconnected blocks to rotate when shearing forces are applied.

Consider now the equilibrium of one of the elementary blocks of Figure 3.5. Let τ_{xy} be the shearing stress on the horizontal faces of the element, and τ_{yx} the complementary shearing stress[2]

[2]Notice that the first suffix x shows the direction, the second the plane on which the stress acts; thus τ_{xy} acts in direction of x axis on planes y = constant.

on vertical faces of the element, Figure 3.6. Suppose a is the length of the element, b its height, and that it has unit thickness. The total shearing force on the upper and lower faces is then

$$\tau_{xy} \times a \times 1 = a\tau_{xy}$$

while the total shearing force on the end faces is

$$\tau_{yx} \times b \times 1 = b\tau_{yx}$$

For rotational equilibrium of the element we then have

$$(a\tau_{xy}) \times b = (b\tau_{yx}) \times a$$

and thus

$$\tau_{xy} = \tau_{yx}$$

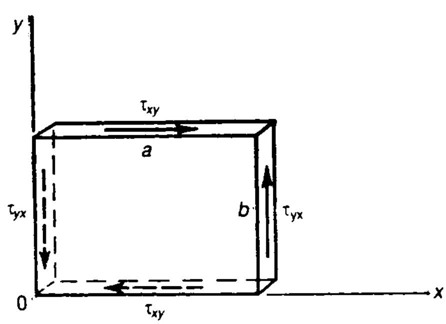


Figure 3.6 Complementary shearing stresses over the faces of a block when they are connected.

We see then that, whenever there is a shearing stress over a plane passing through a given line, there must be an *equal* complementary shearing stress on a plane perpendicular to the given plane, and passing through the given line. The directions of the two shearing stresses must be either both towards, or both away from, the line of intersection of the two planes in which they act.

It is extremely important to appreciate the existence of the complementary shearing stress, for its necessary presence has a direct effect on the maximum stress in the material, as we shall see later in Chapter 5.

3.4 Shearing strain

Shearing stresses in a material give rise to *shearing strains*. Consider a rectangular block of material, Figure 3.7, subjected to shearing stresses τ in one plane. The shearing stresses distort the rectangular face of the block into a parallelogram. If the right-angles at the corners of the face change by amounts γ, then γ is the shearing strain. The angle γ is measured in radians, and is non-dimensional therefore.

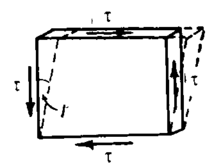


Figure 3.7 Shearing strain in a rectangular block; small values of γ lead to a negligible change of volume in shear straining.

For many materials shearing strain is linearly proportional to shearing stress within certain limits. This linear dependence is similar to the case of direct tension and compression. Within the limits of proportionality

$$\tau = G_{\gamma} , \tag{3.3}$$

where G is the *shearing modulus* or modulus of rigidity, and is similar to Young's modulus E, for direct tension and compression. For most materials E is about 2.5 times greater than G.

It should be noted that no volume changes occur as a result of shearing stresses acting alone. In Figure 3.7 the volume of the strained block is approximately equal to the volume of the original rectangular prism if the angular strain γ is small.

3.5 Strain energy due to shearing actions

In shearing the rectangular prism of Figure 3.7, the forces acting on the upper and lower faces undergo displacements. Work is done, therefore, during these displacements. If the strains are kept within the elastic limit the work done is recoverable, and is stored in the form of strain energy. Suppose all edges of the prism of Figure 3.7 are of unit length; then the prism has unit volume, and the shearing forces on the sheared faces are τ. Now suppose τ is increased by a small amount, causing a small increment of shearing strain $\delta\gamma$. The work done on the prism during this small change is $\tau\delta\gamma$, as the force τ moves through a distance $\delta\gamma$. The total work done in producing a shearing strain γ is then

$$\int_0^{y} \tau d\gamma$$

While the material remains elastic, we have from equation (3.3) that $\tau = G\gamma$, and the work done is stored as strain energy; the strain energy is therefore

$$\int_0^{\gamma} \tau \, d\gamma = \int_0^{\gamma} G\gamma \, d\gamma = \frac{1}{2} G\gamma^2 \qquad (3.4)$$

per unit volume. In terms of τ this becomes

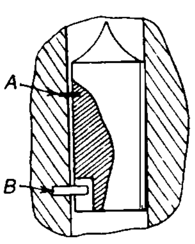
$$\frac{1}{2} G\gamma^2 = \frac{\tau^2}{2G} \quad = \text{ shear strain energy per unit volume} \qquad (3.5)$$

Further problems *(answers on page 691)*

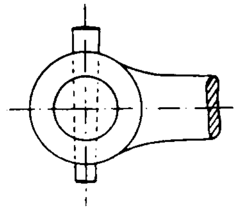
3.4 Rivet holes 2.5 cm diameter are punched in a steel plate 1 cm thick. The shearing strength of the plate is 300 MN/m^2. Find the average compressive stress in the punch at the time of punching.

3.5 The diameter of the bolt circle of a flanged coupling for a shaft 12.5 cm in diameter is 37.5 cm. There are six bolts 2.5 cm diameter. What power can be transmitted at 150 rev/min if the shearing stress in the bolts is not to exceed 60 MN/m^2?

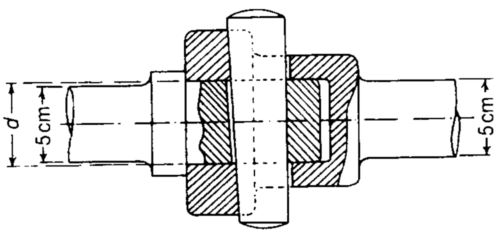
3.6 A pellet carrying the striking needle of a fuse has a mass of 0.1 kg; it is prevented from moving longitudinally relative to the body of the fuse by a copper pin A of diameter 0.05 cm. It is prevented from turning relative to the body of the fuse by a steel stud B. A fits loosely in the pellet so that no stress comes on A due to rotation. If the copper shears at 150 MN/m^2, find the retardation of the shell necessary to shear A. *(RNC)*



3.7 A lever is secured to a shaft by a taper pin through the boss of the lever. The shaft is 4 cm diameter and the mean diameter of the pin is 1 cm. What torque can be applied to the lever without causing the average shearing stress in the pin to exceed 60 MN/m^2.



3.8 A cotter joint connects two circular rods in tension. Taking the tensile strength of the rods as 350 MN/m^2, the shearing strength of the cotter 275 MN/m^2, the permissible bearing pressure between surfaces in contact 700 MN/m^2, the shearing strength of the rod ends 185 MN/m^2, calculate suitable dimensions for the joint so that it may be equally strong against the possible types of failure. Take the thickness of the cotter = *d/4*, and the taper of the cotter 1 in 48.



3.9 A horizontal arm, capable of rotation about a vertical shaft, carries a mass of 2.5 kg, bolted to it by a 1 cm bolt at a distance 50 cm from the axis of the shaft. The axis of the bolt is vertical. If the ultimate shearing strength of the bolt is 50 MN/m^2, at what speed will the bolt snap? *(RNEC)*

3.10 A copper disc 10 cm in diameter and 0.0125 cm thick, is fitted in the casing of an air compressor, so as to blow and safeguard the cast-iron case in the event of a serious compressed air leak. If pressure inside the case is suddenly built up by a burst cooling coil, calculate at what pressure the disc will blow out, assuming that failure occurs by shear round the edges of the disc, and that copper will normally fail under a shearing stress of 120 MN/m^2. *(RNEC)*